Statistical Behavior of the Nonlinear Correlation in Financial Markets

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We graft the volatility clustering in empirical financial time series into the Equiluz and Zimmermann model (EZ model), which was introduced to reproduce the herding behaviors of a financial time series. We examine the universality of the grafting methodology, which is one similar kind of sorting method that has been used to copy the nonlinear correlation structure of the real financial time series and to dress with it the model-based time series. According to the previous work, our grafting method proved that the nonlinear correlation structure of a herding model can be improved. In particular, we expand the same methodology to the high frequency financial time series and confirm that an improvement is achieved. Based on this result, we claim that a gap between real financial data and a model-based one can be narrowed.

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I. INTRODUCTION

In recent years, much attention of physicists has moved to fluctuations observed in the financial markets [1, 2], because fluctuations have much information about the interaction structure among a system's constituents. From the view of complex systems which are composed of many constituents, interactions among elements may generate long-range temporal/spatial correlations in financial markets. So far, to analyze financial markets [3–5], various models and theoretical approaches have been developed. Important examples are percolation models and minority games and their variants. Among them, a dynamic version of the static percolation model, the so-called EZ herding model, has been proposed [6]. In real terms, a financial market is composed of thinking atoms, for example, agents. And they are playing in the market via various investment portfolios and strategies in a very complicated manner. These facts account for the complexity of financial markets. Although suggested herding models well describe possible interactions among traders in a market, there is a limitation in catching a nonlinear correlation structure observed in real financial time series. For example, the magnitude of the returns in the EZ herding model does not show a long-range correlation. To achieve long-range temporal correlation, different methods have been developed until now while one dynamic interaction has been introduced [7], another dynamic interaction was introduced in a different way. But these dynamic interaction functions do not play their roles very well. To improve this limitation, the previous work [8] proposed the grafting methodology, and this is better than the dynamic interaction functions.

In this paper, we will apply the grafting method to financial indices, in particular to the EUR-USD and USD-JPY exchange rates, and more importantly, to investigate the universality of our grafting method. Two financial databases are quoted using min-by-min tick data. First, the EUR-USD exchanges cover the period of the first quarter in 2008, and the USD-JPY exchanges cover the period of the first quarter in 2008, also. From these data, we investigate the nonlinear temporal correlations and graft them into the time series generated from the naive EZ model and its modified version. The latter was suggested for sustaining the cluster structure formed among agents when they place their buying/selling orders.

The organization of this paper is as follows. In Section II, the so-called stylized facts, such as the fat-tailed distribution of returns and long-range correlation of absolute returns, will be analyzed with our databases. The magnitude of nonlinear correlations between price fluctuations, by increasing quotation time lags, is examined in terms of the scaling of the moments for the following three databases: real financial indices, the naive EZ model data, and the modified EZ model data. In Section III, the result of our findings is presented. The final section presents the concluding remarks.

II. THEORETICAL BACKGROUND

First of all, we define the return as

$$R(t,\tau) \equiv \log P(t+\tau) - \log P(t), \tag{1}$$

where τ is the lag time. We use the geometric returns for the multi-period time series. To avoid differences in the magnitude of price fluctuations, we take the normalized return $r(t,\tau)$ as

$$r(t,\tau) \equiv \frac{R(t,\tau) - \langle R(t,\tau)\rangle}{\sigma(t,\tau)}, \tag{2}$$

where $\sigma(t,\tau)$ is the standard deviation of returns $R(t,\tau)$.

Figure 1 shows the price fluctuations of the time series. Both time series exhibit a stable pattern because of the normal market situation at that time. Figure 2 shows the positive and negative complementary cumulative distribution of $r(t,\tau)$ for $\tau = 1$ min. Now, we check the nature of the distribution of $r(t,\tau)$ with an increase in the time scale τ. The central regime of returns exhibits a power-law form in Figure 2. The exponents of both time series are near the stable Levy regime $0 < \alpha < 2$ while the tails are not in the Levy regime. Since these are short-period databases, the exponents are not in Levy regime exactly. But the two databases follow power-law behavior rationally. The tail behavior shows a truncated Levy distribution. Therefore, it is worthwhile to find the convergence of the Gaussian distribution by increasing the time scale τ. In Figure 3, from 1 min up to 100

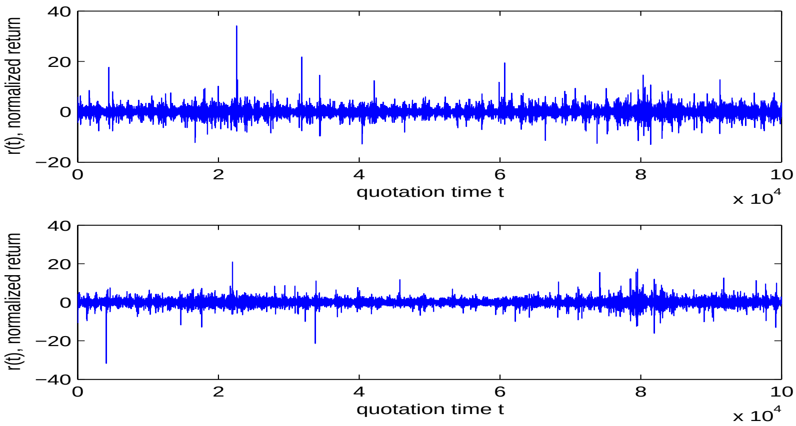

FIG. 1: Normalized returns are presented for the (a) EUR-USD and (b) USD-JPY exchanges.

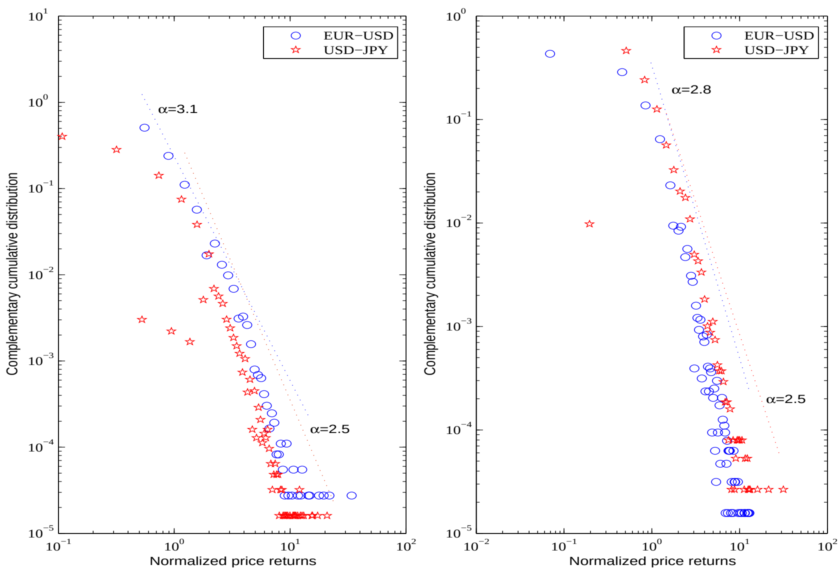

FIG. 2: The complementary cumulative distribution functions $P(r \geq x)$ for: (a) the positive tails, and (b) the negative tails.

min, the return of the EUR-USD exchange rate sustains its functional form of distribution of returns. This is a result of the fact that the central region is in the Levy stable distribution. The USD-JPY exchange data shows the same pattern, and this is exhibited in Figure 4. The limited size of the databases restricts the investigation into further long time scales.

We investigate the temporal correlation between successive returns in this section. This

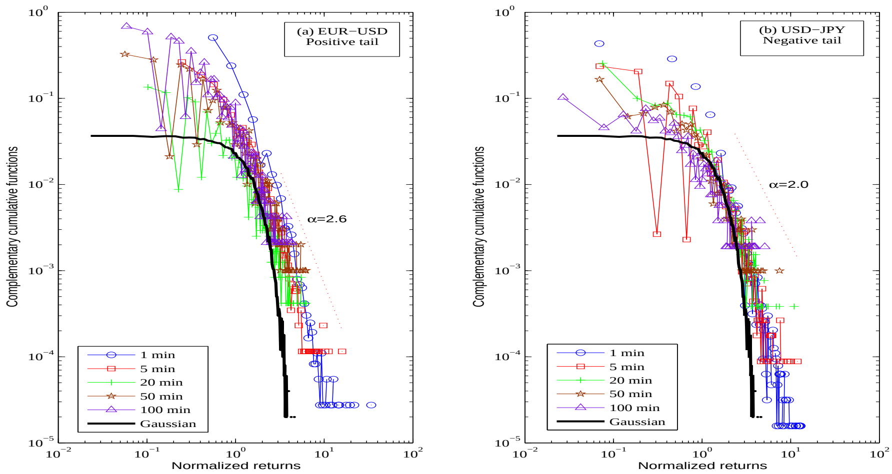

FIG. 3: The complementary cumulative distribution functions $P(r \geq x)$ of EUR-USD exchange is presented for: (a) the positive tails, and (b) the negative tails.

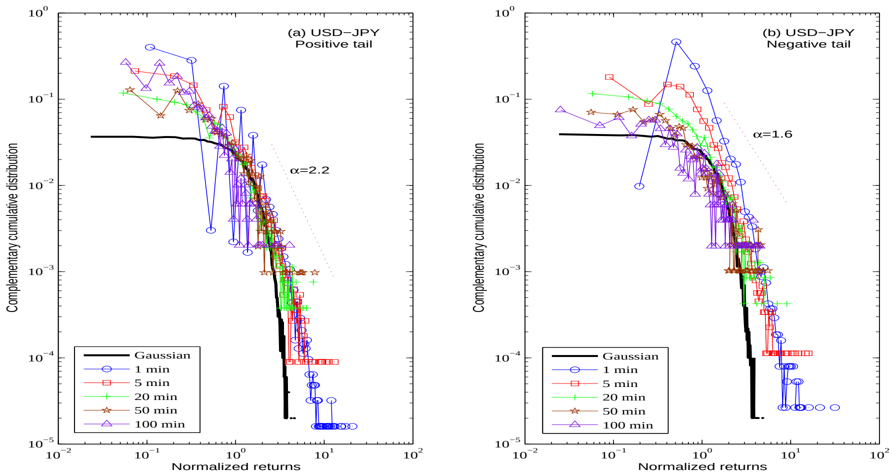

FIG. 4: The complementary cumulative distribution functions $P(r \geq x)$ of returns of USD-JPY exchange is presented for: (a) the positive tails, and (b) the negative tails.

underpins the clustering volatility. To this end, we define the autocorrelation function as

$$C(\tau) \equiv \frac{\langle |r(t+\tau)||r(t)|\rangle - \langle |r(t+\tau)|\rangle\langle |r(t)|\rangle}{\sigma(t+\tau)\sigma(t)}, \tag{3}$$

where τ denotes the lag time. Figure 5 shows the long-range feature of absolute returns for our two databases, the EUR-USD and USD-JPY exchange rates. The exponents are,

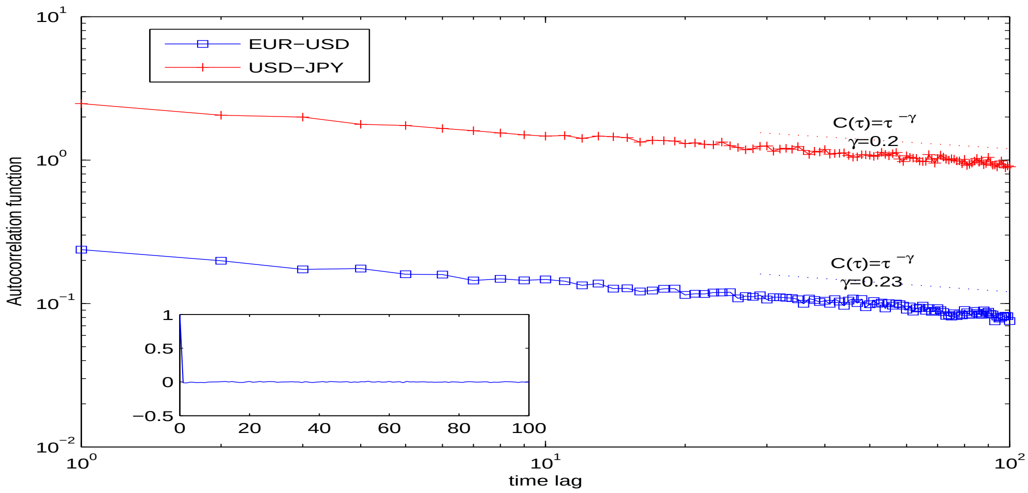

FIG. 5: Autocorrelation functions of absolute returns of two databases, the EUR-USD and USD-JPY exchanges are shown.

respectively, given as 0.23 and 0.20 for the two databases. As shown in the inset of Figure 5, the shuffled returns break down all inherent correlations between successive returns.

Let us compute moments over the different time lags. We use the quotation time t not to be overlapped to avoid a short-range correlation. First, we define the moments of the normalized returns distribution as

$$\mu_k \equiv \langle |r|^k \rangle. \tag{4}$$

We investigate the scaling behavior shown in Figure 4 concerning scaling of the moments. As shown in Figure 6, we examine the convergence of the distribution to the Gaussian distribution in terms of scaling of the moments, as the time scale τ increases. The EUR-USD and USD-JPY exchange rates have explicit similarity. This may be due to the stable market situation at that time. This verification shows a clear behavior of slow convergence to the Gaussian distribution as the time scale τ increases.

We can show the property of real time series by a graph, and this property appears clearly in the numerical values. The numerical values can be found in Table I. According to the table, real time series have long-range correlations based on statistical findings. In addition, we have checked that the alternative hypothesis of the Jarque-Bera test about returns is satisfied at the significant level 0.05 over all lag times concerned. The Jarque-Bera statistic is defined as

$$JB \equiv \frac{6}{n}\left(S^2 + \frac{(K-3)^2}{4}\right), \tag{5}$$

where S denotes the sample skewness and K denotes the sample kurtosis. In statistics, the

EUR-USD exchange rate					
τ	Mean	Standard Deviation	Kurtosis	Skewness	Jarque-Bera
1min	7.09×10^{-7}	1.61×10^{-4}	31.94	0.64	3.50×10^{6}
5min	3.54×10^{-6}	3.52×10^{-4}	16.31	0.29	1.48×10^{5}
20min	1.42×10^{-5}	6.86×10^{-4}	11.02	-0.21	1.34×10^{4}
50min	3.54×10^{-5}	1.10×10^{-3}	8.89	0.08	2.89×10^{3}
100min	7.09×10^{-5}	1.50×10^{-3}	6.95	-0.14	6.54×10^{2}
USD-JPY exchange rate					
τ	Mean	Standard Deviation	Kurtosis	Skewness	Jarque-Bera
1min	-8.50×10^{-7}	2.64×10^{-4}	30.41	-0.33	3.13×10^{6}
5min	-4.25×10^{-6}	5.52×10^{-4}	20.16	-0.26	2.46×10^{5}
20min	-1.70×10^{-5}	1.00×10^{-3}	9.99	-0.13	1.02×10^{4}
50min	-4.25×10^{-5}	1.70×10^{-3}	9.29	-0.02	3.29×10^{3}
100min	-8.50×10^{-5}	$2.3q0 \times 10^{-3}$	6.02	-0.14	3.83×10^{2}

TABLE I: Statistics of the EUR-USD and USD-JPY exchange rates.

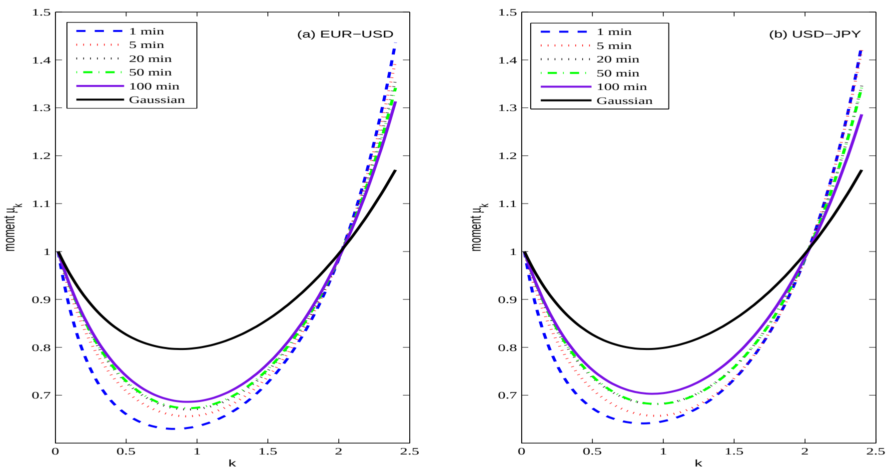

FIG. 6: The scaling of moments is presented for: the (a) EUR-USD, and (b) the USD-JPY exchanges.

Jarque-Bera test is a goodness of the fit measure of departure from the normality based on the sample kurtosis and skewness.

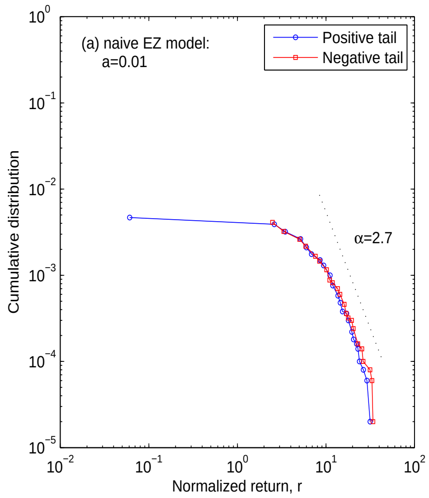

FIG. 7: The complementary cumulative distribution functions of returns generated by the EZ herding model: (a) the naive EZ model, and (b) the modified EZ model.

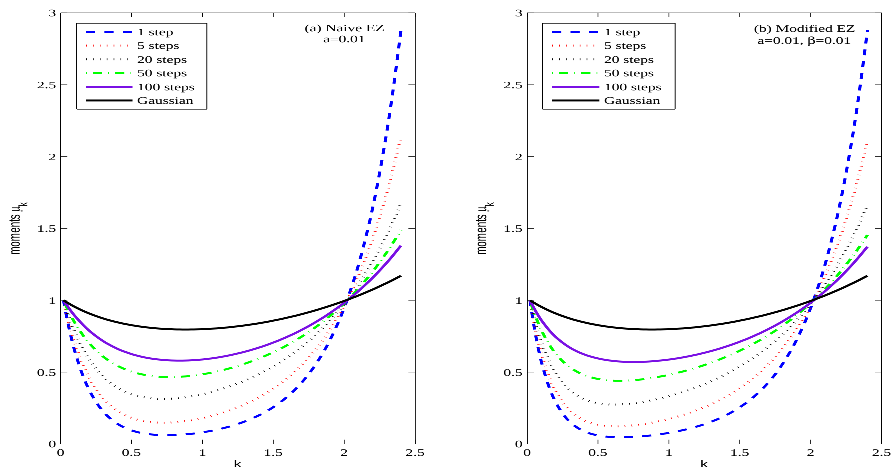

FIG. 8: The scaling of moments is presented for: (a) the naive EZ model, and (b) the modified EZ model.

III. NUMERICAL RESULTS

Fundamental modeling procedure follows the Equiluz and Zimmermann model logic [6]. In detail, we consider a system composed of N agents, represented by nodes in a network where $N = 10000$. The state of agent l is represented by $\phi_l = 0, +1, -1$

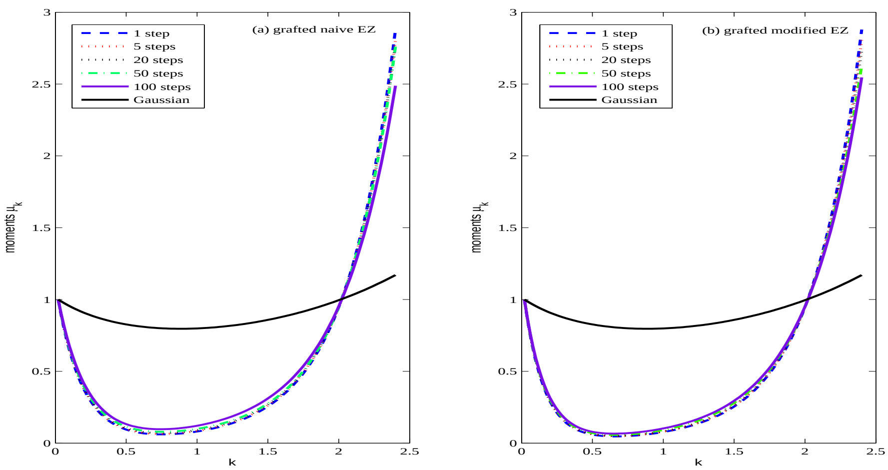

FIG. 9: The scaling of moments is presented for: (a) the naive EZ model, and (b) the modified EZ model, which are both grafted with nonlinear correlations of real returns.

corresponding to an inactive state $\phi_l = 0$ and two active states $\phi_l = +1$, $\phi_l = -1$. $\phi_l = 0$ represents waiting, $\phi_l = +1$ represents buying, $\phi_l = -1$ represents selling. Agents can be isolated or connected through links forming a cluster. The nodes(agents) in the same cluster share the same information. Initially, all N agents are isolated, and each agent is regarded as a cluster with size one. The network of links evolves dynamically in following way:

(1) At a time step t, an agent j with its cluster is selected at random.

(2) With probability a, the state of j becomes active by randomly choosing the state 1 or -1, and instantly all agents belonging to the same cluster follow this same action by imitation. The aggregate state of the system $s_i = s(t_i) = \Sigma_{l=1,N}\phi_l$ and the total size of cluster are computed. After that the cluster is broken up into isolated agents, removing all links inside the cluster, and resetting their state.

(3) With probability (1-a), the state of j remains inactive ($\phi_j = 0$) and, instead, a new link between agent j and any other agents from the whole network is established.

This process is repeated from step (1). Step (3) describes how an agent disperses information to other agents in terms of a linking mechanism, by which a cluster grows. All agents in the same cluster share the same information. Herding behavior then appears in the market. The price P evolves as $P(t_i + 1) = P(t_i)\exp(s_i/\lambda)$, where λ denotes the market liquidity. And the return is defined as $R(t_i) = \ln[P(t_i)] - \ln[P(t_{i-1})]$. The modified version is governed by $\beta = 0.01$. β denotes the fraction of breaking-up of an activated cluster. This condition leads to a less fat-tailed distribution by increasing the probability of selecting smaller clusters instead of forming a larger cluster. The parameter a controls the rate activity/dispersion in the EZ herding model, while the parameter β controls the rate of breaking-up of an activated cluster in the modified EZ herding model [8]. Figure 7 shows the complementary cumulative distribution of those generated returns. As shown on the two graphs, the power-exponent is changed by about 1. Figure 8 shows the tendency of convergence of the distribution of returns to a Gaussian distribution as the time scale τ increases because of the independence of consecutive transactions. Therefore, there is no clear existence of volatility clustering. To see the nonlinear correlations, it is necessary to take a smaller value of β than above. We graft the pattern of long-range correlations of real financial dynamics into the generated return by simulation. Because the structure of nonlinear correlations is defined as the order of returns, grafting methodology is not a simple shuffling of the data but gives real market structure to the generated data by simulation. Therefore, this follows the next procedure. In order to attach the original structure, the arrangement of generated returns is reordered according to the order of real returns. As shown in Figure 9, the grafted time series have the original nonlinear correlation of the returns, the data generated by the grafting method is more strongly related with reality than the data generated by a simulation. Figure 9 shows that the distribution functions of the EZ model based on grafted returns have slower convergence to Gaussian behavior than the non-grafted data in Figure 8. Because the grafted time series have the correlation pattern of real structure, there is a clear tendency of convergence to the Gaussian distribution in terms of the scaling of moments. But Figure 9 shows a clearer tendency of slow convergence than the previous work [8]. Because we use more high frequency data in this work rather than that work, this property appears analytically. This may be due to the robust market structure in the shorter time resolution.

IV. CONCLUSION

In conclusion, we have proved that the empirical property of short-period real returns is different from those noted in other works [7, 9–11]. We found that the real time series follow the power-law. As the lag time τ increases, the power-law scaling is willing to keep on own property. However, the generated returns by simulation have a collapse of the power-law scaling into a single curve as the lag time τ increases. To improve the convergence of the generated data to Gaussian behavior, we proposed two methods: which are the grafting methodology and the modified EZ model. The former improved this problem. But the latter only made the result have a less fat-tailed distribution. By grafting the nonlinear correlation structure of short-period real time series to the simulated returns, we got a slow convergence to a Gaussian behavior in terms of scaling moments. Through this work, we found that the grafting methodology is useful for application to a lot of financial time series. This is a section of universality property. Therefore, this algorithm of long-range correlations can be useful for generating an ensemble of databases which can be used to put a price to various derivatives in terms of Monte-Carlo simulation. Also, based on our surrogating methodology, we can attempt to classify financial securities in terms of the nonlinear correlation structure.

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