Preprint typeset in JHEP style - HYPER VERSION

KYUSHU-HET 125, TU-882

Mixed Mediation of Supersymmetry Breaking with Anomalous $U(1)$ Gauge Symmetry

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Abstract: Models with anomalous $U(1)$ gauge symmetry contain various superfields which can have nonzero supersymmetry breaking auxiliary components providing the origin of soft terms in the visible sector, e.g. the $U(1)$ vector superfield, the modulus or dilaton superfield implementing the Green-Schwarz anomaly cancellation mechanism, $U(1)$-charged but standard model singlet matter superfield required to cancel the Fayet-Iliopoulos term, and finally the supergravity multiplet. We examine the relative strength between these supersymmetry breaking components in a simple class of models, and find that various different mixed mediations of supersymmetry breaking, involving the modulus, gauge, anomaly and D-term mediations, can be realized depending upon the characteristics of D-flat directions and how those D-flat directions are stabilized with a vanishing cosmological constant. We identify two parameters which represent such properties and thus characterize how the various mediations are mixed. We also discuss the moduli stabilization and soft terms in a variant of KKLT scenario, in which the visible sector Kähler modulus is stabilized by the D-term potential of anomalous $U(1)$ gauge symmetry.

Keywords: Anomalous $U(1)$ symmetry, Moduli stabilization, Supersymmetry Breaking.

Contents

1. Introduction

Anomalous $U(1)$ gauge symmetry, which will be referred to as $U(1)_A$ in the following, appears often in 4-dimensional (4D) effective theory of string compactification. It accompanies a modulus T which transforms nonlinearly under $U(1)_A$, and the holomorphic gauge kinetic function of the model depends on T as $f_a \ni k_a T$, where k_a is a real constant. Then the variation of f_a under $U(1)_A$ cancels the anomaly due to the loops of light fermions, realizing the Green-Schwarz (GS) anomaly cancellation mechanism [1]. The $U(1)_A$ gauge boson receives a mass through the Stückelberg mechanism associated with the nonlinear transformation of T, which is typically not far below the string or Planck scale. The nonlinear transformation of T also induces a moduli-dependent Fayet-Iliopoulos (FI) term [2]. It has been noticed that anomalous $U(1)$ gauge symmetry can have a variety of interesting phenomenological implications. It can be used to forbid dangerous interactions such as the ones which lead to a too rapid proton decay, or to explain the smallness of some couplings in the low energy theory. In some cases, it can be identified as a flavor symmetry that explains the observed hierarchical fermion masses [3, 4, 5, 6].

Due to the existence of the GS modulus T and the associated FI term, anomalous $U(1)$ gauge symmetry can play an important role in supersymmetry (SUSY) breaking and its transmission to the supersymmetric standard model [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. In most cases, the FI term has a vacuum expectation value (VEV) far above the weak scale, even close to the Planck scale in some cases. Then, to avoid a too large D-term SUSY breaking, the FI term should be cancelled by other contribution to the $U(1)_A$ D-term due to $U(1)_A$ charged but standard model (SM) singlet matter field X. This SM singlet X can play another important role. In many cases, string models with $U(1)_A$ contain exotic matter fields Φ, Φ^c which are vector-like under the SM gauge group, and these exotic matter fields get a mass far above the weak scale through the Yukawa coupling to X. Then they can be identified as the messenger for gauge-mediated SUSY breaking if the F-component of X develops a nonzero VEV.

Therefore, models with anomalous $U(1)$ gauge symmetry contain various sources of SUSY breaking, including *(i)* the $U(1)_A$ D-term, *(ii)* the F-component of the GS modulus T, *(iii)* the F-component of the chiral matter superfield X whose lowest component cancels the FI term and provides a large mass to exotic matter fields, and finally *(iv)* the auxiliary component of the supergravity (SUGRA) multiplet which is generically of the order of the gravitino mass $m_{3/2}$. Then the visible sector soft terms receive modulus-mediated contribution of the order of F^T and gauge-mediated contribution of the order of $\frac{g^2}{8\pi^2}\frac{F^X}{X}$, as well as the anomaly-mediated contribution of the order of $\frac{g^2}{8\pi^2}m_{3/2}$. Furthermore, there can be D-term contribution to scalar masses for $U(1)_A$ charged matter fields. This means that the four well-known mediation schemes of SUSY breaking, i.e. moduli mediation [17], gauge mediation [18, 19], anomaly mediation [20] and D-term mediation, generically appear together in models with $U(1)_A$.

In this paper we wish to examine the possible pattern of the mediation of SUSY breaking in models with anomalous $U(1)$ gauge symmetry. As we will see, the relative strength between different mediations crucially depends on the characteristics of the D-flat directions, and also on how the D-flat directions are stabilized. Depending upon the detailed form of the Kähler potential and superpotential, various different mixed mediations involving some or all of the moduli, gauge, anomaly and D-term mediations can be realized within a relatively simple class of models.

This paper is organized as follows. In section 2, we discuss generic features of supersymmetry breaking and the resulting pattern of soft terms in models with anomalous $U(1)$ gauge symmetry. In section 3, we consider a set of specific models to examine the stabilization of D-flat directions under the constraint of nearly vanishing cosmological constant, and evaluate the relative strength between different mediations in each model. Section 4 is the conclusion.

2. SUSY breaking in models with anomalous $U(1)$

In this section, we discuss generic aspects of supersymmetry breaking in models with anomalous $U(1)$ symmetry. We first examine the relations between different SUSY breaking auxiliary components in models with $U(1)_A$, and then discuss the resulting pattern of soft terms.

2.1 SUSY breaking auxiliary components

In the presence of anomalous $U(1)$ gauge symmetry, the quantum consistency of the theory is ensured by the GS anomaly cancellation mechanism [1]. This mechanism is implemented by a non-linear variation of the GS modulus

$$T \to T - \frac{\delta_{GS}}{2}\Lambda_A \tag{2.1}$$

under the gauge transformation

$$V_A \to V_A - \frac{1}{2}(\Lambda_A + \Lambda_A^*), \tag{2.2}$$

where V_A is the vector superfield containing the $U(1)_A$ gauge field. For the anomaly cancellation to work, the holomorphic gauge kinetic function of the model should contain a T-dependent piece,

$$f_a = k_a T + \cdots , \tag{2.3}$$

where k_a is a real constant and the ellipsis stands for the T-independent part. In the normalization convention of T for which $k_a = \mathcal{O}(1)$, the anomaly cancellation implies

$$\frac{\delta_{GS}}{2} = \mathcal{O}\left(\frac{1}{8\pi^2}\right). \tag{2.4}$$

Since the $U(1)_A$ invariance forces the modulus Kähler potential K_0 to be a function of the gauge-invariant combination $t_A = T + T^* - \delta_{GS}V_A$, the GS mechanism dynamically induces a modulus-dependent FI term[1]

$$\xi_{FI} = \frac{\delta_{GS}}{2}\partial_T K_0, \tag{2.5}$$

while rendering the vector superfield V_A massive through the Stückelberg mechanism:

$$\Delta M_V^2 = \frac{g_A^2\delta_{GS}^2}{2}\partial_T\partial_{\bar{T}}K_0, \tag{2.6}$$

where g_A is the $U(1)_A$ gauge coupling.

To proceed, let us consider 4D effective SUGRA model with chiral superfields $\Phi_I = \{T_M, \phi_i\}$, where $T_M = \{T, T_\alpha\}$ stand for generic moduli including the GS modulus T and ϕ_i are chiral matter superfields with $U(1)_A$ charge q_i. Under $U(1)_A$, these chiral superfields transform as

$$\delta\Phi_I = \eta^I\Lambda_A, \tag{2.7}$$

where the holomorphic Killing vectors η^I are given by

$$\eta^T = -\frac{1}{2}\delta_{GS}, \quad \eta^{T_\alpha} = 0, \quad \eta^{\phi_i} = q_i\phi_i.$$

Since the moduli stabilization is relatively insensitive to the form of the matter Kähler metric, we assume for simplicity that the Kähler metric of ϕ_i is independent of moduli.

[1] Unless specified, we will use the convention $M_{Pl} = 1$ throughout this paper, where $M_{Pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck scale.

(In fact, most of our results apply well at least qualitatively to the case when the matter Kähler metrics are moduli-dependent.) Then the Kähler potential of the model takes the form:

$$K = K_0(t - \delta_{GS}V_A, T_\alpha, T_\alpha^*) + \sum_i \phi_i^* e^{2q_i V_A}\phi_i, \tag{2.8}$$

where $t = T + T^*$ and we have ignored the terms of higher order in ϕ_i which are presumed to be suppressed by $1/M_{Pl}$. The associated $U(1)_A$ gauge boson mass and D-term are given by

$$\begin{aligned} M_V^2 &= 2g_A^2 \eta^I \eta^J \partial_I \partial_{\bar{J}} K = 2g_A^2 \left(M_{GS}^2 + \sum_i q_i^2 |\phi_i|^2 \right), \\ D_A &= -\eta^I \partial_I K = \xi_{FI} - \sum_i q_i |\phi_i|^2, \end{aligned} \tag{2.9}$$

where

$$\xi_{FI} = \frac{\delta_{GS}}{2}\partial_T K_0, \quad M_{GS}^2 = \frac{\delta_{GS}^2}{4}\partial_T \partial_{\bar{T}} K_0.$$

From this, one easily finds

$$m_{3/2} D_A = \eta^I F^{\bar{J}} \partial_I \partial_{\bar{J}} K = \sum_i q_i |\phi_i|^2 \left(\frac{F^i}{\phi_i} \right)^* - \frac{1}{2}\delta_{GS} \sum_M F^{\bar{M}} \partial_T \partial_{\bar{M}} K_0, \tag{2.10}$$

where $m_{3/2} = e^{K/2}W$ for the superpotential W, and the auxiliary F-component of $\Phi_I = \{T_M, \phi_i\}$ is defined as

$$F^I = -e^{K/2} K^{I\bar{J}} (\partial_J W + W \partial_J K)^*.$$

Combining (2.10) with the stationary condition $\partial_I (V_F + V_D) = 0$, one can derive [15, 21]

$$\left(V_F + 2|m_{3/2}|^2 + \frac{1}{2}M_V^2 \right) D_A = -F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L \partial_L K) + V_D \eta^I \partial_I g_A^2, \tag{2.11}$$

where the F and D term scalar potentials are given by

$$V_F = K_{I\bar{J}} F^I F^{\bar{J}} - 3e^K |W|^2, \quad V_D = \frac{g_A^2}{2} D_A^2. \tag{2.12}$$

This relation can be generalized to the case including an uplifting potential $V_{\rm lift}$ which might be necessary to achieve a vacuum with nearly vanishing cosmological constant. As long as $V_{\rm lift} \ll M_V^2$ (in the unit with $M_{Pl} = 1$), which is always the case in models with low energy SUSY, the effect of $V_{\rm lift}$ in the generalized version of (2.11) can be safely ignored, and we can apply (2.11) to the case with $V_{\rm lift}$ as well. We then find

$$\begin{aligned} g_A^2 D_A &= -\frac{F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} (\eta^L \partial_L K)}{\eta^I \eta^{\bar{J}} \partial_I \partial_{\bar{J}} K} \left(1 + \mathcal{O}\left(\frac{m_{3/2}^2}{M_V^2} \right) \right) \\ &= \frac{\frac{1}{2}\delta_{GS} \sum_{MN} F^M F^{\bar{N}} \partial_T \partial_M \partial_{\bar{N}} K_0 - \sum_i q_i |F^i|^2}{\frac{1}{4}\delta_{GS}^2 \partial_T \partial_{\bar{T}} K_0 + \sum_i q_i^2 |\phi_i|^2} \left(1 + \mathcal{O}\left(\frac{m_{3/2}^2}{M_V^2} \right) \right). \end{aligned} \tag{2.13}$$

In models with low energy SUSY, $m_{2/3}$ and $\sqrt{D_A}$ have a VEV in TeV or multi-TeV range (or lower than TeV). On the other hand, although it depends on the stabilization of D-flat directions, typically there exist some $U(1)_A$ charged (but SM singlet) matter fields having a VEV far above TeV. Also, with the Stückelberg contribution (2.6), the $U(1)_A$ gauge boson mass M_V is rather close to the Planck scale or the string scale, thus is much higher than TeV. Then the relations (2.9), (2.10) and (2.13) give rise to

$$\begin{aligned}
&\frac{1}{2}\delta_{GS}\partial_T K_0 \simeq \sum_i q_i|\phi_i|^2, \\
&\frac{1}{2}\delta_{GS}\sum_M F^M\partial_M\partial_{\bar{T}}K_0 \simeq \sum_i q_i|\phi_i|^2\left(\frac{F^i}{\phi_i}\right), \\
&g_A^2 D_A \simeq \frac{\frac{1}{2}\delta_{GS}\sum_{MN}F^M F^{\bar{N}}\partial_T\partial_M\partial_{\bar{N}}K_0 - \sum_i q_i|\phi_i|^2\left|\frac{F^i}{\phi_i}\right|^2}{\frac{1}{4}\delta_{GS}^2\partial_T\partial_{\bar{T}}K_0 + \sum_i q_i^2|\phi_i|^2},
\end{aligned} \tag{2.14}$$

which in fact correspond to the lowest component, the F-component, and the D-component of the equation of motion for the $U(1)_A$ vector superfield V_A in the limit that V_A receives a large supersymmetric mass.

Eq. (2.14) includes relations between the moduli F-components F^M, the matter F-components F^i, and the $U(1)_A$ D-term. To see the implication of those relations more clearly, let X denote the $U(1)_A$ charged (but SM singlet) matter field with the *largest* VEV, and consider the case that the FI term in D_A is cancelled dominantly by $q_X|X|^2$, so

$$|X|^2 \sim |\xi_{FI}| \gg D_A. \tag{2.15}$$

We further assume that F^T and F^X/X are at least comparable to other moduli and matter F-components, respectively. Then, in the convention with $q_X = -1$, we find

$$\begin{aligned}
\frac{F^X}{X} &\simeq \left(\frac{K_0''}{K_0'}\right)F^T, \\
g_A^2 D_A &\simeq \left(\frac{\left(\frac{K_0''}{K_0'}\right)^2 - \frac{K_0'''}{K_0'}}{1 - \frac{\delta_{GS}}{2}\frac{K_0''}{K_0'}}\right)|F^T|^2, \\
\frac{F^X}{X} &= -m_{3/2}^*\left(1 + \frac{\partial_X W}{X^* W}\right)^*,
\end{aligned} \tag{2.16}$$

where the prime denotes the derivative with respect to $t = T + T^*$, and the last relation is derived from $F^X = -e^{K/2}K^{X\bar{I}}(D_I W)^*$. These relations suggest that the relative ratios between the four SUSY breaking auxiliary components $F^T, F^X, m_{3/2}$ and D_A are determined mostly by

$$R_1 \equiv -\frac{\delta_{GS}}{2}\frac{K_0''}{K_0'}, \quad R_2 \equiv \left(\frac{D_X W}{X^* W}\right)^* = 1 + \left(\frac{\partial_X W}{X^* W}\right)^*, \tag{2.17}$$

where $R_1 > 0$ in our convention with $q_X = -1$. More specifically, in the basis where F^T is real, we obtain

$$F^T : \frac{F^X}{X} : m_{3/2}^* : g_A\sqrt{D_A} \simeq \frac{\delta_{GS}}{2R_1} : -1 : \frac{1}{R_2} : \frac{1}{\sqrt{R_1+1}}, \tag{2.18}$$

where the first three relations are precise, while the relative size of $\sqrt{D_A}$ is approximately estimated under the assumption that K_0'''/K_0'' is comparable to (or smaller than) K_0''/K_0', which holds true in most cases. In the subsections 3.2 and 3.3, we will discuss explicit examples in which the D-flatness is achieved as (2.15), and therefore the ratios between the SUSY breaking auxiliary components are given by (2.18).

Let us now discuss the possible ranges of R_1 and R_2. The first possibility is that $K_0'' \sim K_0'$, which would result in

$$R_1 = \mathcal{O}(\delta_{GS}) = \mathcal{O}\left(\frac{1}{8\pi^2}\right). \tag{2.19}$$

An example for such case is provided by the modulus Kähler potential

$$K_0 \simeq -n_0 \ln(T + T^* - \delta_{GS} V_A) \tag{2.20}$$

for T stabilized at a value of order unity.

Another even more interesting possibility is that T is stabilized at near a point with $\xi_{FI} = \frac{1}{2}\delta_{GS}K_0' = 0$, which would give

$$R_1 = \mathcal{O}(1) \quad \text{or} \quad \gg 1. \tag{2.21}$$

This would be a plausible scenario if the Kähler potential admits a limit with $\xi_{FI} = 0$, or more generally a limit with ξ_{FI} far below M_{Pl}^2, since $\xi_{FI} = \phi_i = 0$ is a point of enhanced (approximate) symmetry[2] and satisfies the equation of motion in the limit to ignore SUSY breaking effects. Then a (local) minimum of the scalar potential with nonzero but small VEV of $|X|/M_{Pl}$ can be developed by small SUSY breaking effects, which results in a tiny VEV of $\xi_{FI}/M_{Pl}^2 \simeq -|X|^2/M_{Pl}^2$. It has been known that many brane models constructed within type IIA or IIB string theory admit supersymmetric brane configuration with smooth background spacetime geometry, which gives rise to an anomalous $U(1)$ gauge symmetry with $\xi_{FI} = 0$ [22]. In addition, it has recently been noticed that heterotic string compactification also can give rise to such a solution with $\xi_{FI} = 0$ [23], which is at the boundary of the Kähler moduli space in which the Hermitian Yang-Mills equations are satisfied. There are also examples that $\xi_{FI} = 0$ correspond to a singular limit of collapsing cycle (or orbifold) [24]. So an anomalous $U(1)$ symmetry with $\xi_{FI} = 0$ somewhere in moduli space is not unusual, but appears quite often in phenomenologically interesting set of string compactifications.

The value of R_2 is determined mostly by the mechanism to stabilize the D-flat directions involving X. In case that X is the dominant matter field which cancels the FI term, there is only one relevant D-flat direction described by $Xe^{-2T/\delta_{GS}}$. However, if there exists additional matter field Y having an opposite $U(1)_A$ charge and comparable VEV, there will be additional D-flat direction described by a $U(1)_A$ invariant holomorphic monomial of X and Y. If the superpotential of the model is independent of these X-dependent D-flat directions, so $\partial_X W = 0$, by definition R_2 has a value close to the unity. In this case, the D-flat directions should be stabilized by nontrivial structure of the Kähler potential which

[2] Note that $D_A = 0$ and the global part of $U(1)_A$ is restored at this point.

might be induced by radiative corrections. Another possibility is that the superpotential contains a higher dimensional term depending on X, e.g. $\Delta W \sim X^3 Y$ for $q_Y = -3q_X$, and then X is stabilized by the competition between the supersymmetric potential $|\partial_X W|^2$ and the SUSY breaking terms controlled by $m_{3/2} \sim W/M_{Pl}^2$, e.g. an A-term of the form $m_{3/2} X \partial_X W$ or a scalar mass term of the form $-m_{3/2}^2 |X|^2$. Such setup stabilizes the D-flat direction at a (local) minimum satisfying

$$|\partial_X W|^2 \sim m_{3/2} X \partial_X W \quad \text{or} \quad m_{3/2}^2 |X|^2, \tag{2.22}$$

for which

$$R_2 = 1 + \left(\frac{\partial_X W}{X^* W}\right)^* = \mathcal{O}(1). \tag{2.23}$$

It is also possible that $|R_2|$ has a value much smaller or much larger than the unity. For instance, if the D-flat direction $X e^{-2T/\delta_{GS}}$ is stabilized by a nonperturbative superpotential $\Delta W \sim X^n e^{-2nT/\delta_{GS}}$ at near the supersymmetric solution with $D_X W = 0$, we have

$$|R_2| \ll 1. \tag{2.24}$$

In other case that the superpotential provides spontaneous SUSY breaking in the global SUSY limit, e.g. through the Polonyi term of the form $\Delta W \sim e^{-2T/\delta_{GS}} X$, it is also possible that

$$|R_2| \gg 1. \tag{2.25}$$

In this case, the condition for vanishing cosmological constant provides an upper limit

$$|R_2| \sim \left|\frac{\partial_X W}{X W}\right| \lesssim \mathcal{O}\left(\frac{M_{Pl}}{\sqrt{|\xi_{FI}|}}\right), \tag{2.26}$$

where we have used $|X|^2 \simeq |\xi_{FI}|$ together with the observation that $|\partial_X W|^2 \lesssim \mathcal{O}(|W|^2/M_{Pl}^2)$ in order for the cosmological constant to be nearly vanishing.

The above discussion implies that even within a relatively simple class models a variety of different patterns of SUSY breaking can be realized, depending on the characteristics of D-flat directions and how those D-flat directions are stabilized. In more complicate situation in which there exist multiple number of moduli and/or of $U(1)_A$-charged matter fields providing non-negligible amount of SUSY breaking, there can be more model-dependent variation in the pattern of SUSY breaking. In the next section, we will examine a set of simple models realizing the scenarios considered above, and evaluate the values of R_1 and R_2 in those models.

2.2 Soft terms

The SUSY-breaking auxiliary components F^T, F^X, $m_{3/2}$ and D_A can generate soft SUSY-breaking masses in the visible sector through various mediation mechanisms as described below.

A) *Modulus mediation*: Since the gauge kinetic function $f_a \ni k_a T$, the F-term of the GS modulus generates the gaugino masses as

$$M_a(\mathrm{MM}) = F^T \partial_T \ln \mathrm{Re}(f_a) = \frac{k_a g_a^2(\Lambda)}{2} F^T, \tag{2.27}$$

at a scale Λ close to the Planck or string scale [17]. Similarly the T-dependence of matter wave functions gives rise to soft scalar masses which are generically of $\mathcal{O}(|F^T|^2)$:

$$m_i^2(\mathrm{MM}) = -|F^T|^2 \partial_T \partial_{\bar{T}} \ln(e^{-K_0/3} Z_i), \tag{2.28}$$

where Z_i is the Kähler metric of the matter superfield ϕ_i.

B) *Gauge mediation*: Soft masses can receive a gauge-mediated contribution if there exist gauge-charged messenger fields which couple to X [18, 19]. Indeed, in most of the known potentially (semi)realistic string models with $U(1)_A$, there exist exotic matter fields Φ, Φ^c which are vector-like under the SM gauge group and become massive through the Yukawa coupling

$$\Delta W \; = \; y_\Phi X \Phi \Phi^c. \tag{2.29}$$

Then, gaugino masses are generated at the messenger scale $\Lambda_\Phi = \lambda_\Phi \langle X \rangle$ according to

$$M_a(\mathrm{GM}) = -\frac{N_\Phi g_a^2(\Lambda_\Phi)}{16\pi^2} \frac{F^X}{X}, \tag{2.30}$$

where N_Φ is the number of $\Phi + \Phi^c$ which are assumed to form $5 + \bar{5}$ of $SU(5)$. Gauge mediation induces soft scalar masses also, giving

$$m_i^2(\mathrm{GM}) = \mathcal{O}\left(\left(\frac{1}{8\pi^2} \frac{F^X}{X} \right)^2 \right). \tag{2.31}$$

An interesting feature of the gauge mediation in models with $U(1)_A$ is the contribution to scalar masses originating from D_A. Because $\Phi\Phi^c$ carries a $U(1)_A$ charge $q_\Phi + q_{\Phi^c} = 1$ (note that we use the convention $q_X = -1$), the supertrace of the messenger mass matrix is non-vanishing due to the D-term contribution. As a result, gauge-mediated soft scalar masses contain the piece [25]

$$\delta m_i^2(\mathrm{GM}) \simeq \sum_a C_i^a g_a^4 \frac{N_\Phi \ln(\Lambda/\Lambda_\Phi)}{(8\pi^2)^2} g_A^2 D_A, \tag{2.32}$$

where C_i^a is the quadratic Casimir of ϕ_i. This piece can be important for matter fields with $q_i = 0$ when $g_A^2 D_A \gtrsim |F^X/X|^2$.

C) *Anomaly mediation*: Supergravity always mediates SUSY breaking through the conformal anomaly [20]. This effect can be described by introducing the supergravity conformal compensator C with an F-component:

$$F^C = m_{3/2}^* + \frac{1}{3} F^I \partial_I K, \tag{2.33}$$

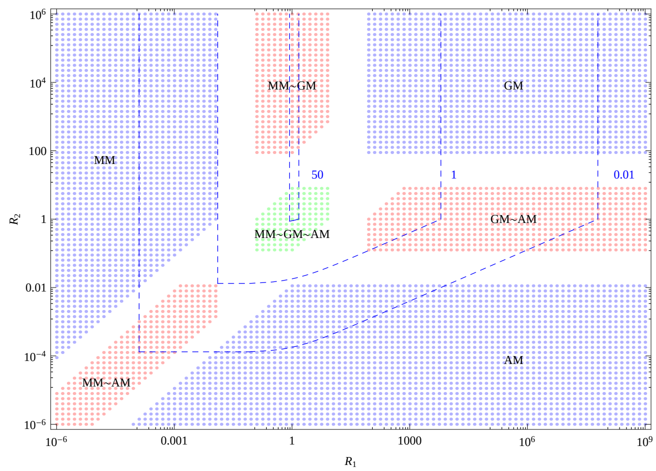

Figure 1: Relative strength of each mediation. Either modulus, gauge, or anomaly mediation dominates over the other two in the blue region, while mixed mediations are realized in the red and green regions. The dashed blue line is the contour for a given value of $m_{\rm soft}({\rm D})/\hat{m}_{\rm soft}$, where $\hat{m}_{\rm soft}$ is the biggest of $m_{\rm soft}({\rm MM, GM, AM})$.

which is generically of the order[3] of $m_{3/2}$. In this formulation, due to the super-Weyl invariance, the C-dependence of physical gauge couplings and wavefunction coefficients is determined by the renormalization group running, which results in

$$M_a({\rm AM}) = \frac{\beta_a}{g_a} F^C = \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right),$$
$$m_i^2({\rm AM}) = -\frac{1}{4}\frac{d\gamma_i}{d\ln\mu}\left|F^C\right|^2 = \mathcal{O}\left(\left(\frac{m_{3/2}}{8\pi^2}\right)^2\right), \qquad (2.34)$$

where $\beta_a = dg_a/d\ln\mu$ and $\gamma_i = d\ln Z_i/d\ln\mu$ are the gauge beta-function and the matter anomalous dimension, respectively.

D) *D-term contribution*: In the presence of anomalous $U(1)_A$, there can be a D-term contribution to the soft scalar mass for $U(1)_A$ charged matter fields:

$$m_i^2({\rm D}) = -q_i g_A^2 D_A, \qquad (2.35)$$

where q_i denotes the $U(1)_A$ charge of the corresponding matter field.

Let us now examine the relative importance of these mediations in the case that

$$|X|^2 \sim |\xi_{FI}| \gg D_A \qquad (2.36)$$

and F^T and F^X/X are at least comparable to other moduli and matter F-components, respectively. In such case, the ratios between auxiliary components are estimated as (2.18), and then we find the following order of magnitude relation between different mediations:

$$m_{\rm soft}({\rm MM}) : m_{\rm soft}({\rm GM}) : m_{\rm soft}({\rm AM}) : m_{\rm soft}({\rm D}) \ \sim \ \frac{1}{R_1} : 1 : \frac{1}{R_2} : \frac{8\pi^2}{\sqrt{R_1+1}}. \qquad (2.37)$$

[3] In some case such as the no-scale model, there can be a cancellation between $m_{3/2}$ and $\frac{1}{3}F^I\partial_I K$, which would result in $|F^C| \ll |m_{3/2}|$.

Note that the ratios between the modulus, gauge and D-term mediations are determined essentially by a single parameter R_1, while the relative importance of anomaly mediation is determined by R_2. Fig. 1 shows the relative strength of modulus, gauge, and anomaly mediation as a function of R_1 and R_2. Also shown in the figure is the contour of the ratio between $m_{\rm soft}(\rm D)$ and the biggest of $m_{\rm soft}(\rm MM)$, $m_{\rm soft}(\rm GM)$ and $m_{\rm soft}(\rm AM)$. As we have noticed, both R_1 and R_2 are quite model-dependent and can have a wide range of values. In the subsections 3.2 and 3.3, we will discuss a set of simple models in which the SUSY breaking auxiliary components are given by (2.18), and therefore the soft masses obey the relation (2.37). In the subsequent subsections, we will consider other type of models which require a separate discussion as the relation (2.18) does not apply.

3. Stabilization of the D-flat direction

In this section, we explore with explicit examples how the D-flat directions in models with $U(1)_A$ can be stabilized at a phenomenologically viable (meta-stable) vacuum with nearly vanishing cosmological constant. We will consider two different types of models, one in which the D-flatness is achieved mostly by the cancellation between ξ_{FI} and the matter field with the largest VEV, and the other in which the D-flatness is achieved mostly by the cancellation between two matter fields with opposite $U(1)_A$ charges. For the first type of models discussed in the subsections 3.2 and 3.3, the simple relations (2.18) and (2.37) are satisfied, so the structure of mixed mediation is determined by R_1 and R_2. On the other hand, (2.18) and (2.37) do not apply to the second type of models discussed in the subsections 3.4 and 3.5. Our results show that various different mixed mediations can be realized within a relatively simple class of models.

3.1 Uplifting potential

Quite often, SUSY breaking by $U(1)_A$ charged fields alone cannot give a nearly vanishing cosmological constant, and then one needs to introduce additional SUSY breaking providing an uplifting potential for de-Sitter or Minkowski vacuum solution. If $U(1)_A$ charged fields all get a mass $\gg m_{3/2}$, the stabilization would not be affected significantly by the uplifting potential. However, in case that some D-flat direction has a mass $\lesssim m_{3/2}$, the uplifting potential can play an important role for the stabilization. This means that one needs to include the uplifting potential explicitly in the analysis in order to draw a reliable conclusion on the stabilization. To be specific, here we will consider a particular form of uplifting potential which originates from a sequestered SUSY breaking sector in which SUSY is non-linearly realized.

Then the full scalar potential of the model can be derived from the following 4D SUGRA action:

$$\int d^4\theta \, C\bar{C}\left(-3e^{-K/3} + C\bar{C}M^4\Lambda^2\bar{\Lambda}^2\right) + \left(\int d^2\theta \, C^3 W + \text{h.c.}\right), \tag{3.1}$$

where C is the chiral compensator superfield introduced to encode the SUSY breaking effects due to the auxiliary component of the SUGRA multiplet, K and W are the conventional Kähler potential and superpotential of the model, and $\Lambda^2\bar{\Lambda}^2$ is the Volkov-Akulov

(VA) action of the Goldstino superfield $\Lambda^\alpha = \theta^\alpha + \frac{1}{M^2}\lambda^\alpha$, where λ^α is the Goldstino fermion. Here the C-dependence of the action is determined by the super-Weyl invariance. The VA action might be a low energy consequence of spontaneous SUSY breaking at some high energy scale, e.g. a low energy realization of the action

$$\int d^4\theta C\bar{C}\left(Z\bar{Z} - \frac{Z^2\bar{Z}^2}{M_1^2}\right) + \left(\int d^2\theta\, C^3 M_2^2 Z + \text{h.c.}\right), \tag{3.2}$$

where Z is a Polonyi field, and $M_1 \sim M_2 \gg m_{3/2}$ are constant mass parameters. Alternatively it might represent the effect of anti-brane stabilized at the tip of warped throat in KKLT-type compactification [26]. In the former case, the Goldstino scale M is determined as $M \sim M_1 \sim M_2$, while in the latter case M is determined by the red-shifted tension of anti-brane. After integrating out all auxiliary components and choosing the Einstein frame condition for the lowest component of the compensator superfield $C_0 = e^{K/6}$, one finds that the full scalar potential is given by

$$V_{\text{TOT}} = V_{\text{SUGRA}} + V_{\text{lift}}, \tag{3.3}$$

where

$$V_{\text{SUGRA}} = V_F + V_D = \left(K_{I\bar{J}}F^I F^{J*} - 3e^K|W|^2\right) + \frac{g_A^2}{2}D_A^2 \tag{3.4}$$

is the conventional SUGRA potential and

$$V_{\text{lift}} = M^4 e^{2K/3} \tag{3.5}$$

is the uplifting potential from the VA action.

3.2 Models with non-perturbative superpotential

A natural source of moduli potential in string theory is non-perturbative effect such as stringy instanton or hidden sector gaugino condensation. So let us discuss first the stabilization of D-flat direction by non-perturbative superpotential. For simplicity, we consider the case that the GS modulus does not have a Kähler mixing with other SUSY breaking moduli, and the FI term is cancelled mostly by the $U(1)_A$ charged matter field X with the largest VEV. Since the moduli-dependence of the matter Kähler metric does not change the essential feature of stabilization, it is sufficient to consider the case of moduli-independent matter Kähler metric. Then the Kähler potential relevant for our discussion is given by

$$K = K_0(t - \delta_{GS}V_A) + X^* e^{-2V_A} X, \tag{3.6}$$

where K_0 can take an arbitrary form. The non-perturbative superpotential generically takes the form

$$W_{\text{np}} \propto e^{-2nT/\delta_{GS}} X^n \tag{3.7}$$

with an integer $n > 0$ and $\delta_{GS} > 0$. Since we need to stabilize X at a scale far above TeV, it is desirable that X has a flat potential in the global SUSY limit when the D-flat direction is mostly X. This is achieved when $n = 1$, so we consider

$$W = \omega_0 + Ae^{-2T/\delta_{GS}} X, \tag{3.8}$$

where A is a constant of order unity[4] and ω_0 is a small constant of $\mathcal{O}(m_{3/2}M_{Pl}^2)$.

For the Kähler potential (3.6) and the superpotential (3.8), it is straightforward to see that the VEV of $\arg(e^{-2T/\delta_{GS}}X)$ is fixed by the F-term potential at

$$\arg(e^{-2T/\delta_{GS}}X) = \begin{cases} \arg\left(\frac{\omega_0}{A}\right) + \pi & \text{for } R_1 < \frac{1-\xi_{FI}}{2-|X|^2}, \\ \arg\left(\frac{\omega_0}{A}\right) & \text{for } R_1 > \frac{1-\xi_{FI}}{2-|X|^2}, \end{cases} \tag{3.9}$$

where $R_1 = -\delta_{GS}K_0''/2K_0'$ as defined in (2.17) and $\xi_{FI} = \delta_{GS}K_0'/2$. Meanwhile, one combination of $\mathrm{Im}(T)$ and $\arg(X)$ remains unfixed, and is absorbed into the $U(1)_A$ gauge boson. After replacing $\arg(e^{-2T/\delta_{GS}}X)$ with its VEV, the remaining $t = T + T^*$ and $|X|$ can be fixed by the stationary condition

$$\partial_{t,|X|}\left(V_{\rm SUGRA} + V_{\rm lift}\right) = 0 \tag{3.10}$$

under the constraint

$$\langle V_{\rm SUGRA} + V_{\rm lift}\rangle = 0. \tag{3.11}$$

To proceed, let us first consider the case with

$$R_1 \ll 1. \tag{3.12}$$

We find that in this case the uplifting potential can be treated as a small perturbation, so one can start with a solution of

$$\partial_{t,|X|}V_{\rm SUGRA} = 0. \tag{3.13}$$

Then the stationary conditions for $V_{\rm SUGRA}$ give rise to

$$\begin{aligned} g_A^2 D_A &= -\frac{1}{\frac{\delta_{GS}}{2}K_0''}\left(V_F' + (\partial_t \ln g_A^2)V_D\right), \\ \dot{V}_F &= \frac{2|X|^2}{\frac{\delta_{GS}}{2}K_0''}\left(V_F' + (\partial_t \ln g_A^2)V_D\right), \end{aligned} \tag{3.14}$$

where the prime and dot denote the derivatives with respect to t and $\ln|X|$, respectively. The second relation above determines how the D-flat direction is fixed by the F-term potential. Neglecting small corrections of $\mathcal{O}(m_{3/2}^4/M_V^4)$, the condition for the solution of (3.14) to be a (local) minimum of $V_{\rm SUGRA}$ reads

$$\ddot{V}_F + 4\frac{|X|^4}{\frac{\delta_{GS}^2}{4}K_0''^2}\left(V_F'' - \frac{K_0'''}{K_0''}V_F'\right) - 4\frac{|X|^2}{\frac{\delta_{GS}}{2}K_0''}\dot{V}_F' > 0. \tag{3.15}$$

Note that a supersymmetric solution of (3.14) leads to $\dot{V}_F = V_F' = 0$ and $D_A = 0$.

[4]Note that A can always be made to be of order unity through a constant shift of T. We have also chosen the normalization convention of T for which $\delta_{GS} = \mathcal{O}(1/8\pi^2)$. Whenever we use an explicit form of the modulus Kähler potential, it is defined in such field basis.

It turns out that, in the limit $R_1 \ll 1$, the equations (3.14) have a unique solution which is supersymmetric and automatically fulfills the (meta)stability condition (3.15) regardless of the form of K_0. The minimum of $V_{\rm SUGRA}$ is given by

$$\begin{aligned} |X|^2 &= -\frac{\delta_{GS}}{2}K_0', \\ \left|\frac{A}{\omega_0}\right| e^{-t/\delta_{GS}} &= \frac{|X|}{1-|X|^2} \simeq |X|, \end{aligned} \tag{3.16}$$

and the vacuum solution of $V_{\rm TOT}$ can be obtained by taking into account the small shift from this solution induced by $V_{\rm lift}$. It is then easy to find that the vacuum solution of $V_{\rm TOT}$ gives

$$\begin{aligned} F^T &\simeq \frac{\delta_{GS}}{2}\frac{3\partial_t \ln V_{\rm lift}}{K_0'} m_{3/2} = \delta_{GS} m_{3/2} = \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right), \\ \frac{F^X}{X} &\simeq \left(\frac{K_0''}{K_0'}\right) F^T \simeq -2R_1 m_{3/2}, \end{aligned} \tag{3.17}$$

where we have used the sequestered uplifting potential $V_{\rm lift} = M^4 e^{2K/3}$. The above result also leads to $R_2 \simeq 2R_1$. The D-flat direction, which is mostly T in this case, acquires a mass of $\sim m_{3/2}\ln(M_{Pl}/m_{3/2}) \gg m_{3/2}$, and this is the reason why $F^T \ll m_{3/2}$.

Another limit for which the analysis is straightforward is the case with

$$R_1 \gg 1. \tag{3.18}$$

Since the non-perturbative superpotential stabilizes the GS modulus at $t/\delta_{GS} \gg 1$ in the field basis with $A = \mathcal{O}(1)$, it is plausible to assume that the modulus Kähler potential satisfies

$$\left|\left(\frac{\delta_{GS}}{2}\right)^{k-2} \frac{\partial_t^k K_0}{K_0''}\right| \lesssim \left(\frac{\delta_{GS}}{t}\right)^{k-2} \ll 1 \qquad (k \geq 3) \tag{3.19}$$

at the stationary point of $V_{\rm SUGRA}$. We then find that $V_{\rm SUGRA}$ can have a SUSY breaking minimum at

$$\begin{aligned} |X|^2 &= -\frac{\delta_{GS}}{2}K_0' + \mathcal{O}\left(\frac{m_{3/2}^2 M_{Pl}^2}{M_V^2}\right), \\ \left|\frac{A}{\omega_0}\right| e^{-t/\delta_{GS}} &= \left(R_1 + \mathcal{O}\left(\frac{M_V^2}{M_{Pl}^2}\right)\right)|X|, \end{aligned} \tag{3.20}$$

for $M_{Pl} \gg M_V \gg (m_{3/2}M_{Pl})^{1/2}$. It is straightforward to show that the above field configuration satisfies the stability condition (3.15) and leads to

$$\langle V_{\rm SUGRA}\rangle = e^K |\omega_0|^2 \left(-3 + R_1\frac{M_{GS}^2}{M_{Pl}^2} + \mathcal{O}\left(\frac{M_{GS}^2}{M_{Pl}^2}\right)\right), \tag{3.21}$$

where the second term in the brackets is the contribution from F^X, and thus $R_2 \simeq R_1$ at the minimum of $V_{\rm SUGRA}$. The above form of the vacuum energy density suggests that one can get a de-Sitter or Minkowski minimum *without* introducing an uplifting potential, if

$$R_1 = \mathcal{O}\left(\frac{M_{\rm Pl}^2}{M_{GS}^2}\right) = \mathcal{O}\left(\frac{1}{\delta_{GS}^2}\right). \tag{3.22}$$

In this case, X (approximately) corresponds to the D-flat direction, and its scalar component acquires a mass of $\mathcal{O}(m_{3/2}M_{Pl}/M_{GS})$, while its fermionic component corresponds to the Goldstino absorbed into the gravitino.

Unlike the case with $R_1 \ll 1$ or $R_1 \gg 1$, the analysis of the vacuum solution of $V_{\rm TOT}$ for $R_1 = \mathcal{O}(1)$ is quite nontrivial. In such case, we need more model-dependent and detailed analysis to make sure that there exists a (local) minimum of $V_{\rm TOT}$ with vanishing cosmological constant.

In the following, to examine explicitly the stabilization of T and X, we consider two different forms of the modulus Kähler potential

$$\begin{aligned} K_0^{(I)} &= -n_0 \ln(t - \delta_{GS} V_A), \\ K_0^{(II)} &= \tfrac{1}{2} K_0''(t_0)(t - t_0 - \delta_{GS} V_A)^2, \end{aligned} \tag{3.23}$$

with the superpotential and the uplifting potential given by

$$\begin{aligned} W &= \omega_0 + AXe^{-2T/\delta_{GS}}, \\ V_{\rm lift} &= M^4 e^{2K/3}. \end{aligned} \tag{3.24}$$

In $K_0^{(I)}$, T might correspond to a dilaton in the weak coupling limit or a volume modulus in the large volume limit. On the other hand, $K_0^{(II)}$ assumes that there exists a point in the moduli space where the FI term vanishes [22, 23, 24],

$$\xi_{FI}(t_0) = \frac{1}{2}\delta_{GS} K_0'(t = t_0) = 0, \tag{3.25}$$

and then the modulus Kähler potential is expanded around $t = t_0$. Using the field redefinition $T \to \alpha T + \beta$ for real α and β, we can choose the convention

$$|A| = 1, \quad \delta_{GS} = \frac{1}{2\pi^2}. \tag{3.26}$$

Generically $K_0''(t_0)$ is expected to be of order unity, however ω_0 is required to be exponentially small to realize low energy SUSY. To be specific, we choose

$$K''(t_0) = 1, \quad |\omega_0| = e^{-38}, \tag{3.27}$$

and examine the following four examples:

$$\begin{aligned} &\text{Model I} &:& \quad K_0 = K_0^{(I)} \quad \text{with } n_0 = 1, \\ &\text{Model II} &:& \quad K_0 = K_0^{(II)} \quad \text{with } t_0 = 2.6, \\ &\text{Model III} &:& \quad K_0 = K_0^{(II)} \quad \text{with } t_0 = 2.1, \\ &\text{Model IV} &:& \quad K_0 = K_0^{(II)} \quad \text{with } t_0 = 1.9. \end{aligned} \tag{3.28}$$

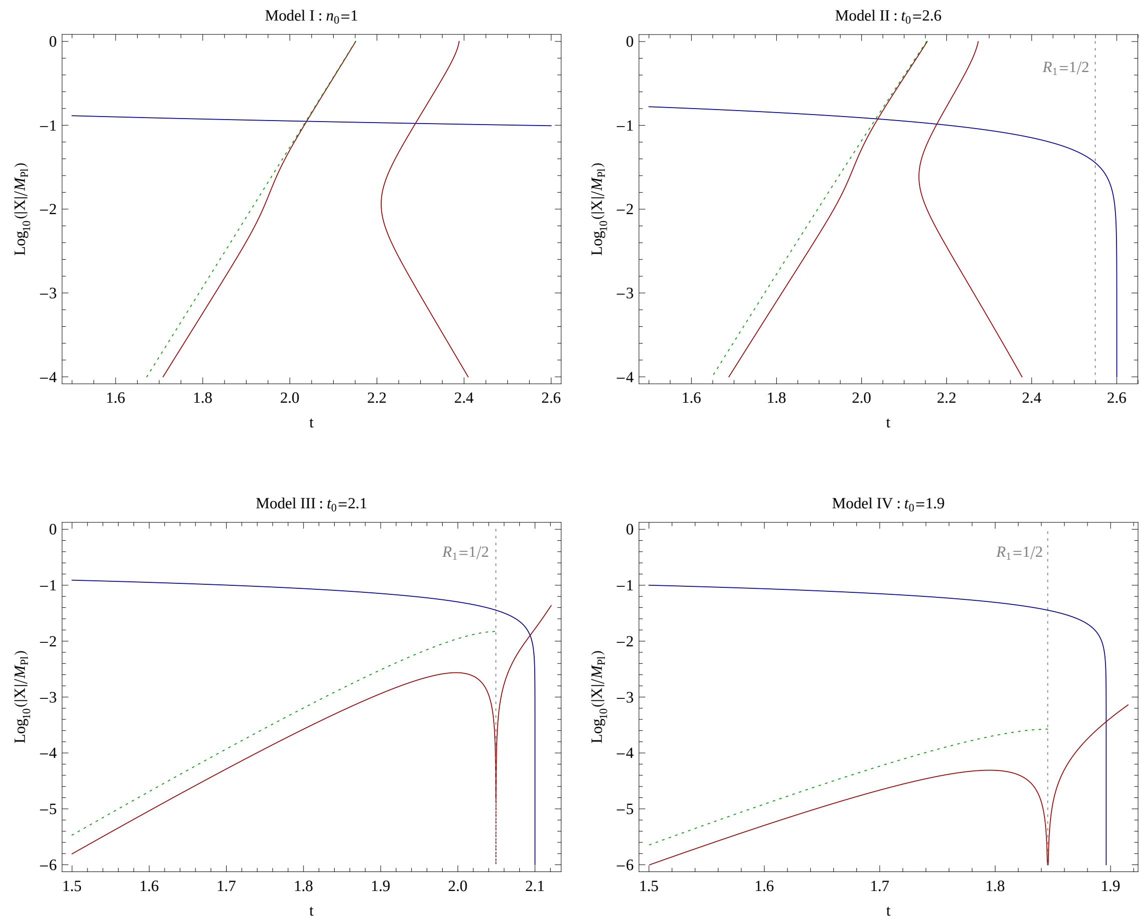

Figure 2: Stabilization of the GS modulus $t = T + T^*$ and the chiral matter $\ln |X|$ in each model. In each figure, the blue curve corresponds to the D-flat condition, while the red curve represents the stationary condition, $(\partial_t D_A)\partial_{|X|}V_{\rm TOT} - (\partial_{|X|}D_A)\partial_t V_{\rm TOT} = 0$. We also plot the dashed green curve along which F^T vanishes. For Model II, III and IV, $R_1 = 1/2$ on the vertical dashed line.

Fig. 2 shows how a minimum of $V_{\rm TOT} = V_{\rm SUGRA} + V_{\rm lift}$ with vanishing cosmological constant is developed in these examples. In each figure, the blue curve corresponds to the D-flat condition, while the red curve represents the stationary condition[5], $(\partial_t D_A)\partial_{|X|}V_{\rm TOT} - (\partial_{|X|}D_A)\partial_t V_{\rm TOT} = 0$. Hence the point where the two curves intersect corresponds to a stationary solution of $V_{\rm TOT}$. In Model I and II, we have two intersecting points, and the point with smaller t is a (local) minimum, while the other point is a saddle point. The vacuum solution can be considered as a small deviation from the supersymmetric solution which is the intersecting point of the blue and dashed green curves in the figure, and this small deviation is due to $V_{\rm lift}$. On the other hand, for Model III and IV, the blue and red curves have a unique intersecting point which is a local minimum of the potential. Here

[5]This stationary condition corresponds to the second relation of (3.14) for Model IV. On the other hand, for other models, one needs to replace V_F with $V_F + V_{\rm lift}$ in (3.14) since the uplifting potential is needed to cancel the cosmological constant.

we have tuned the size of $V_{\rm lift}$ to make the vacuum solution have a vanishing cosmological constant. We then find that the VEVs of

$$\frac{|X|^2}{M_{Pl}^2} \simeq -\frac{\xi_{FI}}{M_{Pl}^2}, \quad R_1 = -\frac{\delta_{GS}}{2}\frac{K_0''}{K_0'}, \quad |R_2| = \left|\frac{D_X W}{XW}\right| \tag{3.29}$$

are given by

$$\begin{array}{llll} \text{Model I} & 1.6\times 10^{-2}, & 1.6\times 10^{-2}, & 3.1\times 10^{-2} \\ \text{Model II} & 1.5\times 10^{-2}, & 4.2\times 10^{-2}, & 8.4\times 10^{-2} \\ \text{Model III} & 1.6\times 10^{-4}, & 4, & 3.8 \\ \text{Model IV} & 1.4\times 10^{-7}, & 4.7\times 10^{3}, & 4.7\times 10^{3} \end{array} \tag{3.30}$$

and therefore the ratios

$$F^T : \frac{F^X}{X} : m_{3/2}^* : g_A\sqrt{D_A} \simeq \frac{\delta_{GS}}{2R_1} : -1 : \frac{1}{R_2} : \frac{1}{\sqrt{R_1+1}} \tag{3.31}$$

are determined as

$$\begin{array}{lccccccc} \text{Model I} & 1.6 & : & -1 & : & 32 & : & 1 \\ \text{Model II} & 0.6 & : & -1 & : & 12 & : & 1 \\ \text{Model III} & 6\times 10^{-3} & : & -1 & : & 0.26 & : & 0.45 \\ \text{Model IV} & 5\times 10^{-6} & : & -1 & : & 2\times 10^{-4} & : & 1.5\times 10^{-2} \end{array} \tag{3.32}$$

With the above results, the ratios between the modulus, gauge, anomaly and D-term mediated soft masses in each model can be read off from the following order of magnitude relation

$$m_{\rm soft}({\rm MM}) : m_{\rm soft}({\rm GM}) : m_{\rm soft}({\rm AM}) : m_{\rm soft}({\rm D}) \sim \frac{1}{R_1} : 1 : \frac{1}{R_2} : \frac{8\pi^2}{\sqrt{R_1+1}}, \tag{3.33}$$

where we have assumed that there exist gauge-charged messengers $\Phi + \Phi^c$ which become massive through the coupling to X:

$$\Delta W = y_\Phi X \Phi \Phi^c. \tag{3.34}$$

Our results indicate that the relative importance of each mediation is quite model-dependent, particularly on the form of the modulus Kähler potential. Models I and II realize mixed modulus-anomaly-D-term mediation, which corresponds to the mirage mediation [27, 28, 29] with additional D-term contribution [15]. On the other hand, in Model III, gaugino masses are determined by mixed modulus-gauge-anomaly mediation, while scalar masses are dominated by the D-term contribution which is one or two orders of magnitudes heavier than gaugino masses. Finally, soft masses in Model IV are determined by mixed gauge-D-term mediation. So various different mixed mediations can be realized even within the simple class of models discussed in this subsection.

Since the $U(1)_A$ vector superfield acquires a large supersymmetric mass, while leaving the D-flat combination of T and $\ln X$ light, we can study the low energy dynamics of model with an effective supergravity in which the massive $U(1)_A$ vector superfield is integrated out. In the limit $R_1 \ll 1$ or $R_1 \gg 1$, the massive and light degrees of freedom can be easily identified, and therefore it is straightforward to derive the effective theory of light fields in this limit. For instance, the relations

$$\frac{M_V^2}{2g_A^2} \simeq \frac{1}{4}\delta_{GS}^2 K_0'' + |X|^2, \quad R_1 \simeq \frac{\delta_{GS}^2}{4}\frac{K_0''}{|X|^2} \tag{3.35}$$

suggest that the Goldstone superfield absorbed into the massive $U(1)_A$ vector superfield is mostly $\ln X$ in the limit $R_1 \ll 1$, while it is mostly T in the opposite limit $R_1 \gg 1$.

Since it provides an efficient way to understand our results, let us derive the effective theory explicitly in the limit $R_1 \ll 1$ or $R_1 \gg 1$. When $R_1 \ll 1$, the massive vector superfield and the light D-flat direction are identified as

$$V_H = V_A - \ln\left(\frac{|X|}{M_{Pl}}\right), \quad \tilde{T} = T - \frac{1}{2}\delta_{GS}\ln\left(\frac{X}{M_{Pl}}\right), \tag{3.36}$$

both of which are invariant under $U(1)_A$. Then it is convenient to introduce the $U(1)_A$-invariant combination of matter fields

$$\tilde{\phi}_i = \left(\frac{X}{M_{Pl}}\right)^{q_i}\phi_i, \tag{3.37}$$

and rewrite the Kähler potential and superpotential in terms of V_H, $\tilde{T}$ and $\tilde{\phi}_i$:

$$\begin{aligned} K &= K_0(t - \delta_{GS}V_A) + X^* e^{-2V_A} X + \phi_i^* e^{2q_iV_A}\phi_i \\ &= K_0(\tilde{t} - \delta_{GS}V_H) + e^{-2V_H} + \tilde{\phi}_i^* e^{2q_iV_H}\tilde{\phi}_i, \\ W &= \omega_0 + AXe^{-2T/\delta_{GS}} + \lambda_{ijk}X^{q_i+q_j+q_k}\phi_i\phi_j\phi_k \\ &= \omega_0 + Ae^{-2\tilde{T}/\delta_{GS}} + \lambda_{ijk}\tilde{\phi}_i\tilde{\phi}_j\tilde{\phi}_k, \end{aligned} \tag{3.38}$$

where $t = T + T^*$ and $\tilde{t} = \tilde{T} + \tilde{T}^*$. Now the massive vector superfield V_H can be integrated out by solving the equation of motion

$$\partial_{V_H} K = 0, \tag{3.39}$$

whose solution is given by

$$\frac{2}{\delta_{GS}} e^{-2V_H} = K_0'(\tilde{t}) + \mathcal{O}(\delta_{GS}). \tag{3.40}$$

Inserting this solution into the Kähler potential, we find the effective Kähler potential of $\tilde{T}$ and $\tilde{\phi}_i$ ($\neq \tilde{X}$) is given by

$$K_{\rm eff} = K_0(\tilde{t}) + \mathcal{O}(\delta_{GS}) + \sum_{i\neq X}\left(\frac{\delta_{GS}}{2}\left(K_0'(\tilde{t}) + \mathcal{O}(\delta_{GS})\right)\right)^{-q_i}|\tilde{\phi}_i|^2, \tag{3.41}$$

from which all the low energy consequences of (3.38), including the SUSY breaking auxiliary components of light fields and the soft terms of visible sector fields, can be derived in the approximation in which the subleading corrections suppressed by δ_{GS} are ignored.

On the other hand, in the other limit $R_1 \gg 1$, the massive vector superfield and the light matter fields (including the D-flat direction $\tilde{X}$) are given by

$$V_H = V_A - \frac{1}{\delta_{GS}}(t - t_0), \quad \tilde{\phi}_i = \phi_i e^{2q_i(T-T_0)/\delta_{GS}}, \tag{3.42}$$

where t_0 is the modulus value for which the FI term vanishes:

$$K_0'(t = t_0) = 0. \tag{3.43}$$

We then have

$$\begin{aligned} K &= K_0(t - \delta_{GS}V_A) + \sum_{I=X,i} \phi_I^* e^{2q_I V_A} \phi_I \\ &= K_0(t_0 - \delta_{GS}V_H) + \sum_{I=X,i} \tilde{\phi}_I^* e^{2q_I V_H} \tilde{\phi}_I, \\ W &= \omega_0 + AXe^{-2T/\delta_{GS}} + \lambda_{ijk} X^{q_i+q_j+q_k} \phi_i\phi_j\phi_k \\ &= \omega_0 + Ae^{-2T_0/\delta_{GS}}\tilde{X} + \lambda_{ijk}\tilde{X}^{q_i+q_j+q_k}\tilde{\phi}_i\tilde{\phi}_j\tilde{\phi}_k, \end{aligned} \tag{3.44}$$

for which the solution of the equation of motion (3.39) is given by

$$V_H = -\frac{\sum_I q_I \tilde{\phi}_I^* \tilde{\phi}_I}{2M_{GS}^2} + \mathcal{O}\left(\frac{|\tilde{\phi}|^4}{M_{GS}^4}\right), \tag{3.45}$$

where $M_{GS}^2 = \delta_{GS}^2 K_0''(t = t_0)/4$. The resulting effective Kähler potential and superpotential are given by

$$\begin{aligned} K_{\rm eff} &= K_0(t_0) + \sum_{I=X,i} |\tilde{\phi}_I|^2 - \sum_{IJ=X,i} \frac{q_I q_J}{2M_{GS}^2} |\tilde{\phi}_I|^2 |\tilde{\phi}_J|^2 + \mathcal{O}\left(\frac{|\tilde{\phi}|^6}{M_{GS}^4}\right), \\ W_{\rm eff} &= \omega_0 + Ae^{-2T_0/\delta_{GS}}\tilde{X} + \lambda_{ijk}\tilde{X}^{q_i+q_j+q_k}\tilde{\phi}_i\tilde{\phi}_j\tilde{\phi}_k. \end{aligned} \tag{3.46}$$

One is thus led to the low energy theory where $\tilde{X}$ acts as a Polonyi field and is stabilized by the same way as in sweet-spot SUSY [30]. In that scenario, the Higgs sector feels SUSY breaking also through direct interactions with the Polonyi field, in addition to gauge-mediated one. But, the higher dimensional operator $q_i|\tilde{X}|^2|\tilde{\phi}_i|^2/M_{GS}^2$ in the above effective Kähler potential can transmit SUSY breaking not only to the Higgs fields but also to the sfermions if they are charged under $U(1)_A$.

3.3 Models with radiative stabilization

In some case, non-perturbative superpotential of the GS modulus might not be available because either $\delta_{GS} < 0$, so it is forbidden by $U(1)_A$, or the corresponding instanton amplitude is vanishing due to the zero mode structure. Even in such case, if the Kähler potential admits a limit with $\xi_{FI} = 0$ or more generally a limit with ξ_{FI} far below M_{Pl}^2, for which the D-flat direction is described by X, the scalar potential of D-flat direction can receive a field-theoretic radiative correction which can fix the VEV of X at a proper value.

As an example of such model, let us consider

$$
\begin{aligned}
K &= K_0(t-\delta_{GS}V_A)+\sum_i \phi_i^* e^{2q_iV_A}\phi_i,\\
W &= \omega_0 + y_\Phi X\Phi\Phi^c,
\end{aligned}
\tag{3.47}
$$

where $\phi_i = \{X, \Phi, \Phi^c\}$ for the exotic matter fields $\Phi + \Phi^c$ which form $5+\bar{5}$ of $SU(5)$ with the $U(1)_A$ charges satisfying $q_\Phi + q_{\Phi^c} = 1$, and K_0 is assumed to satisfy

$$K_0'(t=t_0) = 0. \tag{3.48}$$

It is straightforward to analyze this model if one uses the effective theory in which the massive $U(1)_A$ vector superfield $V_H = V_A - (t-t_0)/\delta_{GS}$ is integrated out. Using the procedure described in the previous section (see Eqs. (3.45) and (3.46)), one easily finds

$$V_H = \frac{|\tilde{X}|^2}{2M_{GS}^2} + \cdots\,, \tag{3.49}$$

and the resulting effective Kähler potential takes the form

$$K_{\rm eff} = |\tilde{X}|^2 - \frac{1}{2}\frac{|\tilde{X}|^4}{M_{GS}^2} + \cdots\,, \tag{3.50}$$

where the ellipses denote the terms involving Φ, Φ^c. Then the scalar potential of $|X|$ at tree level includes a quartic term originating from the quartic term in $K_{\rm eff}$:

$$(V_{\rm SUGRA}+V_{\rm lift})|_{\rm tree} = \frac{1}{2}\frac{m_{3/2}^2}{M_{GS}^2}|\tilde{X}|^4 + \cdots\,. \tag{3.51}$$

There are also radiative corrections to the scalar potential for $|X|$, in particular the anomaly mediated soft scalar mass associated with the Yukawa coupling $y_\Phi X\Phi\Phi^c$, which is given by

$$\Delta V = N_\Phi y_\Phi^2\Big(5(5N_\Phi+2)y_\Phi^2 - 16g_3^2 - 6g_2^2 - 2g_1^2\Big)\left(\frac{m_{3/2}}{16\pi^2}\right)^2|\tilde{X}|^2, \tag{3.52}$$

where N_Φ is the number of $\Phi+\Phi^c$, and g_a are the SM gauge couplings at the scale $y_\Phi|X|$. In order for X to develop nonzero VEV, we need

$$5(5N_\Phi+2)y_\Phi^2 < 16g_3^2 + 6g_2^2 + 2g_1^2 \tag{3.53}$$

at the scale $y_\Phi|X|$. Then, the VEV of $|\tilde{X}|$ is determined as

$$\langle|\tilde{X}|\rangle = \Big(N_\Phi y_\Phi^2\Big(16g_3^2+6g_2^2+2g_1^2-5(5N_\Phi+2)y_\Phi^2\Big)\Big)^{1/2}\frac{M_{GS}}{16\pi^2}, \tag{3.54}$$

for which

$$R_1 = \frac{M_{GS}^2}{|\tilde{X}|^2} = \frac{(16\pi^2)^2}{N_\Phi y_\Phi^2(16g_3^2+6g_2^2+2g_1^2-5(5N_\Phi+2)y_\Phi^2)}. \tag{3.55}$$

In fact, this model possesses an anomalous global Peccei-Quinn (PQ) symmetry

$$U(1)_{PQ}: \quad \tilde{\phi}_i \to e^{q_i \alpha} \tilde{\phi}_i \tag{3.56}$$

which can solve the strong CP problem through the axion mechanism [31, 32]. Then the phase degree of freedom of $\tilde{X}$ can be identified as the QCD axion with a decay constant $v_{PQ} = \langle |\tilde{X}| \rangle$ which is constrained as $10^9\text{GeV} \lesssim v_{PQ} \lesssim 10^{12}\text{GeV}$. The axion scale of the model can be in this range if

$$10^{-5} \lesssim y_\Phi \lesssim 10^{-2}. \tag{3.57}$$

On the other hand, for this range of y_Φ, we have

$$R_1 \gg (8\pi^2)^2, \quad R_2 = 1 + \left(\frac{\partial_X W}{X^* W}\right)^* \simeq 1. \tag{3.58}$$

Applying this result to

$$F^T : \frac{F^X}{X} : m_{3/2}^* : g_A \sqrt{D_A} \simeq \frac{\delta_{GS}}{2R_1} : -1 : \frac{1}{R_2} : \frac{1}{\sqrt{R_1 + 1}}, \tag{3.59}$$

we find the modulus, gauge, anomaly and D-term mediated soft masses are estimated as

$$m_{\text{soft}}(\text{GM}) \sim m_{\text{soft}}(\text{AM}) \gg m_{\text{soft}}(\text{D}) \gg m_{\text{soft}}(\text{MM}), \tag{3.60}$$

and therefore soft masses in this model are determined by mixed gauge-anomaly mediation [33, 34]. The radial scalar and fermion components of $\tilde{X}$ correspond to the saxion s and the axino $\tilde{a}$, respectively. Their masses are given by

$$m_s \sim \frac{m_{3/2}}{\sqrt{R_1}}, \quad m_{\tilde{a}} \sim \frac{m_{3/2}}{R_1}, \tag{3.61}$$

which suggest that the axino is the LSP in this model.

3.4 Models with non-renormalizable superpotential

In this subsection, we consider a model with additional $U(1)_A$-charged matter Y with $q_Y > 0$, which can cancel the D-term contribution from X. Then there are two D-flat directions in the model, parameterized by $Xe^{-2T/\delta_{GS}}$ and $X^{q_Y} Y$. The superpotential is allowed to contain these two $U(1)_A$-invariant holomorphic operators. Here we discuss only the case when $q_Y = -3q_X = 3$, and the superpotential contains $X^3 Y$, but no nonperturbative term involving $Xe^{-2T/\delta_{GS}}$.

Then the Kähler potential and superpotential of the model are given by

$$\begin{aligned} K &= K_0(t - \delta_{GS} V_A) + X^* e^{-2V_A} X + Y^* e^{6V_A} Y, \\ W &= \omega_0 + \lambda \frac{X^3 Y}{M_{Pl}} + y_\Phi X \Phi \Phi^c, \end{aligned} \tag{3.62}$$

where the last term in the superpotential is not crucial for the stabilization of D-flat directions, but is introduced for the gauge mediation of SUSY breaking by F^X. A key assumption on the model is that the Kähler potential of the GS modulus admits a point with vanishing FI term:

$$K_0'(t = t_0) = 0. \tag{3.63}$$

We also assume for simplicity that the matter Kähler metric of X and Y are independent of the GS modulus, however our results equally apply to the case with moduli-dependent matter Kähler metric.

Now the $U(1)_A$ D-term is given by

$$g_A^2 D_A = \xi_{FI} + |X|^2 - 3|Y|^2, \tag{3.64}$$

where $\xi_{FI} = \delta_{GS} K_0'/2$. In section 2 and also in the previous subsections, we were focusing on the case that D-flatness is achieved through the cancellation between ξ_{FI} and $|X|^2$, yielding $|\xi_{FI}| \simeq |X|^2 \gg g_A^2 D_A$. Note that X was defined as the $U(1)_A$-charged matter field with the largest VEV. In fact, this model realizes a different scenario with

$$|X|^2 \sim |Y|^2 \gg \xi_{FI}, \tag{3.65}$$

and as a consequence does not obey the relations in (2.16) except for the last one.

For the above Kähler potential and superpotential, one of the stationary conditions for scalar potential takes the form

$$\begin{aligned} \partial_t \left(V_{\rm SUGRA} + V_{\rm lift}\right) = & \left(V_F + \frac{2}{3}V_{\rm lift} + \left(2 - \frac{K_0' K_0'''}{K_0''^2}\right) e^K |W|^2\right) K_0' \\ & + \frac{\delta_{GS}}{2} K_0'' g_A^2 D_A + (\partial_t \ln g_A^2) \frac{g_A^2}{2} D_A^2 = 0, \end{aligned} \tag{3.66}$$

which is satisfied by the D-flat direction given by

$$t = t_0, \quad |X|^2 = 3|Y|^2. \tag{3.67}$$

In the region with $|X|^2 \ll M_{GS}^2 = \delta_{GS}^2 K_0''/4$, the scalar potential along this D-flat direction is written as

$$V_{\rm TOT} \simeq e^K \left(4|\lambda|^2 |X|^6 - \frac{2}{\sqrt{3}} |\lambda\omega_0| |X|^4 - 3|\omega_0|^2\right), \tag{3.68}$$

for $\text{Arg}(X^3 Y)$ fixed at $\text{Arg}(\lambda^* \omega_0) + \pi$ by the minimization condition. Here we have set $M_{Pl} = 1$, and neglected small corrections suppressed by $|X|^2/M_{Pl}^2$. This determines the VEVs of X and Y as

$$|X|^2 = 3|Y|^2 \simeq \frac{\sqrt{3}}{9|\lambda|} m_{3/2} M_{Pl}. \tag{3.69}$$

The resulting SUSY breaking auxiliary components are given by

$$\frac{F^X}{X} = \frac{F^Y}{Y} \simeq -\frac{2}{3} m_{3/2}, \quad F^T = D_A = 0, \tag{3.70}$$

so soft masses are determined by mixed gauge-anomaly mediation:

$$m_{\rm soft}({\rm GM}) \sim m_{\rm soft}({\rm AM}) \gg m_{\rm soft}({\rm MM}),\, m_{\rm soft}({\rm D}). \tag{3.71}$$

For X stabilized at $|X| \ll M_{GS}$, the longitudinal component of the massive $U(1)_A$ vector superfield comes mostly from T. The non-renormalizable superpotential term provides masses to X and Y, which include two radial scalars $s_{1,2}$, the massive angular scalar a_h, and two fermions $\tilde{a}_{1,2}$:

$$\begin{aligned} &m_{s_1} \simeq \frac{1}{3} m_{3/2}, \quad m_{s_2} \simeq \frac{\sqrt{3}}{3} m_{3/2}, \quad m_{a_h} \simeq \frac{\sqrt{6}}{3} m_{3/2}, \\ &m_{\tilde{a}_1} \simeq 0.1 m_{3/2}, \quad m_{\tilde{a}_2} \simeq 0.8 m_{3/2}. \end{aligned} \tag{3.72}$$

One combination of Arg(X) and Arg(Y) remains unfixed.

We note that the above model possesses a PQ symmetry:

$$U(1)_{PQ}: \quad \phi_i \to e^{q_i \alpha} \phi_i \tag{3.73}$$

which is spontaneously broken by the VEVs of X and Y. The corresponding axion scale v_{PQ} can take naturally an intermediate scale value

$$v_{PQ} \sim \sqrt{m_{3/2} M_{Pl}} \sim 10^{11} {\rm GeV}, \tag{3.74}$$

when $m_{3/2} \sim 10^4$GeV for which $m_{\rm soft} \sim \frac{g^2}{8\pi^2} m_{3/2}$ has a weak scale size.

3.5 KKLT with D-term stabilization

In this subsection, we discuss a class of models in which multiple number of moduli, including the GS modulus, participate in SUSY breaking. As a concrete example, we consider a variant of KKLT scenario with multiple number of Kähler moduli, in which the visible sector Kähler modulus T is stabilized by the D-term potential of $U(1)_A$, while the other Kähler moduli are stabilized by nonperturbative superpotential as in the original KKLT scenario [26].

The motivation for this variant is the observation that instantons wrapping the visible sector 4-cycle in KKLT setup have SM-charged zero modes of chiral fermions, and as a result the corresponding nonperturbative superpotential of T should involve a gauge invariant product of SM-charged chiral matter superfields [35]. This would effectively make the nonperturbative superpotential of T vanish, and then one needs other mechanism to stabilize the visible sector Kähler modulus. As we will see, if the model contains an anomalous $U(1)_A$ with T being the GS modulus, and the moduli Kähler potential admits a limit in which the FI term has a value far below M_{Pl}^2, all Kähler moduli can be stabilized even in the absence of nonperturbative superpotential of T.

Since the generalization to the case with more moduli is straightforward, here we consider a simple case with two Kähler moduli T and T', where $t = T + T^*$ corresponds to the volume of 4-cycle supporting the visible sector, while $t' = T' + T'^*$ stands for hidden sector 4-cycle. We assume that there exists a nonperturbative superpotential of T'

generated by stringy instanton wrapping the hidden cycle, while no nonperturbative term of T due to the fermion zero modes on the visible sector cycle. In Type IIB string theory for KKLT compactification, the Kähler potential of $T_\alpha = \{T, T'\}$ takes the no-scale form at the leading order in the α' and string loop expansion, so we consider a no-scale Kähler potential obeying

$$K_0(\gamma t_A, \gamma t') = K_0(t_A, t') - 3\ln\gamma \tag{3.75}$$

for arbitrary real constant γ, where $t_A = t - \delta_{GS}V_A$. We further assume that there exists a solution in the moduli space with vanishing FI term:

$$\partial_t K_0 = 0, \tag{3.76}$$

and explore the (local) minimum of the scalar potential near this solution[6].

In the absence of nonperturbative effects breaking the axionic shift symmetry $T \to T + i\beta$ ($\beta =$ real constant), the $U(1)_A$ symmetry leads to an anomalous global symmetry

$$U(1)_{PQ}: \quad \phi_i \to e^{i\alpha}\phi_i, \tag{3.77}$$

which can be identified as a PQ symmetry solving the strong CP problem. Then, this PQ symmetry should be broken spontaneously by the VEV of SM singlet but $U(1)_A$ charged matter fields at a scale between 10^9GeV and 10^{12}GeV. As for those matter fields, we consider the example of the previous subsection. Then the Kähler potential and superpotential of the model are given by

$$\begin{aligned} K &= K_0(t_A, t') + X^* e^{-2V_A} X + Y^* e^{6V_A} Y, \\ W &= \omega_0 + A e^{-aT'} + \lambda \frac{X^3 Y}{M_{Pl}} + y_\Phi X \Phi \Phi^c. \end{aligned} \tag{3.78}$$

Here we assume for simplicity that the matter Kähler metric of X and Y are independent of moduli, however our results equally apply to the case with moduli-dependent matter Kähler metric. For a no-scale moduli Kähler potential, we have

$$K_0^{\alpha\bar\beta}\partial_{\bar\beta}K_0 = -(T^\alpha + T^{\alpha *}), \quad K_0^{\alpha\bar\beta}(\partial_\alpha K_0)\partial_{\bar\beta}K_0 = 3, \tag{3.79}$$

and

$$2(\partial_t K_0)\partial_t K_0^{T\bar T'} + (\partial_{t'}K_0)\partial_t K_0^{T'\bar T'} = 0, \tag{3.80}$$

where $K_0^{\alpha\bar\beta}$ is the inverse of the moduli Kähler metric $K_{0\alpha\bar\beta} = \partial_\alpha\partial_{\bar\beta}K_0$. With these relations, one can find

$$\begin{aligned} \partial_t\left(V_{\rm SUGRA} + V_{\rm lift}\right) = &\left(V_F + \frac{2}{3}V_{\rm lift} - 2\frac{\partial_t K_0^{TT'}}{\partial_{t'}K_0}e^K|W_{T'}|^2\right)\partial_t K_0 \\ &+ \frac{\delta_{GS}}{2}(\partial_t^2 K_0)g_A^2 D_A + (\partial_t \ln g_A^2)\frac{g_A^2}{2}D_A^2, \end{aligned} \tag{3.81}$$

[6] A simple example of such Kähler potential is the one discussed in [23]: $K_0 = -\ln((t_1 - \delta_{GS}V_A)(t_2 + \delta_{GS}V_A)^2 + \frac{1}{6}(t_2 + \delta_{GS}V_A)^3)$, where $t = t_1 - t_2$, $t' = t_1 + t_2$, and the Kähler cone is defined by $t_{1,2} > 0$. Note that $\xi_{FI} = \delta_{GS}(\partial_1 K_0 - \partial_2 K_0)/2 = 0$ on the wall in the Kähler cone defined by $t_2 = 4t_1$.

where

$$D_A = \frac{\delta_{GS}}{2}\partial_t K_0 + |X|^2 - 3|Y|^2. \tag{3.82}$$

The stationary condition $\partial_t(V_{\rm SUGRA} + V_{\rm lift}) = 0$ is thus satisfied by the field configuration satisfying

$$\partial_t K_0 = 0, \quad |X|^2 = 3|Y|^2, \tag{3.83}$$

for which $D_A = 0$. On the other hand, as in the original KKLT scenario, the stationary condition $\partial_{t'}(V_{\rm SUGRA} + V_{\rm lift}) = 0$ leads to

$$D_{T'}W = \frac{(\partial_{t'}K_0)W}{\ln(M_{Pl}/m_{3/2})}\left(1 + \mathcal{O}\left(\frac{1}{\ln(M_{Pl}/m_{3/2})}\right)\right). \tag{3.84}$$

We then find t and t' are fixed by the conditions

$$\partial_t K_0 = 0, \quad \partial_{t'}K_0 = \frac{aAe^{-aT'}}{w_0}\left(1 + \mathcal{O}\left(\frac{1}{\ln(M_{Pl}/m_{3/2})}\right)\right), \tag{3.85}$$

and X and Y are fixed at

$$|X|^2 = 3|Y|^2 \simeq \frac{\sqrt{3}}{9|\lambda|}m_{3/2}M_{Pl}, \quad {\rm Arg}(X^3Y) = {\rm Arg}(\lambda^*\omega_0) + \pi. \tag{3.86}$$

At this minimum, the SUSY breaking auxiliary components have VEVs as

$$\begin{aligned} \frac{F^X}{X} &= \frac{F^Y}{Y} \simeq -\frac{2}{3}m_{3/2}, \quad D_A = 0, \\ \frac{F^T}{T+T^*} &\simeq \frac{F^{T'}}{T'+T'^*} \simeq \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}, \end{aligned} \tag{3.87}$$

where the relation between F^T and $F^{T'}$ is derived from the no-scale relation $K_0^{\alpha\bar{\beta}}\partial_{\bar{\beta}}K_0 = -(T^\alpha + T^{\alpha *})$. Therefore, this variant of KKLT setup gives rise to a mixed modulus-gauge-anomaly mediation[7]:

$$m_{\rm soft}({\rm MM}) \sim m_{\rm soft}({\rm GM}) \sim m_{\rm soft}({\rm AM}) \gg m_{\rm soft}({\rm D}), \tag{3.88}$$

which was dubbed as deflected (or axionic) mirage mediation [37, 38]. As in the model discussed in the previous subsection, the Goldstone boson from spontaneously broken $U(1)_{PQ}$ can play the role of the QCD axion with the axion scale given by $v_{PQ} \sim \sqrt{m_{3/2}M_{Pl}}$.

[7] It is worth noting that, if it is not charged under $U(1)_A$, T can alternatively be stabilized by the uplifting potential [36]. This gives a similar pattern of SUSY breaking, while the axion scale is around the GUT or string scale.

4. Conclusion

There can be various sources of SUSY breaking in models with anomalous $U(1)$ gauge symmetry: the $U(1)$ D-term, the F-components of the Green-Schwarz modulus T and the chiral matter X introduced to cancel the Fayet-Iliopoulos term, and also the SUGRA auxiliary component of the order of $m_{3/2}$. Then the visible sector soft masses generically receive a modulus-mediated contribution of the order of F^T and a D-term contribution of the order of $\sqrt{D_A}$ as well as the anomaly-mediated contribution of the order of $\frac{g^2}{8\pi^2}m_{3/2}$. Most of the known (semi) realistic string models with $U(1)_A$ include also exotic SM gauge charged matter fields Φ, Φ^c which become massive through the Yukawa coupling to X, and therefore play the role of messenger for the gauge mediation of SUSY breaking by F^X. In such case, soft masses also receive a gauge-mediated contribution of the order of $\frac{g^2}{8\pi^2}\frac{F^X}{X}$.

In this paper, we have examined the relative strength of these modulus, gauge, anomaly and D-term mediations in a simple class of models, and find that various different mixed mediation scenarios can be realized depending upon the characteristics of the D-flat directions and how those D-flat directions are stabilized. A key quantity which would determine the characteristics of the D-flat directions is the ratio between the Stückelberg contribution to the $U(1)_A$ gauge boson mass-square and the Fayet-Iliopoulos term. Our results suggest that although its accurate structure is quite model-dependent, it is quite common that soft terms in models with $U(1)_A$ are not dominated by a single mediation, but determined by a proper mixture of the moduli, gauge, anomaly and D-term mediations.

Acknowledgement

We thank A. Uranga for useful discussions. KC is supported by the KRF Grants funded by the Korean Government (KRF-2008-314-C00064 and KRF-2007-341-C00010) and the KOSEF Grant funded by the Korean Government (No. 2009-0080844). KSJ is supported by the JSPS Grant-in-Aid 21-09224 for JSPS Fellows. KO is supported by the Grant-in-aid for Scientific Research No.21740155 and No.18071001 from MEXT of Japan.

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