# Code Acquisition for DS/SS Communications in Non-Gaussian Impulsive Channels

Seokho Yoon, *Member, IEEE*, Iickho Song, *Senior Member, IEEE*, and Sun Yong Kim, *Senior Member, IEEE*

***Abstract*—In this letter, we propose a new detector for code acquisition systems in non-Gaussian noise channels. Modeling the acquisition problem as a hypothesis testing problem, a detector is derived for non-Gaussian, symmetric $\alpha$-stable noise, based on the locally optimum detection technique. Numerical results show that the proposed detector can offer robustness and substantial performance improvement over the conventional schemes in non-Gaussian channels.**

***Index Terms*—Code acquisition, locally optimum (LO) detection, non-Gaussian impulsive channel, spread-spectrum system.**

## I. Introduction

RAPID establishment of code synchronization (acquisition) is an important technical issue in direct-sequence spread-spectrum (DS/SS) systems. The basic unit in an acquisition system is a decision-making device or a detector. Most schemes proposed for rapid code acquisition employ the *squared-sum (SS) detector* with a noncoherent in-phase/quadrature-phase (I-Q) correlator [1]. This is because the SS detector is optimized for Gaussian noise channels, and the statistics due to fading in acquisition systems can usually be modeled as Gaussian processes by virtue of the central limit theorem. However, many real-life noises, such as atmospheric and man-made noise arising in indoor/outdoor mobile communication systems, is experimentally known to be non-Gaussian or *impulsive* [2]–[4]. It is clear that the acquisition system using the SS detector is inadequate in such non-Gaussian environments.

In this letter, we propose to employ locally optimum (LO) detectors [5], [6] for code acquisition systems in non-Gaussian channels. The motivation of using the LO detector is as follows. First, since an LO detector has the maximum slope of its power function when the signal-to-noise ratio (SNR) approaches zero, it is expected to have good performance when the SNR is low. Second, an LO detector can always be obtained and is usually much easier to implement than other detectors, including uniformly most powerful and optimum detectors.

Paper approved by M. Brandt-Pearce, the Editor for Modulation and Signal Design of the IEEE Communications Society. Manuscript received January 11, 2002; revised September 21, 2002 and April 10, 2003. This work was supported by Korea Science and Engineering Foundation (KOSEF) under Grant R01-2000-000-00259-0.

S. Yoon is with the School of Information and Communication Engineering, Sungkyunkwan University, Suwon, Kyunggi-Do, 440-746, Korea.

I. Song is with the Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 305-701, Korea.

S. Y. Kim is with the Department of Electronics Engineering, Konkuk University, Seoul 143-701, Korea (e-mail: kimsy@konkuk.ac.kr).

Digital Object Identifier 10.1109/TCOMM.2003.822694

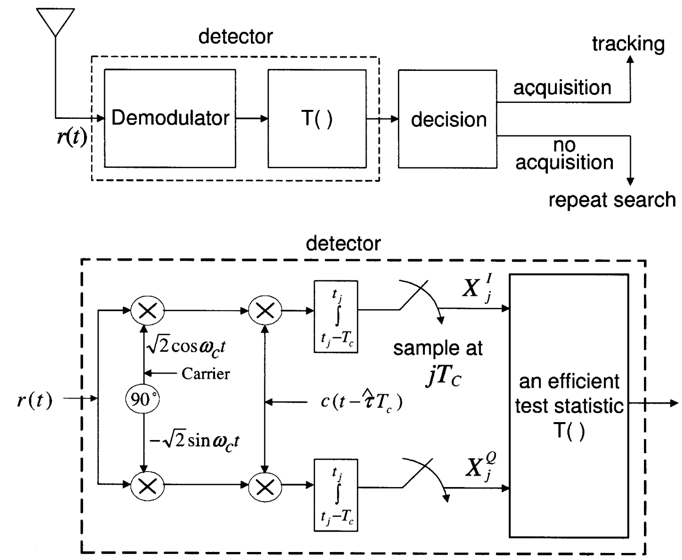


Fig. 1. Structure of PN code-acquisition systems.

## II. Observation Model

In a DS/SS system, the received signal can be expressed as

$$r(t) = \sqrt{2E}d(t-\tau T_c)c(t-\tau T_c)\cos(\omega_c t+\phi)+n(t). \quad (1)$$

In (1), $E$ is the energy per chip; $d(t)$ is the data sequence waveform; $T_c$ is the chip duration; $c(t) = \sum_{i=-\infty}^{\infty} c_i p_{T_c}(t-iT_c)$ with $c_i \in \{-1,+1\}$ the $i$th chip of a pseudonoise (PN) code sequence of period $L$ and $p_{T_c}(t)$ the PN code waveform defined as a unit rectangular pulse over $[0, T_c]$; $\tau$ is the time delay normalized to $T_c$; $\omega_c$ is the carrier angular frequency; $\phi$ is the phase distributed uniformly over $[0, 2\pi)$; and $n(t)$ is the ambient white non-Gaussian channel noise. In this letter, it is assumed that there is a preamble for acquisition so that no data modulation is present during acquisition (i.e., $d(t)=1$).

A typical structure of the PN code acquisition system with a noncoherent I-Q correlator is shown in Fig. 1. We consider the serial search scheme with single dwell [1]. The $j$th sampled I-Q components $X_j^I$ and $X_j^Q$ can be obtained as, for $j = 1, 2, \cdots, N$

$$X_j^I = \int_{t_j-T_c}^{t_j} r(t)c(t-\hat{\tau}T_c)\sqrt{2}\cos(\omega_c t)dt \quad (2)$$

$$X_j^Q = \int_{t_j-T_c}^{t_j} r(t)c(t-\hat{\tau}T_c)(-\sqrt{2})\sin(\omega_c t)dt \quad (3)$$

respectively, where $N$ is the correlation length, $\hat{\tau}$ is the time delay (normalized to the chip duration $T_c$) of the locally generated PN code, and $t_j = t_0 + jT_c$. Here, $t_0$ is an initial time. A test statistic is evaluated with the vectors $X^I = (X_1^I, X_2^I, \ldots, X_N^I)$ and $X^Q = (X_1^Q, X_2^Q, \ldots, X_N^Q)$, and then compared with a threshold. In the conventional systems, the test statistic is, for example, the SS statistic $T_{\text{SS}}(X^I, X^Q) = \left(\sum_{i=1}^N X_i^I\right)^2 + \left(\sum_{i=1}^N X_i^Q\right)^2$.

We can regard the PN code-acquisition problem as a hypothesis testing problem: given $X^I$ and $X^Q$, a decision is to be made between the null hypothesis $H$ and alternative hypothesis $K$, where $H : |\tau - \hat{\tau}| \geq 1$ and $K : |\tau - \hat{\tau}| < 1$. Under $K$, each sampled correlation value between the locally generated and received PN codes is $\sqrt{E}(1 - |\delta|)$, where $\delta$ is the residual shift (normalized to $T_c$) between the two PN codes, with the value ranging in the interval $(-1, +1)$. For simplicity, we assume that the system is chip synchronous (that is, $\delta = 0$) as in most other studies (an investigation of the effect of a nonzero $\delta$ has been considered in [7]). Thus, each sampled correlation value is $\sqrt{E}$. On the other hand, each sampled correlation value is $+1$ or $-1$ with equal probability, and the mean value of the sampled correlation is 0 under $H$. From these results and (2) and (3), we can alternatively express $H$ and $K$ as

$$H : \left(X_j^I = N_j^I, X_j^Q = N_j^Q\right), \quad j = 1, 2, \cdots, N \tag{4}$$

$$K : \left(X_j^I = \theta\cos\phi + N_j^I, X_j^Q = \theta\sin\phi + N_j^Q\right), \quad j = 1, 2, \cdots, N \tag{5}$$

or simply as

$$H : \theta = 0 \tag{6}$$

$$K : \theta > 0. \tag{7}$$

In (4)–(7), $\theta = \sqrt{E}$ is the signal strength parameter, and $N^I = (N_1^I, N_2^I, \ldots, N_N^I)$ and $N^Q = (N_1^Q, N_2^Q, \ldots, N_N^Q)$ are the I-Q noise sample vectors, respectively. We model $N^I$ and $N^Q$ with symmetric $\alpha$-stable (S$\alpha$S) distributions, which have been proved to be very useful in modeling non-Gaussian noise [8], [9]. The joint probability density function (pdf) of $N_j^I$ and $N_j^Q$ can be most conveniently defined by the inverse Fourier transform (IFT) of their characteristic function [8]

$$f_{\alpha,\gamma,\beta_1,\beta_2}(x_1, x_2) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[i(\beta_1\omega_1 + \beta_2\omega_2) - \gamma\left(\omega_1^2 + \omega_2^2\right)^{\frac{\alpha}{2}}\right] \mathrm{e}^{-i(x_1\omega_1 + x_2\omega_2)} d\omega_1 d\omega_2 \tag{8}$$

where the parameters $\alpha$ and $\gamma$ are termed the *characteristic exponent* and *dispersion*, respectively, and $\beta_1$ and $\beta_2$ are *location parameters*. The characteristic exponent $\alpha$ ranges in the interval $0 < \alpha \leq 2$ with a smaller $\alpha$ indicating heavier tails (more impulsive behavior). The dispersion $\gamma$ is a positive constant relating to the spread of the pdf. The location parameters $\beta_1$ and $\beta_2$ are the symmetry axes of the marginal S$\alpha$S distributions: we assume $\beta_1 = \beta_2 = 0$, without loss of generality. Unfortunately, no closed-form expression exists for (8), except for the special cases of $\alpha = 1$ and $\alpha = 2$

$$f_{\alpha,\gamma}(x_1, x_2) = \begin{cases} \frac{\gamma}{2\pi\left(x_1^2 + x_2^2 + \gamma^2\right)^{\frac{3}{2}}}, & \text{for } \alpha = 1 \\ \frac{1}{4\pi\gamma}\exp\left(-\frac{x_1^2 + x_2^2}{4\gamma}\right), & \text{for } \alpha = 2. \end{cases} \tag{9}$$

Because of such a lack of closed-form expressions, we concentrate on the case $\alpha = 1$: nonetheless, we shall see in Section IV that the system obtained for $\alpha = 1$ is not only robust to the variation of $\alpha$, but also outperforms the conventional system for most values of $\alpha$.

From (9), the joint pdf $f(\theta; j)$ of $X_j^I$ and $X_j^Q$ as a function of $\theta$ is

$$f(\theta; j) = \frac{\gamma}{2\pi\left[\left(x_j^I - \theta\cos\phi\right)^2 + \left(x_j^Q - \theta\sin\phi\right)^2 + \gamma^2\right]^{\frac{3}{2}}}. \tag{10}$$

The joint pdf of the $2N$ sampled in-phase and quadrature observations $\{X_i^I, X_i^Q\}$, $i = 1, \cdots, N$, is then $f_{X^I,X^Q}(x^I, x^Q) = E_\phi\{\prod_{j=1}^N f(\theta; j)\}$ under the assumption that the samples $(X_j^I, X_j^Q)$ of the bivariate noise process form a sequence of independent random vectors for given $\phi$, where $E_\phi$ denotes the expectation over $\phi$.

## III. LO Detector Test Statistic

The test statistic of an LO detector is obtained from [5], [6]

$$T_{\text{LO}}(X^I, X^Q) = \frac{1}{f_{X^I,X^Q}(x^I, x^Q)|_{\theta=0}} \times \left.\frac{d^\nu f_{X^I,X^Q}(x^I, x^Q)}{d\theta^\nu}\right|_{\theta=0} \tag{11}$$

where $\nu$ is the order of the first nonzero derivative of $f_{X^I,X^Q}(x^I, x^Q)$ at $\theta = 0$. It can be shown, as in the Appendix, that the test statistic of the LO detector is

$$T_{\text{LO}}(X^I, X^Q) = \sum_{i=1}^N \left\{h\left(x_i^I\right) + h\left(x_i^Q\right)\right\} + \sum_{i=1}^N \sum_{j\neq i, j=1}^N \left\{g\left(x_i^I\right)g\left(x_j^I\right) + g\left(x_i^Q\right)g\left(x_j^Q\right)\right\} \tag{12}$$

where $h(x_i^k) = (\partial^2 f(0; i)/\partial(x_i^k)^2)/f(0; i)$ and $g(x_i^k) = -(\partial f(0; i)/\partial x_i^k)/f(0; i)$ for $k = I, Q$. It is noteworthy that we can obtain the conventional SS detector by substituting $h(x) = x^2/4\gamma^2 - 1/2\gamma$ and $g(x) = x/2\gamma$ into (12) when $\alpha = 2$ (Gaussian).

After the appropriate substitutions of $h(x_i^{I(Q)}) = (12x_i^{I(Q)} - 3x_i^{Q(I)} - 3\gamma^2)/\rho^2$ and $g(x_i^{I(Q)}) = 3x_i^{I(Q)}/\rho$ into (12) and some manipulations, where $\rho = (x_i^I)^2 + (x_i^Q)^2 + \gamma^2$, we have

$$T_{\text{LO}}(X^I, X^Q) = 3\left[\left(\sum_{i=1}^N \frac{X_i^I}{\left(X_i^I\right)^2 + \left(X_i^Q\right)^2 + \gamma^2}\right)^2 + \left(\sum_{i=1}^N \frac{X_i^Q}{\left(X_i^I\right)^2 + \left(X_i^Q\right)^2 + \gamma^2}\right)^2\right] - 2\left(\sum_{i=1}^N \frac{\gamma^2}{\left\{\left(X_i^I\right)^2 + \left(X_i^Q\right)^2 + \gamma^2\right\}^2}\right). \tag{13}$$

The proposed test statistic (13) contains terms which take the form of normalized SS, where the samples $(X_i^I, X_i^Q)$ are normalized by their squared magnitude with offset $\gamma^2$ prior to the SS detector. The normalization operation can prohibit the erroneous increase of the SS due to the noise samples of large magnitude, which are more frequent in an impulsive non-Gaussian noise environment than in a Gaussian noise environment. A similar idea can be found in [10], where the rate of the impulsive jamming has the order of correlation rate, and the normalization is accomplished correlation by correlation.

## IV. Simulation Results

The performance of the proposed and conventional schemes is simulated with a PN code of $L = 1023$ chips, generated from an $m$ sequence with the primitive polynomial $1+z^3+z^{10}$ when $N = 64$. Noise samples $\{N_j^I\}$ and $\{N_j^Q\}$ are generated from

$$\gamma^{\frac{1}{\alpha}} \frac{\sin(\alpha A)}{(\cos A)^{\frac{1}{\alpha}}} \left( \frac{\cos[(1-\alpha)A]}{B} \right)^{\frac{1-\alpha}{\alpha}} \tag{14}$$

where $A$ is uniform on $(-\pi/2, \pi/2)$ and $B$ is exponential with mean 1 [8]. Since S$\alpha$S noise with $\alpha < 2$ has no finite variance, making the standard SNR measure inconsistent, a new scale parameter is used to indicate the strength of the S$\alpha$S noise [11].

The new SNR, which provides a mathematically and conceptually valid characterization of the relative strength between the information-bearing signal and channel noise with infinite variance, is defined as

$$\frac{1}{2C_g} \left( \frac{\sqrt{E}}{S_0} \right)^2 \tag{15}$$

where $S_0 = ((C_g \gamma)^{1/\alpha} / C_g)$ with $C_g \simeq 1.78$ the exponential of the Euler's constant $(\lim_{n\to\infty}(\sum_{k=1}^{n}(1/k) - \ln n))$. The normalizing constant $2C_g$ ensures that the definition of the SNR coincides with that of the standard SNR in the Gaussian case. Because $\gamma$ can be easily and exactly estimated using only the sample mean and variance of observations $X^I$ or $X^Q$ [12], it may be regarded as a known value: in our simulations, we assume $\gamma = 1$.

Fig. 2 shows the detection probabilities of the conventional and proposed schemes for some values of $\alpha$ when the false alarm probability $P_F = 10^{-2}$. Fig. 3 shows the detection probabilities plotted as a function of $\alpha$ for some values of SNR/chip when $P_F = 10^{-2}$. We can clearly see that the proposed detector significantly outperforms the conventional detector for most values of $\alpha$. Only when $\alpha$ is close to two (e.g., $\alpha = 2, 1.95$), the conventional detector performs slightly better. Another important observation is that the performance of the proposed detector is robust to the variation of the value of $\alpha$. In addition, the proposed detector performs better as the impulsiveness becomes higher. This can be explained as follows. Impulses which have large amplitudes are clipped in the proposed detector. Therefore, the effective noise variance at the output of the proposed detector is smaller than the total input noise variance.

In interference-limited code-division multiple-access (CDMA) systems, multiple-access interference (MAI) is often the main factor limiting the performance of CDMA systems. The Gaussian assumption for MAI is appropriate for the reverse link in cellular CDMA, where power control prevents any one mobile from dominating. On the other hand, for CDMA network systems without power control (e.g., peer-to-peer packet radio network where there is no central base station), MAI is well modeled as non-Gaussian S$\alpha$S distributions [13], [14]. Thus, noting that S$\alpha$S distributions include Gaussian distributions as a special case, the analysis and results in this letter should be applicable to realistic interference-limited CDMA systems also.

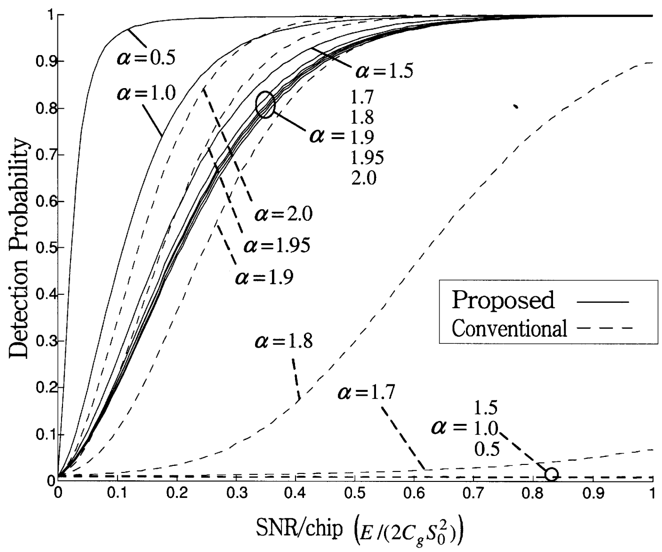


Fig. 2. Detection probability of the conventional and proposed schemes for various values of $\alpha$ when $P_F = 10^{-2}$, $L = 1023$, and $N = 64$.

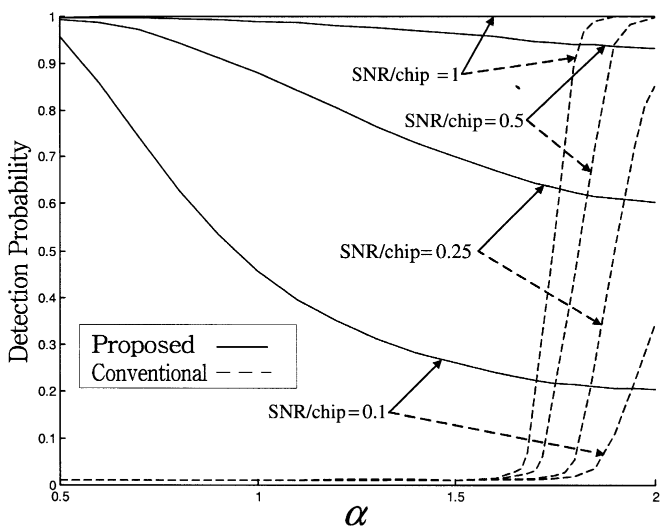


Fig. 3. Detection probability of the conventional and proposed schemes plotted as a function of $\alpha$ for some values of SNR/chip when $P_F = 10^{-2}$, $L = 1023$, and $N = 64$.

## Appendix
## Derivation of the LO Detector Test Statistic

The first derivative of $f_{X^I,X^Q}(x^I, x^Q)$ at $\theta = 0$ is

$$\left. \frac{df_{X^I,X^Q}(x^I, x^Q)}{d\theta} \right|_{\theta=0} = E_\phi \left\{ \sum_{i=1}^{N} f'(0;i) \prod_{j\neq i, j=1}^{N} f(0;j) \right\} \tag{16}$$

vanishes identically since $E_\phi\{f'(0;i)\} = E_\phi\{-\cos\phi(\partial f(0;i)/\partial x_i^I) - \sin\phi(\partial f(0;i)/\partial x_i^Q)\} = 0$. We thus obtain the second derivative of $f_{X^I,X^Q}(x^I,x^Q)$ at $\theta = 0$

$$\left.\frac{d^2 f_{X^I,X^Q}(x^I,x^Q)}{d\theta^2}\right|_{\theta=0} = E_\phi\left\{\sum_{i=1}^{N}\left[f''(0;i)\prod_{j\neq i,j=1}^{N} f(0;j) + f'(0;i)\sum_{j\neq i,j=1}^{N} f'(0;j)\prod_{k\neq j,k\neq i,k=1}^{N} f(0;k)\right]\right\}. \quad (17)$$

Noting that

$$f''(0;i) = \cos^2\phi\frac{\partial^2 f(0;i)}{\partial {x_i^I}^2} + \sin^2\phi\frac{\partial^2 f(0;i)}{\partial {x_i^Q}^2} + \sin\phi\cos\phi\frac{\partial^2 f(0;i)}{\partial x_i^I\partial x_i^Q} + \sin\phi\cos\phi\frac{\partial^2 f(0;i)}{\partial x_i^Q\partial x_i^I} \quad (18)$$

the first term of (17) is

$$E_\phi\left\{\sum_{i=1}^{N} f''(0;i)\prod_{j\neq i,j=1}^{N} f(0;j)\right\} = \frac{1}{2}\sum_{i=1}^{N}\left\{h\left(x_i^I\right) + h\left(x_i^Q\right)\right\}\prod_{j=1}^{N} f(0;j) \quad (19)$$

where $h(x_i^a) = (1/f(0;i))(\partial^2 f(0;i)/\partial(x_i^a)^2$ for $a = I, Q$. Similarly, noting that

$$f'(0;i)f'(0;j) = \cos^2\phi\frac{\partial f(0;i)}{\partial x_i^I}\frac{\partial f(0;j)}{\partial x_j^I} + \sin^2\phi\frac{\partial f(0;i)}{\partial x_i^Q}\frac{\partial f(0;j)}{\partial x_j^Q} + \sin\phi\cos\phi \times\left(\frac{\partial f(0;i)}{\partial x_i^I}\frac{\partial f(0;j)}{\partial x_j^Q} + \frac{\partial f(0;i)}{\partial x_i^Q}\frac{\partial f(0;j)}{\partial x_j^I}\right) \quad (20)$$

the second term of (17) is

$$E_\phi\left\{\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N} f'(0;i)f'(0;j)\prod_{k\neq j,k\neq i,k=1}^{N} f(0;k)\right\} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i,j=1}^{N}\left\{g\left(x_i^I\right)g\left(x_j^I\right) + g\left(x_i^Q\right)g\left(x_j^Q\right)\right\} \times\prod_{k=1}^{N} f(0;k) \quad (21)$$

where $g(x_i^a) = (-(1/f(0;i))(\partial f(0;i)/\partial x_i^a)$ for $a = I, Q$.

Using (19) and (21) in (17), and then dividing the result by $f_{X^I,X^Q}(x^I,x^Q)|_{\theta=0} = \prod_{i=1}^{N} f(0;i)$, we get (12).

## REFERENCES


[1] R. R. Rick and L. B. Milstein, "Parallel acquisition in mobile DS-CDMA systems," *IEEE Trans. Commun.*, vol. 45, pp. 1466–1476, Nov. 1997.

[2] S. Nakamura, "A study of errors caused by impulsive noise and a simple estimation method for digital mobile communications," *IEEE Trans. Veh. Technol.*, vol. 45, pp. 310–317, May 1996.

[3] T. K. Blankenship and T. S. Rappaport, "Characteristics of impulsive noise in the 450-MHz band in hospitals and clinics," *IEEE Trans. Antennas Propagat.*, vol. 46, pp. 194–203, Feb. 1998.

[4] K. S. Kim, I. Song, S. I. Park, and B.-H. Chung, "Analysis and performance comparison of the trellis coded and convolutional coded DS/SSMA systems in impulsive noise environment," *Proc. IEEE—Commun.*, vol. 145, pp. 283–287, Aug. 1998.

[5] S. A. Kassam, *Signal Detection in Non-Gaussian Noise*. New York: Springer-Verlag, 1987.

[6] I. Song, J. Bae, and S. Y. Kim, *Advanced Theory of Signal Detection*. Berlin, Germany: Springer-Verlag, 2002.

[7] S. Yoon, I. Song, S. Y. Kim, and S. R. Park, "A DS/CDMA code-acquisition scheme robust to residual code phase offset variation," *IEEE Trans. Veh. Technol.*, vol. 49, pp. 2405–2418, Nov. 2000.

[8] G. Samorodnitsky and M. S. Taqqu, *Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance*. New York: Chapman & Hall, 1994.

[9] C. L. Nikias and M. Shao, *Signal Processing With $\alpha$-Stable Distributions and Applications*. New York: Wiley, 1995.

[10] K. Choi, K. Cheun, and T. Jung, "Adaptive PN code acquisition using instantaneous power-scaled detection threshold under Rayleigh fading and pulsed Gaussian noise jamming," *IEEE Trans. Commun.*, vol. 50, pp. 1232–1235, Aug. 2002.

[11] T. C. Chuah, B. S. Sharif, and O. R. Hinton, "Nonlinear decorrelator for multiuser detection in non-Gaussian impulsive environments," *Electron. Lett.*, vol. 36, pp. 920–922, May 2000.

[12] X. Ma and C. L. Nikias, "Parameter estimation and blind channel identification in impulsive signal environments," *IEEE Trans. Signal Processing*, vol. 43, pp. 2884–2897, Dec. 1995.

[13] E. S. Sousa, "Performance of a spread spectrum packet radio network link in a Poisson field of interferers," *IEEE Trans. Inform. Theory*, vol. 38, pp. 1743–1754, Nov. 1992.

[14] J. Ilow and D. Hatzinakos, "Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers," *IEEE Trans. Signal Processing*, vol. 46, pp. 1601–1611, June 1998.