# Noncoherent Constant False-Alarm Rate Schemes With Receive Diversity for Code Acquisition Under Homogeneous and Nonhomogeneous Fading Circumstances

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***Abstract*—In this paper, we address the cell averaging (CA), greatest of (GO), and smallest of (SO) constant false-alarm rate (CFAR) processors for code acquisition in a homogeneous and a nonhomogeneous environment. The performance characteristics of the CA, GO, and SO processors are analyzed and compared when receiving antenna diversity is employed in the pseudonoise code acquisition of direct-sequence code-division multiple-access systems. From the simulation results, it is observed that the GO CFAR scheme has the best performance in a nonhomogeneous environment and has almost the same performance as the CA CFAR scheme in a homogeneous environment.**

***Index Terms*—Code acquisition, constant false-alarm rate (CFAR) processor, homogeneous environment, nonhomogeneous environment, receive diversity, signal detection.**

## I. Introduction

AMONG VARIOUS types of detectors, an attractive class that can be used under varying channel conditions is the class of constant false-alarm rate (CFAR) [1] processing schemes, which has been used predominantly in radar systems. The threshold in a CFAR detector is determined on a cell-by-cell basis using the noise power estimated by processing a group of surrounding reference cells.

In the past decades, the CFAR processors have been applied to code-acquisition problems for estimating the noise variance in one-antenna direct-sequence code-division multiple-access (DS/CDMA) systems [1]–[3]. It has been shown that one-antenna DS/CDMA systems with a cell averaging (CA) CFAR processor exhibit excellent performance in a homogeneous environment, where the power of interference noise varies slowly as time changes. On the other hand, in a nonhomogeneous environment, where the variance of interference noise varies rapidly as the occasion demands, the one-antenna DS/CDMA systems with a greatest of (GO) CFAR processor is shown [3] to have the best performance. Practically, when the number of users changes abruptly or deep fading and shadowing occur, an abrupt change of the noise variance is observed. A base station affected by the users on a high-speed train is one such example.

In the meantime, the use of multiple antennas in DS/CDMA systems has been widely recognized as a useful means to enhance the signal-to-noise ratio (SNR) and increase the capacity of wireless systems [4]–[10]. Several different types of systems using multiple antennas have been proposed to exploit the attractive features of multiple antennas [4], [7], [9], [10]. A number of spatial processing techniques have also been developed, and their performance has been analyzed [5], [6], [8]. Nonetheless, the performance improvement of initial synchronization (code acquisition and tracking) using multiple antennas has rarely been considered.

In the study in [10], an adaptive hybrid code-acquisition scheme with the CA CFAR algorithm and antenna diversity has been considered in a homogeneous environment but not in a nonhomogeneous environment. It is well known [1] that the performance of the CA CFAR processor deteriorates considerably when the assumption of homogeneous environment is violated. To overcome such a deterioration of the performance resulting from a nonhomogeneous environment, modifications of the CA CFAR schemes can be considered at the cost of slightly additional hardware complexity.

In this paper, we address hybrid code acquisition with multiple antennas incorporating both the modified CA CFAR processors and receive antenna diversity in the acquisition of the pseudonoise (PN) code for DS/CDMA systems. Specifically, not only the GO and smallest of (SO) detectors are analyzed in a homogeneous environment, but the GO, SO, and CA detectors are also taken into account in a nonhomogeneous environment. The contribution and novelty of this paper lies in that we have analytically obtained and compared the performance of the modified CA CFAR schemes in a scenario where the following three elements are taken into account simultaneously: 1) CA, GO, and SO CFAR schemes, 2) receive diversity, and 3) homogeneous and nonhomogeneous environment. As far as we perceive, such an attempt has not yet been reported in other

Manuscript received August 18, 2005; revised June 18, 2006 and October 8, 2006. This work was supported by the National Research Laboratory (NRL) Program of the Korea Science and Engineering Foundation (KOSEF), Ministry of Science and Technology (MOST), under Grant R0A-2005-000-10005-0. The review of this paper was coordinated by Prof. L. Lampe.

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Digital Object Identifier 10.1109/TVT.2007.897647

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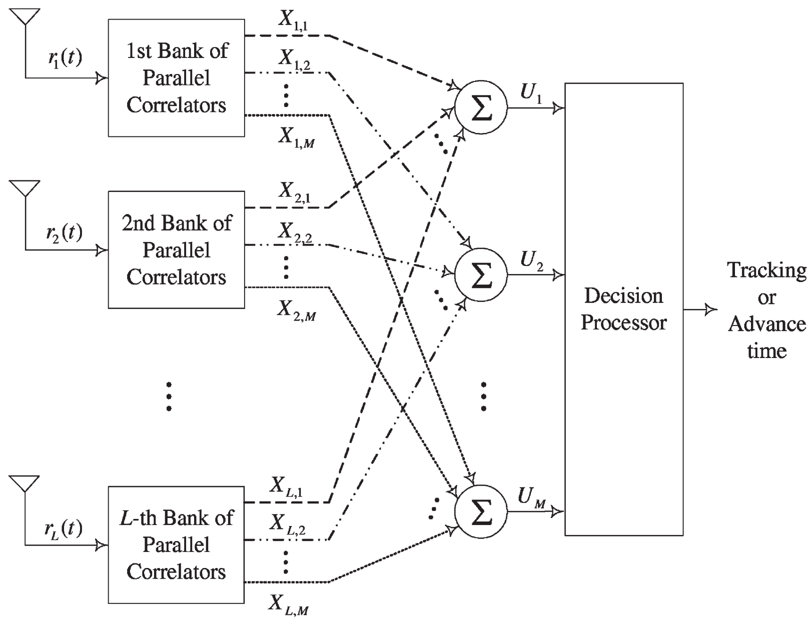


Fig. 1. Hybrid code-acquisition system with multiple antennas.

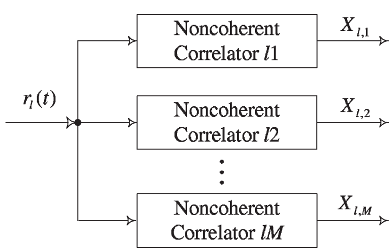


Fig. 2. $l$th bank of parallel correlators in Fig. 1.

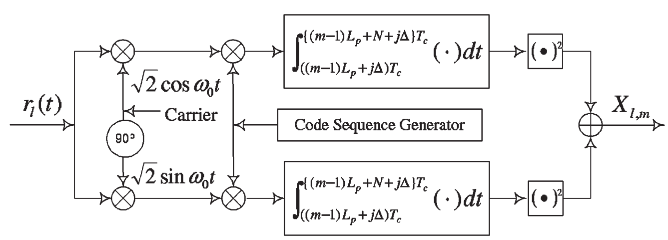


Fig. 3. $m$th correlator in Fig. 2.

published work, although subsets of the three elements have previously been considered partially in several investigations of acquisition problems. Particularly, this paper is distinguishable from the study in [10] in that this paper discusses the acquisition problem with the CA, GO, and SO CFAR schemes in a nonhomogeneous environment as well as in a homogeneous environment, while Oh *et al.* [10] considers the problem with the CA CFAR processor, assuming only a homogeneous environment.

The organization of this paper is as follows. In Section II, a description of the system model is presented. The performance characteristics of several acquisition systems are analyzed in Section III, and simulation results are provided in Section IV. Finally, Section V concludes this paper.

## II. System Model

Fig. 1 shows the hybrid acquisition system with $L$ antenna elements. Each antenna element is followed by a bank of $M$ correlators in a parallel structure, as shown in Fig. 2. The distance between the adjacent antenna elements is assumed to be sufficiently large (about ten times the carrier wavelength $\lambda$ [8]). As a result, signals received at different antennas experience independent fading, which would eventually allow the multiple-antenna system to have a higher SNR (and, consequently, a higher probability to acquire the PN sequence correctly) than the single-antenna systems.

Strictly speaking, the signals received at different antennas will have different propagation delays because they traverse different paths. Nonetheless, the effect on the performance of the acquisition system induced by this delay difference is negligible if the separation distance $d$ is much smaller than $c/B$, where $c$ is the speed of light, and $B$ is the bandwidth of the transmitted signal. This condition is easily satisfied in many wireless systems. For example, in the case of IS-95 for which $B \approx 1.25$ MHz, supposing $d = 10\lambda$ and the carrier frequency is 900 MHz, we have 3.3 m $= d \ll c/B = 240$ m. The propagation delay difference between two received signals is, thus, less than 0.014 chip. In other words, the time required for the received signal associated with a given transmission path to propagate across the antenna array is typically much smaller than the inverse of the bandwidth of the transmitted signal. It is consequently reasonable to assume that the signals received at different antennas have the same timing delay.

The whole uncertainty region of length (the code length) $L_c$ is divided into $M$ subregions of equal length $L_p = \lceil L_c/M \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$. In any bank of $M$ parallel correlators, the $m$th correlator serially searches all the cells in the $m$th subregion, as shown in Fig. 3. In Fig. 3, $N$ is the length of a correlator (the partial correlation length), $j = 0, 1, \ldots, L_p/\Delta - 1$, $T_c$ is the chip duration, and $\Delta$ is the advancing step size. The value of $\Delta$ is normally chosen to be 1 or 1/2, meaning that the local timing

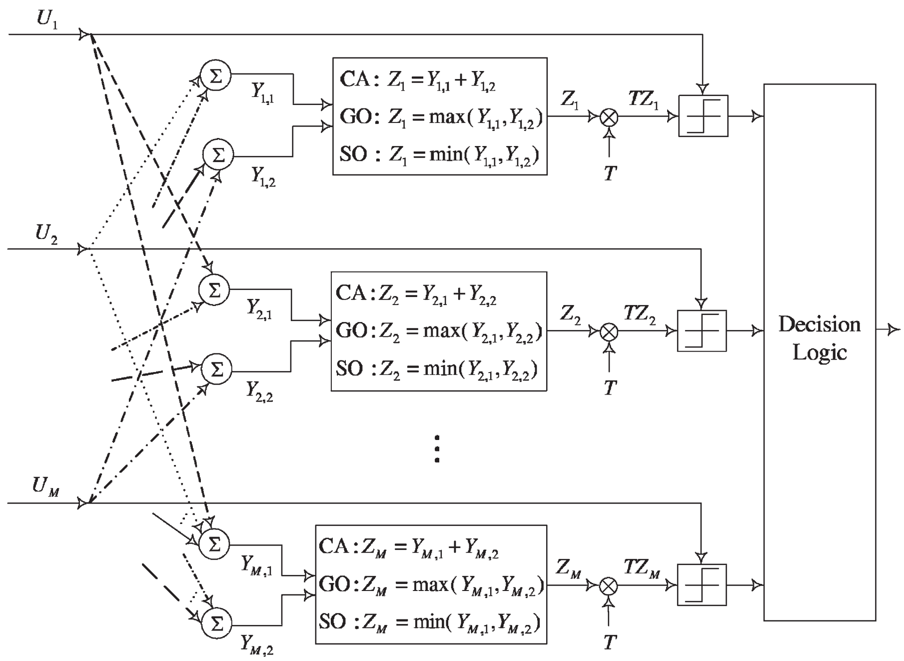


Fig. 4. Structure of the decision processor in Fig. 1.

is updated at intervals of $T_c$ or $T_c/2$, respectively. In this paper, $\Delta$ is set to one, which is the usual practice with no fractional chip timing uncertainty in the system. The outputs of the $m$th correlators of the $L$ banks are summed to produce

$$U_m = \sum_{l=1}^{L} X_{l,m}, \qquad m = 1, 2, \ldots, M \tag{1}$$

and then used as the inputs to the decision processor.

Fig. 4 shows a block diagram of the decision processor in Fig. 1. The threshold $TZ_m$ of the $m$th branch is the product of the CFAR output $Z_m$ and the scale parameter $T$ determined to attain the false-alarm rate $P_{\mathrm{FA}}$. Here, the CFAR output $Z_m$ is obtained by one of the three CFAR processors based on the "upper" reference branch output $Y_{m,1}$ and "lower" reference branch output $Y_{m,2}$, where

$$\begin{aligned} Y_{m,1} &= \sum_{\substack{j=1,\\ j\neq m}}^{\frac{M+1}{2}} U_j \\ Y_{m,2} &= \sum_{j=\frac{M+1}{2}+1}^{M} U_j \qquad \text{for } m \leq \frac{M+1}{2} \end{aligned} \tag{2}$$

and

$$\begin{aligned} Y_{m,1} &= \sum_{j=1}^{\frac{M+1}{2}-1} U_j \\ Y_{m,2} &= \sum_{\substack{j=\frac{M+1}{2},\\ j\neq m}}^{M} U_j \qquad \text{for } m > \frac{M+1}{2}. \end{aligned} \tag{3}$$

Although we have assumed that $M$ is an odd number in (2) and (3), it can be any positive integer in principle: When $M$ is an even number, we need to modify (2) and (3) only slightly.

The essence of the CFAR scheme lies in the use and processing of a group of reference signals when we estimate the noise power for a cell under test. In conventional serial acquisition systems with one antenna [2], [3], in which the correlator outputs are sent serially (timewise) into a shift register, the reference signals are split into "leading" and "lagging" parts symmetrically around the cell under test. In the hybrid acquisition systems considered in this paper, on the other hand, it is more adequate to estimate the noise power by processing reference signals split into "upper" and "lower" parts (i.e., the reference signals are formed and used spacewise and not timewise), since a set of correlation values are obtained at every processing time of the correlators.

In the decision logic of Fig. 4, if only one of $\{U_m\}_{m=1}^{M}$ exceeds the threshold, the code is acquired; otherwise, the phases of the local PN generators are advanced by $\Delta T_c$, and the acquisition process is repeated.

Now, let the hypothesis $H_1$ denote the case under which the phases of the incoming and local PN signals are aligned to within $\Delta T_c$, and the hypothesis $H_0$ denote the case under which the phase difference between the incoming and local PN signals is greater than or equal to the duration of one chip. Then, we can regard the PN code-acquisition process as a binary hypothesis-testing problem. Note that, in CFAR algorithms, all reference signals $\{U_j\}_{j=1,j\neq m}^{M}$ are assumed to be under $H_0$ when the test signal is $U_m$.

## III. Analysis of Decision Processors

In this section, the performance of the CA, GO, and SO CFAR processors is analyzed in homogeneous and nonhomogeneous backgrounds, obtaining closed-form performance expressions. The derivations of the detection, missed detection, and false-alarm probabilities in this section are based on the following assumptions.

1) There is only one sample corresponding to the correct phase ($H_1$ cell).
2) The correlator outputs are independent.
3) The length $N$ of a correlator is large enough so that the correlation of the received and local codes yield zero when they are not in phase ($H_0$ cells).
4) The effect of random data is negligible.

The detection probability $P_D$ is the probability that the output value of the correct branch ($H_1$ branch) in Fig. 4 exceeds the threshold and that the output values of all the other branches ($H_0$ branches) are not larger than the threshold. In other words, we have

$$P_D = \frac{1}{M}\sum_{j=1}^{M}\left\{P(U_j > TZ_j|H_1)\prod_{\substack{m=1,\\ m\neq j}}^{M} P(U_m < TZ_m|H_0)\right\}. \tag{4}$$

It is easy to see that the false-alarm probability can be written as

$$P_{\mathrm{FA}} = \frac{1}{L_p}P_{\mathrm{FA}|H_1} + \frac{L_p - 1}{L_p}P_{\mathrm{FA}|H_0} \tag{5}$$

where

$$\begin{aligned} P_{\mathrm{FA}|H_1} = \frac{1}{M}\sum_{j=1}^{M}\Bigg[&P(U_j > TZ_j|H_0) \\ &\times \sum_{\substack{k=1,\\ k\neq j}}^{M}\Bigg\{P(U_k < TZ_k|H_1) \\ &\cdot \prod_{\substack{m=1,\\ m\neq j, m\neq k}}^{M} P(U_m < TZ_m|H_0)\Bigg\}\Bigg] \end{aligned} \tag{6}$$

is the probability that the $H_1$ signal $U_k$ is less than the threshold, exactly one of $\{U_m\}_{m=1,m\neq k}^{M}$ exceeds the threshold when $H_1$ is true, and

$$P_{\mathrm{FA}|H_0} = \frac{1}{M}\sum_{j=1}^{M}\left\{P(U_j > TZ_j|H_0)\prod_{\substack{m=1,\\ m\neq j}}^{M} P(U_m < TZ_m|H_0)\right\} \tag{7}$$

is the probability that only one of $\{U_m\}_{m=1}^{M}$ exceeds the threshold value when $H_0$ is true. Note that $P_{\mathrm{FA}|H_1}$ can be called the probability of error. Using (4) and (6), the missed detection probability can be evaluated as

$$P_M = 1 - P_D - P_{\mathrm{FA}|H_1}. \tag{8}$$

It is noteworthy that $P_D$, $P_{\mathrm{FA}|H_1}$, $P_{\mathrm{FA}|H_0}$, and $P_M$ can be easily evaluated once $P(U_m > TZ_m|H_1)$ is obtained since

$$P\left(U_m > TZ_m|H_0\right) = P\left(U_m > TZ_m|H_1\right)|_{\text{signal strength}=0} \tag{9}$$

$$P\left(U_m < TZ_m|H_1\right) = 1 - P\left(U_m > TZ_m|H_1\right) \tag{10}$$

and

$$P\left(U_m < TZ_m|H_0\right) = 1 - P\left(U_m > TZ_m|H_0\right). \tag{11}$$

Now, let us try to take the nonhomogeneous environment into account when multiple antennas are available. First, let $\{\varphi_i\}_{i=1}^{P}$ be a set of $P$ integers such that $\sum_{i=1}^{P}\varphi_i = L$ and $1 \le \varphi_i \le L$, where $1 \le P \le L$. Then, assume the values of the noise variance at the $L$ antennas are such that $\varphi_1$ values are the same, $\varphi_2$ values are the same, ..., $\varphi_P$ values are the same. This formulation allows us to accommodate all possibilities of the noise variance in a nonhomogeneous environment, e.g., {all $\varphi_i = 1$} corresponds to the case where the $L$ noise variances are all distinct. Clearly, the homogeneous environment can be considered as a special case of the nonhomogeneous environment with $P = 1$ and $\varphi_1 = L$.

Assuming the reception of flat Rayleigh faded-signals in additive white Gaussian noise (AWGN), the output $X_{l,m}$ of the square-sum unit in Fig. 3 has the exponential probability density function

$$p_{X_{l,m}}(x) = \beta_l e^{-\beta_l x}, \qquad x \ge 0,\ l = 1, 2, \ldots, L. \tag{12}$$

It is well known [11] that we have the null hypothesis

$$H_0 : \beta_l = \frac{1}{\mu_0(1 + \mathcal{C}_l)} \tag{13}$$

and alternative hypothesis

$$H_1 : \beta_l = \frac{1}{\mu_0(1 + \mathcal{C}_l)(1 + S_l)} \tag{14}$$

with $\mu_0$ as the variance of AWGN, $\mathcal{C}_l$ as the interference-to-AWGN ratio, and $S_l$ as the signal-to-interference-and-noise ratio (SINR) of the output $X_{l,m}$.

Now, since the moment generating function (mgf) of the random variable $X_{l,m}$ in Fig. 1 is

$$M_{X_{l,m}}(t) = \frac{\beta_l}{\beta_l + t} \tag{15}$$

$U_m = \sum_{l=1}^{L} X_{l,m}$, and the random variables $\{X_{l,m}\}_{l=1}^{L}$ are independent, the mgf of $U_m$ can be obtained as

$$M_{U_m}(t) = \prod_{i=1}^{P}\left(\frac{\beta_i}{\beta_i + t}\right)^{\varphi_i}. \tag{16}$$

Using the partial fraction expansion and inverse Laplace transform, the pdf of $U_m$ can be expressed as

$$p_{U_m}(u) = \sum_{i=1}^{P}\sum_{j=1}^{\varphi_i}\frac{R_{i,j}}{(j-1)!}u^{j-1}e^{-\beta_i u} \tag{17}$$

from (16), where $R_{i,j}$ is defined in (27) of the Appendix. Once the pdf $p_{U_m}(u)$ is obtained from (17), the probabilities $P_D$ and $P_{\mathrm{FA}}$ can be evaluated from (4) and (5) using (9)–(11).

## IV. Performance Evaluation

In this section, we evaluate and compare the performance of the CA, GO, and SO CFAR processors when multiple antennas are available in a homogeneous and a nonhomogeneous environment. Note that an adaptive hybrid acquisition scheme with the CA CFAR algorithm and antenna diversity has been studied in [10] for the homogeneous environment but not in a nonhomogeneous environment.

### A. Performance Measure and Conditions

To evaluate and compare the performance of various hybrid acquisition schemes, we obtain the mean acquisition time

$$E\{T_{\mathrm{acq}}\} = \frac{NT_c}{2P_D} \cdot [2 + (1 + KP_{\mathrm{FA}|H_0})(L_p - 1). \cdot \{2(P_M + P_{\mathrm{FA}|H_1}) + P_D\} + 2KP_{\mathrm{FA}|H_1}] \quad (18)$$

of the hybrid acquisition system, which can be derived using the signal-flow technique [12], [13]. In evaluating the performance, we set the chip duration $T_c = 1\ \mu$s, code-sequence length $L_c = 2047$ chips, advancing step size $\Delta = 1$, penalty time factor $K = 10^4$, and desired false-alarm probability $P_{\mathrm{FA}} = 10^{-3}$.

We assume that the total noise is composed of AWGN with variance $N_0/2$ and multiple-access interference (MAI). The Gaussian assumption for the total noise is based on the fact that the number of MAI sources increases in proportion to the number of antennas, and thus, the Gaussian assumption can be more easily justified in the multiple-antenna environment than in the single-antenna environment: Note that the Gaussian assumption is widely accepted to be reasonable, even in the single-antenna environment [11].

Then, we have

$$\mu_0 = NN_0 \quad (19)$$

$$\begin{aligned} \mathcal{C}_l &= \frac{1}{3}\sum_{i=1}^{K_l} E_i \Big/ \frac{N_0}{2} \\ &= \frac{1}{3}K_l E_0 \Big/ \frac{N_0}{2} \\ &= \frac{K_l \cdot (\mathrm{SNR/chip})}{3} \end{aligned} \quad (20)$$

and

$$\begin{aligned} S_l &= \frac{2\sigma^2 N E_0 R^2(\tau)}{N_0 + \frac{2}{3}\sum_{i=1}^{K_l} E_i} \\ &= \frac{2\sigma^2 N E_0}{N_0 + \frac{2}{3}K_l E_0} \\ &= \frac{\sigma^2 N \cdot (\mathrm{SNR/chip})}{1 + \frac{K_l}{3} \cdot (\mathrm{SNR/chip})} \end{aligned} \quad (21)$$

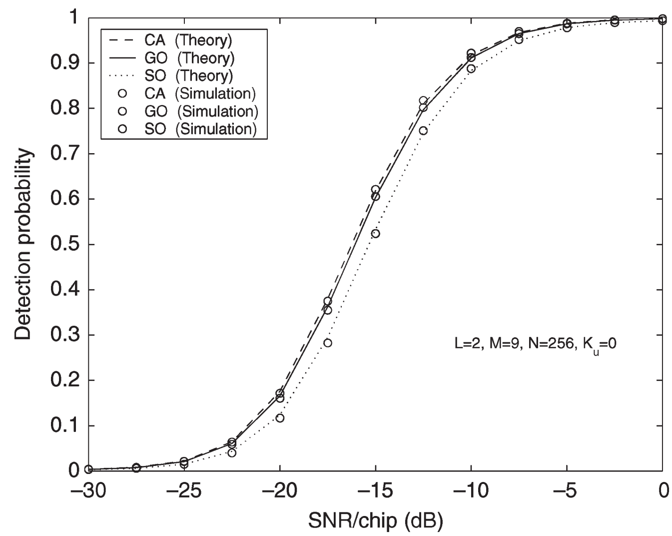


Fig. 5. Detection probability in additive Gaussian homogeneous environment with Rayleigh fading ($L = 2$, $M = 9$, $N = 256$, and $K_u = 0$).

from the study in [11]. In (20) and (21), $K_l$ is the number of interfering users whose signals compose the MAI at the $l$th receiving antenna, $E_i$ is the chip energy of the $i$th interfering user, $E_0$ is the chip energy of the desired user, $\sigma > 0$ is the parameter of Rayleigh pdf $p(\alpha) = (2\alpha/\sigma^2)e^{-\alpha^2/\sigma^2}$

$$R(\tau) = \begin{cases} 1 - \frac{\tau}{T_c}, & \text{if } |\tau| < T_c \\ 0, & \text{if } |\tau| \geq T_c \end{cases} \quad (22)$$

is the autocorrelation function of the PN sequence, $\tau$ is the timing error defined as the time difference between the incoming spreading and local despreading PN code sequences of the desired user, and SNR/chip $= (E_0/(N_0/2))$ is the chip SNR. We have assumed $E_i = E_0$ for all $i$, and there is no residual code phase offset so that $R(\tau) = 1$ in (20) and (21). From (21), it is clear that the SINR $S_l$ has the limiting value $3\sigma^2 N/K_l$ when the SNR per chip tends to infinity.

### B. Simulation Results and Discussion

In the simulation results of this paper, we employ the chip SNR instead of SINR as the measure of the signal strength. The number of Monte Carlo runs for each point in the figures shown here is $10^6$, and the dashed, solid, and dotted lines are used to denote the performance of the CA, GO, and SO CFAR schemes, respectively.

*1) Performance in Homogeneous Environment:* In the homogeneous environment, $\{K_l\}_{l=1}^{L}$ are all equal to a value denoted as $K_u$, and the conventional scheme indicates a serial acquisition scheme with $L = 1$ using a CFAR processor or a hybrid acquisition scheme with $L \geq 1$ using the CA CFAR processor.

Figs. 5 and 6 show the detection and missed detection probabilities, respectively, when the number of antennas is 2 ($L = 2$), the number of correlators is 9 ($M = 9$), the partial correlation length is 256 ($N = 256$), and the number of interfering users is 0 ($K_u = 0$). Clearly, the CA CFAR processor has the best performance, confirming that it is the optimum CFAR

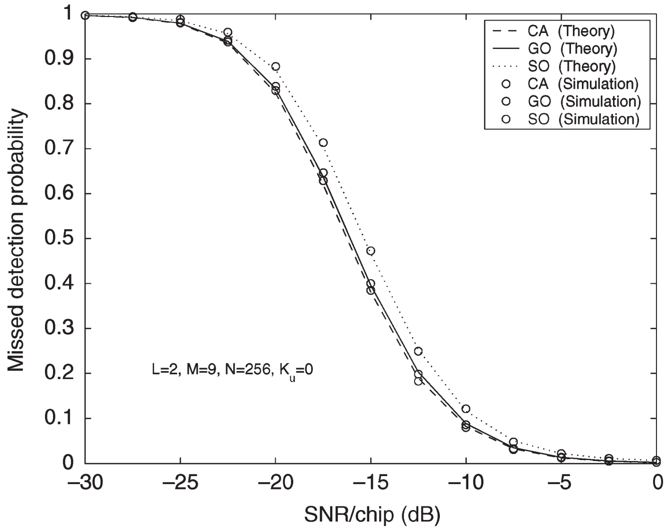


Fig. 6. Missed detection probability in an additive Gaussian homogeneous environment with Rayleigh fading ($L = 2$, $M = 9$, $N = 256$, and $K_u = 0$).

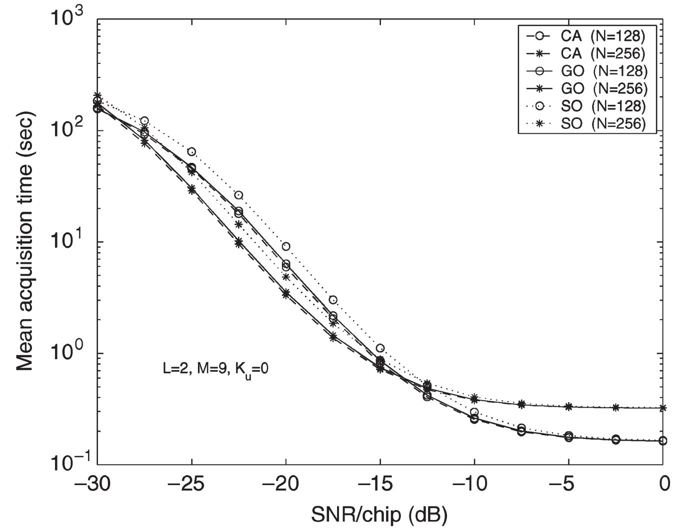


Fig. 7. Mean acquisition time for various values of $N$ in an additive Gaussian homogeneous environment with Rayleigh fading ($L = 2$, $M = 9$, and $K_u = 0$).

processor in a homogeneous environment. The GO CFAR processor performs better than the SO CFAR processor and has almost the same performance as the CA CFAR processor. Note that the GO and SO CFAR processors estimate the noise power by the greatest and smallest, respectively, of the sums in the upper and lower parts, while the CA CFAR processor estimates the noise power by using the sum of all reference signals. Thus, the noise power in the modified CFAR schemes is estimated less efficiently in a homogeneous environment, and therefore, some loss of detection is introduced compared with the CA CFAR processor. In these figures, close agreements between the analysis and simulation results can clearly be observed, implying that (at least partially) the proposed analytical model is reasonably justifiable. This observation, we think, forms a justification that we may obtain the mean acquisition time from (18) with the probabilities $P_D$, $P_m$, $P_{\mathrm{FA}|H_1}$, and $P_{\mathrm{FA}|H_0}$ obtained from simulations.

Fig. 7 shows the mean code-acquisition time for various values of the partial correlation length $N$ when $L = 2$, $M = 9$, and $K_u = 0$. We can observe that the mean acquisition time is shorter when the partial correlation length is larger (smaller) if the SNR/chip is lower (higher) than $-13$ dB because the detection probability converges to one, regardless of the partial correlation length when the SNR per chip is sufficiently high, making the mean acquisition time increase in proportion to the partial correlation length, as shown in (18). For the results with other values of $L$, $M$, and $K_u$ (not shown here due to space restrictions), we can also make the same observations.

Fig. 8 shows the mean acquisition time of various acquisition schemes when $N = 256$ and $K_u = 0$, where the structures of the conventional one-antenna serial acquisition systems with CFAR schemes are adopted from the study in [3]. It should be noted that the mean acquisition times for the serial acquisition schemes are obtained by a formula [12] similar to (18). It is observed that the hybrid acquisition schemes outperform the serial acquisition schemes at all values of the SNR per chip, and the conventional hybrid scheme with the CA CFAR processor is superior to the other schemes. As is easily anticipated, the

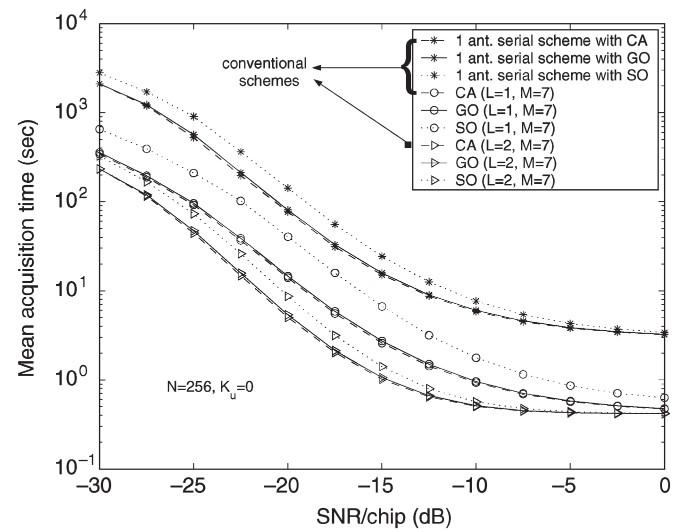


Fig. 8. Mean acquisition time comparison among various schemes in an additive Gaussian homogeneous environment with Rayleigh fading ($N = 256$ and $K_u = 0$).

performance gap between the hybrid and serial acquisition schemes becomes larger as the number of antenna increases.

*2) Performance in Nonhomogeneous Environment:* Unlike in the homogeneous environment, the conventional schemes in a nonhomogeneous environment are serial acquisition schemes with $L = 1$ using CFAR processors. First, we design the CFAR schemes assuming $K_u = 32$, and then, the performance of the hybrid schemes is evaluated when $K_1 = 2$ and $K_2 = 62$. Here, we use $K_u = 32$ as the maximum number of interfering users, since a base station in the commercial CDMA can accommodate up to 32 multiple-access users to maintain the required quality of service [14]. Note that, for both $\{K_u = 32$ in homogeneous environment$\}$ and $\{K_1 = 2$ and $K_2 = 62$ in nonhomogeneous environment$\}$, the number of interfering users at a test cell equals to 64. The results in Fig. 9 show the mean acquisition time when $L = 2$, $M = 9$, and $N = 256$.

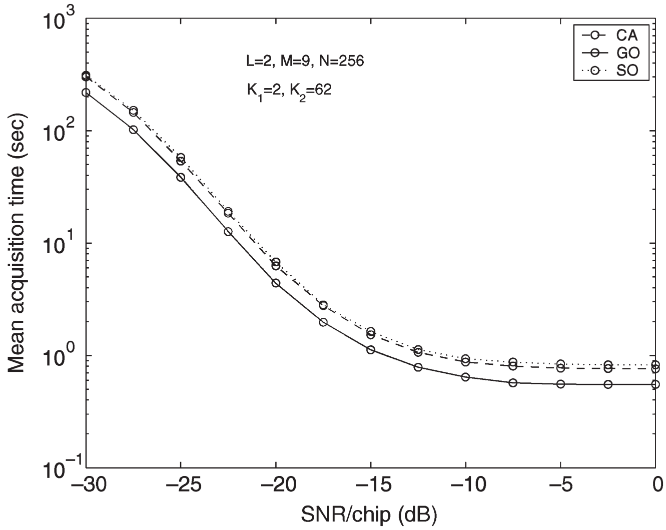


Fig. 9. Mean acquisition time in an additive Gaussian nonhomogeneous environment with Rayleigh fading ($L = 2$, $M = 9$, $N = 256$, $K_1 = 2$, and $K_2 = 62$).

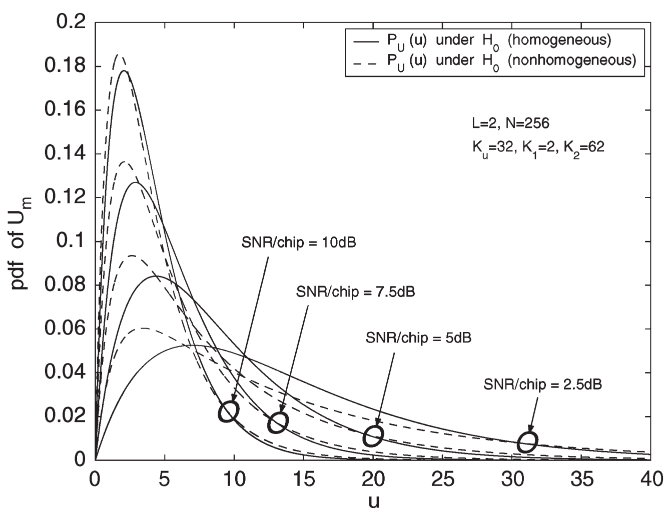


Fig. 10. PDF of $U_m$ under $H_0$ in a homogeneous and a nonhomogeneous environment when $L = 2$ and $N = 256$.

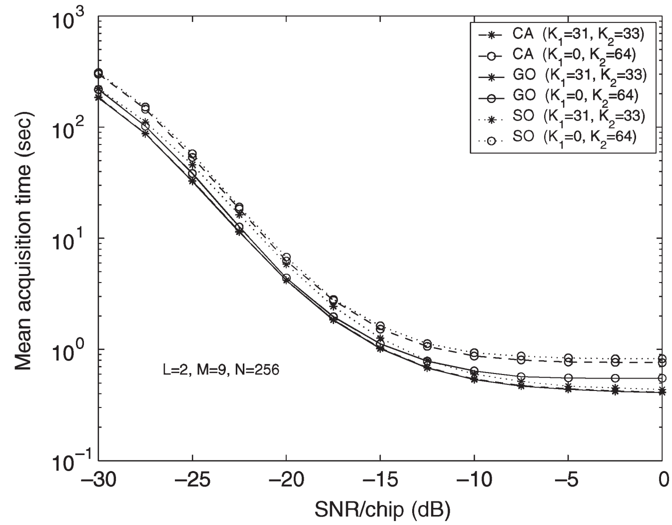


Fig. 11. Mean acquisition time for various values of the number of interfering users in an additive Gaussian nonhomogeneous environment with Rayleigh fading ($L = 2$, $M = 9$, and $N = 256$).

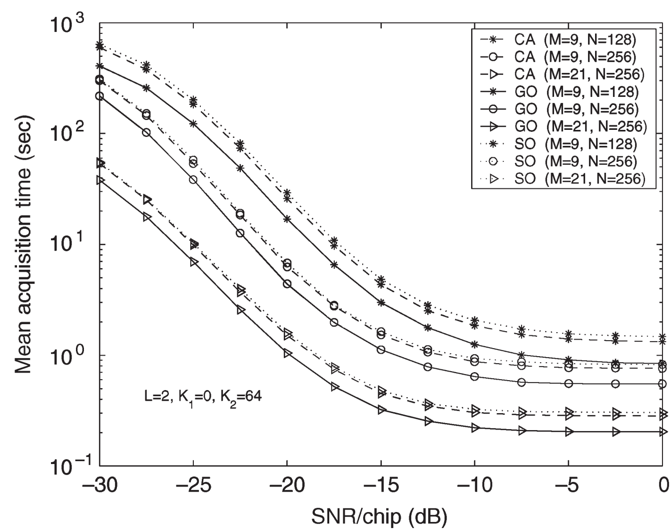


Fig. 12. Mean acquisition time for various values of $N$ and $M$ in an additive Gaussian nonhomogeneous environment with Rayleigh fading ($L = 2$, $K_1 = 0$, and $K_2 = 64$).

It is observed that, in a nonhomogeneous environment, the GO CFAR processor outperforms the other processors, and the CA CFAR processor performs slightly better than the SO CFAR processor. This phenomenon can be explained as follows. First, the pdf of $U_m$ under $H_1$ not only changes barely when the environment changes but also influences little the variation of the mean acquisition time since most of the uncertainty phase corresponds to $H_0$. Second, the pdf of $U_m$ under $H_0$ in a nonhomogeneous environment becomes more impulsive [15] than that in a homogeneous environment, as shown in Fig. 10, as an example when $L = 2$ and $N = 256$. Thus, the GO CFAR processor which estimates the noise power by the GO the sums in the upper and lower windows would perform best, while the noise estimates of the CA and SO CFAR processors exploits the impulsive nature less effectively.

Next, we examine the influence of the number of interfering users on the performance. Fig. 11 shows the mean acquisition time for various values of the number of interfering users when $L = 2$, $M = 9$, and $N = 256$. We can observe that the mean acquisition time becomes longer when the number of interfering users is more biased to one antenna. In addition, the performance gap between the GO and CA CFAR processors is negligible when $K_1 = 31$ and $K_2 = 33$, but it is noticeable when $K_1 = 0$ and $K_2 = 64$. Fig. 12 shows the mean acquisition time for various values of the partial correlation length $N$ and the number $M$ of correlators when $L = 2$, $K_1 = 0$, and $K_2 = 64$. It is observed that the performance of the CFAR processors becomes better as $N$ or $M$ increases. Unlike in a homogeneous environment (Fig. 7), a larger value of $N$ results in better acquisition performance, regardless of the SNR per chip since there always exists MAI in a nonhomogeneous environment. This is because, in the presence of the MAI, a longer correlation length is necessary to ensure a sufficiently high SNR of the decision variables. It is also observed that the performance is saturated when $M > 21$. This can be explained as follows. With the hybrid search, a decision is made based on $M$ variables

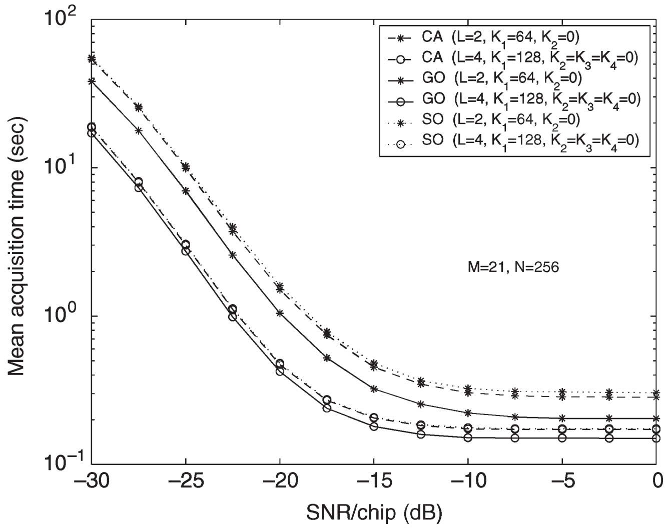


Fig. 13. Mean acquisition time for various values of $L$ in an additive Gaussian nonhomogeneous environment with Rayleigh fading ($M = 21$ and $N = 256$).

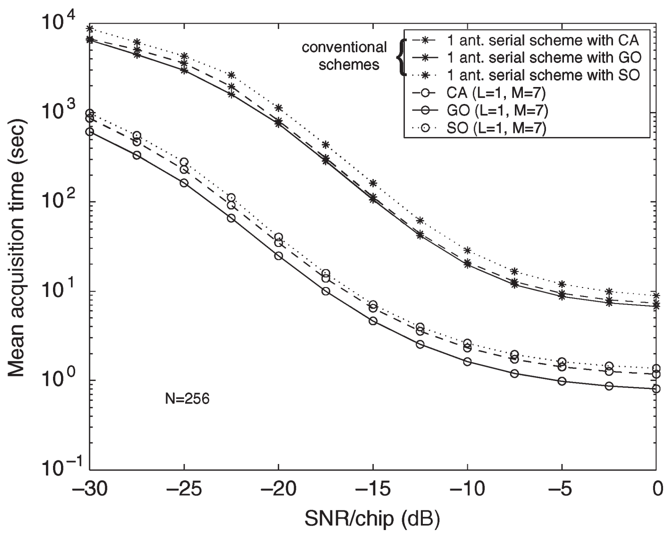


Fig. 14. Comparison of the mean acquisition time among various schemes when the number of interfering users increases from 0 to 32 abruptly in an additive Gaussian nonhomogeneous environment with Rayleigh fading ($N = 256$).

$\{U_m\}_{m=1}^M$. Among the $M$ variables, only one corresponds to the correct PN code phase ($H_1$ state). Therefore, when $M$ becomes larger, the decision variables corresponding to an $H_0$ state increase the possibility for a false-alarm event to occur.

Fig. 13 shows the mean acquisition time for various values of the number $L$ of antennas when $M = 21$ and $N = 256$. Clearly, the CFAR processors performs better as the value of $L$ increases. An additional antenna clearly implies better performance. Yet, when the number of antennas is above four, only a mediocre improvement of performance can be achieved. The reason is that much of the diversity gain has already been achieved by using four receive antennas. In addition, hardware costs and system-design constraints would limit the value of $L$ in practice.

Finally, Fig. 14 shows the mean acquisition time of various acquisition schemes when $N = 256$ and the number of interfering users increases from 0 to 32 abruptly. As in a homogeneous environment, we can observe that the hybrid schemes have better performance than the serial schemes, regardless of the value of SNR per chip.

*3) Some Discussions:* It is anticipated that the relative performance of the GO, SO, and CA processors would be the same whether there exists frequency-selective fading or not, although the absolute performance of the GO, SO, and CA processors will be different for frequency-selective fading channels (from those for frequency-nonselective fading channels). This can be confirmed in other studies such as [16] and [17].

We would also like to add that, if we consider multiple $H_1$ cells, the absolute performance of the CFAR processors will be better than that we have obtained and shown in this paper. Yet, the relative performance among the GO, SO, and CA schemes is expected to be the same as in the single $H_1$ cell environment.

## V. Conclusion

In this paper, adaptive hybrid acquisition schemes with mean-level CFAR processors and antenna diversity have been addressed for PN code acquisition in the presence of MAI under homogeneous and nonhomogeneous circumstances. Since the performance of the CA CFAR processor is known to deteriorate significantly in nonhomogeneous environment, the GO and SO CFAR processors have been applied to alleviate the performance degradation at the cost of negligible hardware complexity.

The performance of the CA, GO, and SO CFAR processors has been analyzed by obtaining the closed-form expressions and compared in a homogeneous and a nonhomogeneous environment with simulation results. Numerical results have indicated that the GO CFAR scheme has the best performance in a nonhomogeneous environment and has almost the same performance as the CA CFAR scheme in a homogeneous environment.

The acquisition schemes considered in this paper are applicable to the single-input–single-output, single-input–multiple-output, multiple-input–single-output (MISO), and multiple-input–multiple-output (MIMO) channels. For the MISO and MIMO channels, we might be able to gain further performance improvement if we, in addition, take transmit diversity into account. For example, if we use several codes for each user to exploit transmit diversity (which seems to be the only plausible way to attain the transmit diversity), we may end up with so many expressions similar to (38) and an increase in the number of the banks of parallel correlators in Fig. 1. We have left the issue of transmit diversity as a topic for further study.

## Appendix
## Detection and False Alarm Probabilities of the CFAR Schemes

To find the inverse Laplace transform, let us express the mgf (16) of $U_m$ in terms of the partial fraction expansion as

$$M_{U_m}(t) = \sum_{i=1}^{P} \sum_{j=1}^{\varphi_P} \frac{R_{i,j}}{(\beta_i + t)^j}. \tag{23}$$

In (23), $R_{i,j},\ i=1,2,\ldots,P,\ j=1,2,\ldots,\varphi_i$ can be computed as

$$\begin{aligned} R_{i,j} &= \frac{1}{(\varphi_i-j)!}\frac{d^{\varphi_i-j}}{dt^{\varphi_i-j}}\left\{(\beta_i+t)^{\varphi_i}M_{U_m}(t)\right\}\Big|_{t=-\beta_i} \\ &= \frac{\prod_{k=1}^{P}\beta_k^{\varphi_k}}{(\varphi_i-j)!}\frac{d^{\varphi_i-j}}{dt^{\varphi_i-j}} \\ &\quad\times\left[\sum_{l_1=0}^{\varphi_1}\cdots\sum_{l_{i-1}=0}^{\varphi_{i-1}}\sum_{l_{i+1}=0}^{\varphi_{i+1}}\cdots\sum_{l_P=0}^{\varphi_P}\left\{\prod_{\substack{k=1,\\k\neq i}}^{P}\binom{\varphi_k}{l_k}\beta_k^{\varphi_k-l_k}\right\}\right. \\ &\qquad\left.\times t^{\sum_{k=1,k\neq i}^{P}l_k}\right]^{-1}\Bigg|_{t=-\beta_i}. \end{aligned} \tag{24}$$

Now, note that [18]

$$\begin{aligned} \frac{d^j}{dt^j}F(\zeta(t)) &= \sum_{\{\eta\}_1}\frac{j!}{\eta_1!\eta_2!,\ldots,\eta_n!}\frac{d^qF(y)}{dy^q} \\ &\quad\times\left(\frac{y^{(1)}}{1!}\right)^{\eta_1}\left(\frac{y^{(2)}}{2!}\right)^{\eta_2},\ldots,\left(\frac{y^{(n)}}{n!}\right)^{\eta_n} \end{aligned} \tag{25}$$

where $y=\zeta(t)$ and the symbol $\sum_{\{\eta\}_1}$ represents the summation over all non-negative integer solutions $\{\eta_m\}$ of the simultaneous equations $\sum_{m=1}^{n} m\eta_m = j$ and $\sum_{m=1}^{n}\eta_m = q$. Now, letting

$$y=\sum_{l_1=0}^{\varphi_1}\cdots\sum_{l_{i-1}=0}^{\varphi_{i-1}}\sum_{l_{i+1}=0}^{\varphi_{i+1}}\cdots\sum_{l_P=0}^{\varphi_P}\left\{\prod_{\substack{k=1,\\k\neq i}}^{P}\binom{\varphi_k}{l_k}\beta_k^{\varphi_k-l_k}\right\}t^{\sum_{\substack{k=1,\\k\neq i}}^{P}l_k} \tag{26}$$

in (24), we can obtain $F(y)=1/y$ and $F^{(q)}(y)=(-1)^q q!/y^{q+1}$ easily. Therefore, we have

$$\begin{aligned} R_{i,j} &= \sum_{\{\eta\}_2}\frac{1}{\eta_1!\eta_2!,\ldots,\eta_n!}\frac{(-1)^q q!\prod_{k=1}^{P}\beta_k^{\varphi_k}}{\prod_{\substack{k=1,\\k\neq i}}^{P}(\beta_k-\beta_i)^{\varphi_k(q+1)}}\cdot\prod_{h=1}^{n}\left(\frac{1}{h!}\right)^{\eta_h} \\ &\quad\times\left[\sum_{l_1=0}^{\varphi_1}\cdots\sum_{l_{i-1}=0}^{\varphi_{i-1}}\sum_{l_{i+1}=0}^{\varphi_{i+1}}\cdots\sum_{l_P=0}^{\varphi_P}\left\{\prod_{\substack{k=1,\\k\neq i}}^{P}\binom{\varphi_k}{l_k}\beta_k^{\varphi_k-l_k}\right\}\right. \\ &\qquad\left.\cdot\left(\sum_{\substack{k=1,\\k\neq i}}^{P}l_k\right)!\Bigg/\left\{\left(\sum_{\substack{k=1,\\k\neq i}}^{P}l_k\right)-h\right\}!(-\beta_i)^{\left(\sum_{\substack{k=1,\\k\neq i}}^{P}l_k\right)-h}\right]^{\eta_h} \end{aligned} \tag{27}$$

from (24)–(26). Here, the symbol $\sum_{\{\eta\}_2}$ represents the summation over all non-negative integer solutions $\{\eta_m\}$ of the simultaneous equations $\sum_{m=1}^{n} m\eta_m = \varphi_i - j$ and $\sum_{m=1}^{n}\eta_m = q$.

Consequently, the pdf of $U_m$ is obtained as

$$p_{U_m}(u)=\sum_{i=1}^{P}\sum_{j=1}^{\varphi_i}\frac{R_{i,j}}{(j-1)!}u^{j-1}e^{-\beta_i u} \tag{28}$$

from the inverse Laplace transform of (23). Now, we will evaluate

$$\begin{aligned} P(U_k>TZ_k|H_1) &= \int_0^\infty\int_{Tz}^\infty p_{U_k}(u|H_1)du\cdot p_{Z_k}(z)dz \\ &= \sum_{i=1}^{P}\sum_{j=1}^{\varphi_i}\sum_{m=0}^{j-1}\frac{R_{i,j}T^m}{m!\beta_i^{j-m}}E_{Z_k}\{Z_k{}^m e^{-\beta_i T Z_k}\} \end{aligned} \tag{29}$$

for each of the three CFAR schemes, where $\beta_i$ is $1/\{\mu_0(1+\mathcal{C}_i)(1+S_i)\}$ under $H_1$. Here, we have used the subscript $Z_k$ in $E_{Z_k}$ to specify that the expectation is over $Z_k$.

### A. CA CFAR Decision Processor

The output $Z_m$ of the CA processor is given by

$$Z_m=\sum_{\substack{j=1,\\j\neq m}}^{M}U_j,\qquad m=1,2,\ldots,M. \tag{30}$$

Thus, using (16), the mgf of $Z_m$ in the CA CFAR scheme is

$$\begin{aligned} M_{Z_m}(t) &= \{M_{U_m}(t)\}^{M-1} \\ &= \prod_{i=1}^{P}\left(\frac{\beta_i}{\beta_i+t}\right)^{(M-1)\varphi_i}. \end{aligned} \tag{31}$$

Comparing (31) with (16), we can obtain the pdf of $Z_m$ directly as

$$\begin{aligned} p_{Z_m}(z) &= p_{U_m}(z)|_{\varphi_i\to(M-1)\varphi_i} \\ &= \sum_{i=1}^{P}\sum_{j=1}^{(M-1)\varphi_i}\frac{W_{i,j}}{(j-1)!}z^{j-1}e^{-\beta_i z} \end{aligned} \tag{32}$$

where

$$\begin{aligned} W_{i,j} &= \sum_{\{\eta\}_3}\frac{1}{\eta_1!\eta_2!,\ldots,\eta_n!} \\ &\quad\times\frac{(-1)^q q!\prod_{k=1}^{P}\beta_k^{(M-1)\varphi_k}}{\prod_{\substack{k=1,\\k\neq i}}^{P}(\beta_k-\beta_i)^{(M-1)(q+1)\varphi_k}}\cdot\prod_{h=1}^{n}\left(\frac{1}{h!}\right)^{\eta_h} \\ &\quad\times\left[\sum_{l_1=0}^{(M-1)\varphi_1}\cdots\sum_{l_{i-1}=0}^{(M-1)\varphi_{i-1}}\sum_{l_{i+1}=0}^{(M-1)\varphi_{i+1}}\cdots\sum_{l_P=0}^{(M-1)\varphi_P}\right. \end{aligned}$$

$$\times \left\{ \prod_{\substack{k=1,\\k\neq i}}^{P} \binom{(M-1)\varphi_k}{l_k} \beta_k^{(M-1)\varphi_k - l_k} \right\}$$
$$\times \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right)! \Bigg/ \left\{ \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right) - h \right\}!$$
$$\left. \cdot \left(-\beta_i\right)^{\left(\sum_{k=1,k\neq i}^{P} l_k\right) - h} \right]^{\eta_h} \quad (33)$$

for $i = 1, 2, \ldots, P$; $j = 1, 2, \ldots, (M-1)\varphi_i$; $\beta_i = 1/\{\mu_0(1+\mathcal{C}_i)\}$. In (33), the symbol $\sum_{\{\eta\}_3}$ represents the summation over all non-negative integer solutions $\{\eta_m\}$ of the simultaneous equations $\sum_{m=1}^{n} m\eta_m = (M-1)\varphi_i - j$ and $\sum_{m=1}^{n} \eta_m = q$.

From (29) and (32), we thus get

$$P(U_k > TZ_k|H_1) = \sum_{i_1=1}^{P} \sum_{j_1=1}^{\varphi_{i_1}} \sum_{m=0}^{j_1-1} \sum_{i_2=1}^{P} \sum_{j_2=1}^{(M-1)\varphi_{i_2}} \binom{j_2+m-1}{m}$$
$$\times\ T^m \cdot R_{i_1,j_1} W_{i_2,j_2}\ \frac{(T\beta_{i_1} + \beta_{i_2})^{-(j_2+m)}}{\beta_{i_1}^{j_1-m}}. \quad (34)$$

With $\beta_{i_1} = 1/\{\mu_0(1+\mathcal{C}_{i_1})(1+S_{i_1})\}$ and $\beta_{i_2} = 1/\{\mu_0(1+\mathcal{C}_{i_2})\}$, (34) can be rewritten as

$$P(U_k > TZ_k|H_1) = \sum_{i_1=1}^{P} \sum_{j_1=1}^{\varphi_{i_1}} \sum_{m=0}^{j_1-1} \sum_{i_2=1}^{P} \sum_{j_2=1}^{(M-1)\varphi_{i_2}} \binom{j_2+m-1}{m}$$
$$\times T^m \cdot \frac{\{(1+\mathcal{C}_{i_1})(1+S_{i_1})\}^{j_1-m}}{\left\{\frac{T}{(1+\mathcal{C}_{i_1})(1+S_{i_1})} + \frac{1}{(1+\mathcal{C}_{i_2})}\right\}^{j_2+m}}$$
$$\cdot R_{i_1,j_1}(\underline{S_P}) \cdot W_{i_2,j_2} \cdot (\mu_0)^{j_1+j_2} \quad (35)$$

where $\underline{S_P} = (S_1, S_2, \ldots, S_P)$ is the vector of SINR

$$R_{i,j}(\underline{S_P}) = \sum_{\{\eta\}_2} \frac{\prod_{k=1}^{P} \left\{\frac{1}{(1+\mathcal{C}_k)(1+S_k)}\right\}^{\varphi_k}}{\eta_1!\eta_2!, \ldots, \eta_n!}$$
$$\cdot \frac{(-1)^q q! \cdot (\mu_0)^{-L+(q+1)\sum_{k=1,k\neq i}^{P} \varphi_k}}{\prod_{\substack{k=1,\\k\neq i}}^{P} \left\{\frac{1}{(1+\mathcal{C}_k)(1+S_k)} - \frac{1}{(1+\mathcal{C}_i)(1+S_i)}\right\}^{(q+1)\varphi_k}}$$
$$\cdot \prod_{h=1}^{n} \left(\frac{1}{h!}\right)^{\eta_h}$$
$$\times \left[ \sum_{l_1=0}^{\varphi_1} \cdots \sum_{l_{i-1}=0}^{\varphi_{i-1}} \sum_{l_{i+1}=0}^{\varphi_{i+1}} \cdots \sum_{l_P=0}^{\varphi_P} \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right)! \right.$$
$$\times \left\{ \prod_{\substack{k=1,\\k\neq i}}^{P} \binom{\varphi_k}{l_k} \{\mu_0(1+\mathcal{C}_k)(1+S_k)\}^{l_k - \varphi_k} \right\} \Bigg/$$
$$\left\{ \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right) - h \right\}!$$
$$\left. \cdot \{-\mu_0(1+\mathcal{C}_i)(1+S_i)\}^{h - \left(\sum_{k=1,k\neq i}^{P} l_k\right)} \right]^{\eta_h} \quad (36)$$

and

$$W_{i,j} = \sum_{\{\eta\}_3} \frac{(-1)^q q! \cdot (\mu_0)^{(M-1)\left\{-L+(q+1)\sum_{k=1,k\neq i}^{P} \varphi_k\right\}}}{\eta_1!\eta_2!, \ldots, \eta_n!}$$
$$\cdot \frac{\prod_{k=1}^{P} \left\{\frac{1}{(1+\mathcal{C}_k)}\right\}^{(M-1)\varphi_k}}{\prod_{\substack{k=1,\\k\neq i}}^{P} \left\{\frac{1}{(1+\mathcal{C}_k)} - \frac{1}{(1+\mathcal{C}_i)}\right\}^{(M-1)(q+1)\varphi_k}} \cdot \prod_{h=1}^{n} \left(\frac{1}{h!}\right)^{\eta_h}$$
$$\times \left[ \sum_{l_1=0}^{(M-1)\varphi_1} \cdots \sum_{l_{i-1}=0}^{(M-1)\varphi_{i-1}} \sum_{l_{i+1}=0}^{(M-1)\varphi_{i+1}} \cdots \sum_{l_P=0}^{(M-1)\varphi_P} \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right)! \right.$$
$$\cdot \left\{ \prod_{\substack{k=1,\\k\neq i}}^{P} \binom{(M-1)\varphi_k}{l_k} \cdot \{\mu_0(1+\mathcal{C}_k)\}^{l_k - (M-1)\varphi_k} \right\} \Bigg/$$
$$\left\{ \left( \sum_{\substack{k=1,\\k\neq i}}^{P} l_k \right) - h \right\}!$$
$$\left. \cdot \{-\mu_0(1+\mathcal{C}_i)\}^{h - \left(\sum_{k=1,k\neq i}^{P} l_k\right)} \right]^{\eta_h}. \quad (37)$$

Here, if we let

$$I_{\mathrm{CA}}(\underline{S_P}) = \sum_{i_1=1}^{P} \sum_{j_1=1}^{\varphi_{i_1}} \sum_{m=0}^{j_1-1} \sum_{i_2=1}^{P} \sum_{j_2=1}^{(M-1)\varphi_{i_2}} \binom{j_2+m-1}{m} T^m$$
$$\cdot \frac{\{(1+\mathcal{C}_{i_1})(1+S_{i_1})\}^{j_1-m}}{\left\{\frac{T}{(1+\mathcal{C}_{i_1})(1+S_{i_1})} + \frac{1}{(1+\mathcal{C}_{i_2})}\right\}^{j_2+m}}$$
$$\cdot R_{i_1,j_1}(\underline{S_P}) \cdot W_{i_2,j_2} \cdot (\mu_0)^{j_1+j_2} \quad (38)$$

the detection and false-alarm probabilities of the CA CFAR scheme can be written as

$$P_D = I_{\mathrm{CA}}(\underline{S_P}) \left\{1 - I_{\mathrm{CA}}(\underline{0_P})\right\}^{M-1} \quad (39)$$

and

$$P_{\mathrm{FA}} = \frac{1}{L_p} \left[ I_{\mathrm{CA}}(\underline{0_P}) \left\{1 - I_{\mathrm{CA}}(\underline{S_P})\right\} \left\{1 - I_{\mathrm{CA}}(\underline{0_P})\right\}^{M-2} \right]$$
$$+ \frac{L_p - 1}{L_p} \left[ I_{\mathrm{CA}}(\underline{0_P}) \left\{1 - I_{\mathrm{CA}}(\underline{0_P})\right\}^{M-1} \right] \quad (40)$$

respectively, where $\underline{0}_P$ is the all-zero vector with $P$ elements. Since $L_p$ is generally large ($> 100$), the false-alarm probability $P_{\mathrm{FA}}$ shown in (40) can be approximated as a constant

$$P_{\mathrm{FA}} \approx I_{\mathrm{CA}}(\underline{0}_P) \left\{1 - I_{\mathrm{CA}}(\underline{0}_P)\right\}^{M-1}. \tag{41}$$

Since $P = 1$ and $\varphi_P = L$ in a homogeneous environment, the vectors $\underline{S}_P$ and $\underline{0}_P$ both become scalars: $i_1 = i_2 = 1$, $\varphi_{i_1} = \varphi_{i_2} = L$, $R_{i_1,j_1}(\underline{S}_P) = 0$ for $j_1 = 1, 2, \ldots, L-1$, $R_{i_1,L}(\underline{S}_P) = \beta_{i_1}^L$, $W_{i_2,j_2} = 0$ for $j_2 = 1, 2, \ldots, L(M-1)-1$, and $W_{i_2,L(M-1)} = \beta_{i_2}^{L(M-1)}$. Thus, (38) can be rewritten as

$$I_{H,\mathrm{CA}}(\alpha) = \sum_{m=0}^{L-1} \left[ \binom{L(M-1)+m-1}{m} \left\{\frac{T}{1+\alpha}\right\}^m \cdot \left\{1 + \frac{T}{1+\alpha}\right\}^{-L(M-1)-m} \right] \tag{42}$$

which is the same result as that in the study in [10]. Here, the subscripts $H$ and CA of $I_{H,\mathrm{CA}}$ are used to denote "homogeneous" and "CA," respectively.

### B. GO CFAR Decision Processor

In the GO CFAR processor, the statistic $Z_m$ is formed as

$$Z_m = \max(Y_{m,1}, Y_{m,2}), \qquad m = 1, 2, \ldots, M. \tag{43}$$

Since $Y_{m,1}$ and $Y_{m,2}$ are independent and identically distributed, the pdf of $Z_m$ is given by [19]

$$\begin{aligned} p_{Z_m}(z) &= p_{Y_1}(z) F_{Y_2}(z) + p_{Y_2}(z) F_{Y_1}(z) \\ &= 2 p_{Y_1}(z) F_{Y_1}(z) \end{aligned} \tag{44}$$

where $p_{Y_i}(z)$ and $F_{Y_i}(z)$ are the pdf and cumulative distribution function (cdf) of $Y_{m,i}$, respectively, for $i = 1, 2$.

Since $Y_{m,i}$ is the sum of $(M-1)/2$ independent random variables $\{U_m\}$, the mgf of $Y_{m,i}$ is given by

$$M_{Y_i}(t) = \prod_{i=1}^{P} \left(\frac{\beta_i}{\beta_i + t}\right)^{\frac{(M-1)\varphi_i}{2}}. \tag{45}$$

Comparing (45) with (16), it is easy to see that the pdf of $Y_{m,i}$ is

$$p_{Y_i}(z) = \sum_{i=1}^{P} \sum_{j=1}^{\frac{(M-1)\varphi_i}{2}} \frac{V_{i,j}}{(j-1)!} z^{j-1} e^{-\beta_i z} \tag{46}$$

and consequently, the cdf of $Y_{m,i}$ is

$$F_{Y_i}(z) = 1 - \sum_{i=1}^{P} \sum_{j=1}^{\frac{(M-1)\varphi_i}{2}} \sum_{r=0}^{j-1} \frac{V_{i,j}}{r! \beta_i^{j-r}} z^r e^{-\beta_i z}. \tag{47}$$

Here

$$\begin{aligned} V_{i,j} = \sum_{\{\eta\}_4} & \frac{1}{\eta_1! \eta_2!, \ldots, \eta_n!} \\ & \times \frac{(-1)^q q! \prod_{k=1}^{P} \beta_k^{\frac{(M-1)\varphi_i}{2}}}{\prod_{k=1, k\neq i}^{P} (\beta_k - \beta_i)^{\frac{(M-1)(q+1)\varphi_i}{2}}} \cdot \prod_{h=1}^{n} \left(\frac{1}{h!}\right)^{\eta_h} \\ & \times \left[ \sum_{l_1=0}^{\frac{(M-1)\varphi_1}{2}} \cdots \sum_{l_{i-1}=0}^{\frac{(M-1)\varphi_{i-1}}{2}} \sum_{l_{i+1}=0}^{\frac{(M-1)\varphi_{i+1}}{2}} \cdots \sum_{l_P=0}^{\frac{(M-1)\varphi_P}{2}} \right. \\ & \times \left\{ \prod_{k=1, k\neq i}^{P} \binom{\frac{(M-1)\varphi_k}{2}}{l_k} \beta_k^{\frac{(M-1)\varphi_k}{2} - l_k} \right\} \\ & \cdot \left(\sum_{k=1, k\neq i}^{P} l_k\right)! \Big/ \left\{ \left(\sum_{k=1, k\neq i}^{P} l_k\right) - h \right\}! \\ & \left. \cdot (-\beta_i)^{\left(\sum_{k=1,k\neq i}^{P} l_k\right) - h} \right]^{\eta_h} \end{aligned} \tag{48}$$

for $i = 1, 2, \ldots, P$; $j = 1, 2, \ldots, [(M-1)\varphi_i/2]$; and $\beta_i = 1/\{\mu_0(1 + \mathcal{C}_i)\}$. In (48), the symbol $\sum_{\{\eta\}_4}$ represents the summation over all non-negative integer solutions $\{\eta_m\}$ of the simultaneous equations $\sum_{m=1}^{n} m\eta_m = [(M-1)\varphi_i/2] - j$ and $\sum_{m=1}^{n} \eta_m = q$.

Now, as we have done for the CA CFAR scheme, we first calculate $E_{Z_k}\{Z_k^m e^{-\beta_{i_1} T Z_k}\}$ using (46) and (47), and then obtain $P(U_k > TZ_k | H_1)$. Then, with $\beta_{i_1} = 1/\{\mu_0(1 + \mathcal{C}_{i_1})(1 + S_{i_1})\}$, $\beta_{i_2} = 1/\{\mu_0(1 + \mathcal{C}_{i_2})\}$, and $\beta_{i_3} = 1/\{\mu_0(1 + \mathcal{C}_{i_3})\}$, the detection and false-alarm probabilities of the GO CFAR scheme can be expressed as

$$P_D = I_{\mathrm{GO}}(\underline{S}_P) \left\{1 - I_{\mathrm{GO}}(\underline{0}_P)\right\}^{M-1} \tag{49}$$

and

$$\begin{aligned} P_{\mathrm{FA}} &= \frac{1}{L_p} \left[ I_{\mathrm{GO}}(\underline{0}_P) \left\{1 - I_{\mathrm{GO}}(\underline{S}_P)\right\} \left\{1 - I_{\mathrm{GO}}(\underline{0}_P)\right\}^{M-2} \right] \\ &\quad + \frac{L_p - 1}{L_p} \left[ I_{\mathrm{GO}}(\underline{0}_P) \left\{1 - I_{\mathrm{GO}}(\underline{0}_P)\right\}^{M-1} \right] \\ &\approx I_{\mathrm{GO}}(\underline{0}_P) \left\{1 - I_{\mathrm{GO}}(\underline{0}_P)\right\}^{M-1} \end{aligned} \tag{50}$$

respectively, where

$$
\begin{aligned}
&I_{\mathrm{GO}}(\underline{S_P})\\
&= 2\sum_{i_1=1}^{P}\sum_{j_1=1}^{\varphi_{i_1}}\sum_{m=0}^{j_1-1}\sum_{i_2=1}^{P}\sum_{j_2=1}^{\frac{(M-1)\varphi_{i_2}}{2}}\binom{j_2+m-1}{m}T^m\\
&\cdot\frac{\{(1+\mathcal{C}_{i_1})(1+S_{i_1})\}^{j_1-m}\cdot R_{i_1,j_1}(\underline{S_P})\cdot V_{i_2,j_2}\cdot(\mu_0)^{j_1+j_2}}{\left\{\frac{T}{(1+\mathcal{C}_{i_1})(1+S_{i_1})}+\frac{1}{(1+\mathcal{C}_{i_2})}\right\}^{j_2+m}}\\
&-2\sum_{i_1=1}^{P}\sum_{j_1=1}^{\varphi_{i_1}}\sum_{m=0}^{j_1-1}\sum_{i_2=1}^{P}\sum_{j_2=1}^{\frac{(M-1)\varphi_{i_2}}{2}}\sum_{i_3=1}^{P}\sum_{j_3=1}^{\frac{(M-1)\varphi_{i_3}}{2}}\sum_{r=0}^{j_3-1}(\mu_0)^{j_1+j_2+j_3}\\
&\cdot\frac{\{(1+\mathcal{C}_{i_1})(1+S_{i_1})\}^{j_1-m}\{(1+\mathcal{C}_{i_3})\}^{j_3-r}}{\left\{\frac{T}{(1+\mathcal{C}_{i_1})(1+S_{i_1})}+\frac{1}{(1+\mathcal{C}_{i_2})}+\frac{1}{(1+\mathcal{C}_{i_3})}\right\}^{j_2+r+m}}\\
&\cdot\frac{(j_2+r+m-1)!}{(j_2-1)!r!m!}\cdot T^m\cdot R_{i_1,j_1}(\underline{S_P})\cdot V_{i_2,j_2}\cdot V_{i_3,j_3}
\end{aligned}
\tag{51}
$$

and

$$
\begin{aligned}
V_{i,j}=&\sum_{\{\eta\}_4}\frac{(-1)^q q!\cdot(\mu_0)^{\frac{(M-1)}{2}\left\{-L+(q+1)\sum_{k=1,k\neq i}^{P}\varphi_k\right\}}}{\eta_1!\eta_2!,\ldots,\eta_n!}\\
&\cdot\frac{\prod_{k=1}^{P}\left\{\frac{1}{(1+\mathcal{C}_k)}\right\}^{\frac{(M-1)\varphi_k}{2}}}{\prod_{k=1,k\neq i}^{P}\left\{\frac{1}{(1+\mathcal{C}_k)}-\frac{1}{(1+\mathcal{C}_i)}\right\}^{\frac{(M-1)(q+1)\varphi_k}{2}}}\cdot\prod_{h=1}^{n}\left(\frac{1}{h!}\right)^{\eta_h}\\
&\times\left[\sum_{l_1=0}^{\frac{(M-1)\varphi_1}{2}}\cdots\sum_{l_{i-1}=0}^{\frac{(M-1)\varphi_{i-1}}{2}}\sum_{l_{i+1}=0}^{\frac{(M-1)\varphi_{i+1}}{2}}\cdots\sum_{l_P=0}^{\frac{(M-1)\varphi_P}{2}}\right.\\
&\quad\times\left(\sum_{k=1,k\neq i}^{P}l_k\right)!\cdot\left\{\prod_{k=1,k\neq i}^{P}\binom{\frac{(M-1)\varphi_k}{2}}{l_k}\right.\\
&\quad\left.\times\{\mu_0(1+\mathcal{C}_k)\}^{l_k-\frac{(M-1)\varphi_k}{2}}\right\}\Big/\\
&\quad\left.\left\{\left(\sum_{k=1,k\neq i}^{P}l_k\right)-h\right\}!\{-\mu_0(1+\mathcal{C}_i)\}^{h-\left(\sum_{k=1,k\neq i}^{P}l_k\right)}\right]^{\eta_h}.
\end{aligned}
\tag{52}
$$

For a homogeneous environment, (51) can be rewritten as

$$
\begin{aligned}
I_{H,\mathrm{GO}}(\alpha)=2\sum_{m=0}^{L-1}&\left[\binom{\frac{L(M-1)}{2}+m-1}{m}\right.\\
&\times\left\{1+\frac{T}{1+\alpha}\right\}^{-\frac{L(M-1)}{2}-m}\\
&\cdot\left\{\frac{T}{1+\alpha}\right\}^m-\sum_{r=0}^{\frac{L(M-1)}{2}-1}\\
&\times\frac{\left(\frac{L(M-1)}{2}+m+r-1\right)!}{\left(\frac{L(M-1)}{2}-1\right)!m!r!}\left\{\frac{T}{1+\alpha}\right\}^m\\
&\left.\cdot\left\{2+\frac{T}{1+\alpha}\right\}^{-\frac{L(M-1)}{2}-m-r}\right]
\end{aligned}
\tag{53}
$$

following steps similar to those from (38) to (42).

### C. SO CFAR Decision Processor

In the SO CFAR scheme, we have

$$Z_m=\min(Y_{m,1},Y_{m,2}),\qquad m=1,2,\ldots,M \tag{54}$$

the pdf of which is given by [19]

$$
\begin{aligned}
p_{Z_m}(z)&=p_{Y_1}(z)+p_{Y_2}(z)-p_{Y_1}(z)F_{Y_2}(z)-p_{Y_2}(z)F_{Y_1}(z)\\
&=2p_{Y_1}(z)\left\{1-F_{Y_1}(z)\right\}.
\end{aligned}
\tag{55}
$$

After several manipulations using some of the results obtained in the derivations for the GO CFAR scheme, we can easily obtain the detection and false-alarm probabilities for the SO CFAR scheme as

$$P_D=I_{\mathrm{SO}}(\underline{S_P})\left\{1-I_{\mathrm{SO}}(\underline{0_P})\right\}^{M-1} \tag{56}$$

and

$$
\begin{aligned}
P_{\mathrm{FA}}=&\frac{1}{L_p}\left[I_{\mathrm{SO}}(\underline{0_P})\left\{1-I_{\mathrm{SO}}(\underline{S_P})\right\}\left\{1-I_{\mathrm{SO}}(\underline{0_P})\right\}^{M-2}\right]\\
&+\frac{L_p-1}{L_p}\left[I_{\mathrm{SO}}(\underline{0_P})\left\{1-I_{\mathrm{SO}}(\underline{0_P})\right\}^{M-1}\right]\\
\approx&\, I_{\mathrm{SO}}(\underline{0_P})\left\{1-I_{\mathrm{SO}}(\underline{0_P})\right\}^{M-1}
\end{aligned}
\tag{57}
$$

respectively, where

$$
\begin{aligned}
&I_{\mathrm{SO}}(\underline{S_P})\\
&=2\sum_{i_1=1}^{P}\sum_{j_1=1}^{\varphi_{i_1}}\sum_{m=0}^{j_1-1}\sum_{i_2=1}^{P}\sum_{j_2=1}^{\frac{(M-1)\varphi_{i_2}}{2}}\sum_{i_3=1}^{P}\sum_{j_3=1}^{\frac{(M-1)\varphi_{i_3}}{2}}\sum_{r=0}^{j_3-1}\\
&\times\frac{\{(1+\mathcal{C}_{i_1})(1+S_{i_1})\}^{j_1-m}\{(1+\mathcal{C}_{i_3})\}^{j_3-r}}{\left\{\frac{T}{(1+\mathcal{C}_{i_1})(1+S_{i_1})}+\frac{1}{(1+\mathcal{C}_{i_2})}+\frac{1}{(1+\mathcal{C}_{i_3})}\right\}^{j_2+r+m}\Big/(\mu_0)^{j_1+j_2+j_3}}\\
&\cdot\frac{(j_2+r+m-1)!}{(j_2-1)!r!m!}\cdot T^m\cdot R_{i_1,j_1}(\underline{S_P})\cdot V_{i_2,j_2}\cdot V_{i_3,j_3}.
\end{aligned}
\tag{58}
$$

The function $I_{\mathrm{SO}}(\underline{S_P})$ can be deduced from $I_{\mathrm{GO}}(\underline{S_P})$ by noting the similarity between (44) and (55). In a homogeneous environment, we can rewrite (55) as

$$
\begin{aligned}
I_{H,\mathrm{SO}}(\alpha)=2\sum_{m=0}^{L-1}\sum_{r=0}^{\frac{L(M-1)}{2}-1}&\frac{\left(\frac{L(M-1)}{2}+m+r-1\right)!}{\left(\frac{L(M-1)}{2}-1\right)!m!r!}\\
&\cdot\left\{2+\frac{T}{1+\alpha}\right\}^{-\frac{L(M-1)}{2}-m-r}\left\{\frac{T}{1+\alpha}\right\}^m.
\end{aligned}
\tag{59}
$$

Here, the subscripts $H$ and SO of $I_{H,\mathrm{SO}}$ are used to denote "homogeneous" and "SO," respectively.

## ACKNOWLEDGMENT

The authors would like to thank the invaluably constructive suggestions and helpful comments from the anonymous reviewers and the Associate Editor.

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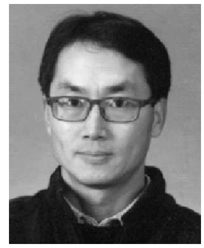

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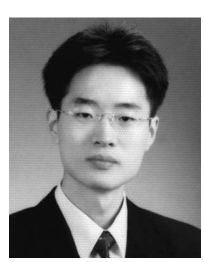

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