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Parametrizing the power spectrum: beyond the truncated Taylor expansion

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Abstract. The power spectrum is traditionally parametrized by a truncated Taylor series: $\ln \mathcal{P}(k) = \ln \mathcal{P}_* + (n_* - 1)\ln(k/k_*) + \frac{1}{2}n'_* \ln^2(k/k_*)$. It is reasonable to truncate the Taylor series if $|n'_* \ln(k/k_*)| \ll |n_* - 1|$, but it is not if $|n'_* \ln(k/k_*)| \gtrsim |n_* - 1|$. We argue that there is no good theoretical reason to prefer $|n'_*| \ll |n_* - 1|$, and show that current observations are consistent with $|n'_* \ln(k/k_*)| \sim |n_* - 1|$ even for $|\ln(k/k_*)| \sim 1$. Thus, there are regions of parameter space, which are both theoretically and observationally relevant, for which the traditional truncated Taylor series parametrization is inconsistent, and hence it can lead to incorrect parameter estimations. Motivated by this, we propose a simple extension of the traditional parametrization, which uses no extra parameters, but that, unlike the traditional approach, covers well motivated inflationary spectra with $|n'_*| \sim |n_* - 1|$. Our parametrization therefore covers not only standard slow-roll inflation models but also a much wider class of inflation models. We use this parametrization to perform a likelihood analysis for the cosmological parameters.

Keywords: cosmological perturbation theory, inflation, power spectrum

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Contents

1. Introduction

In estimating the cosmological parameters and the primordial power spectrum using the observational data, such as WMAP and SDSS [1]–[3], one introduces free parameters to parametrize the power spectrum $\mathcal{P}(k)$. There is some arbitrariness in how to do this beyond simply parametrizing the amplitude of a scale-invariant spectrum. It has become traditional to consider a truncated Taylor expansion about some particular pivot scale k_*, including two derivative terms, the spectral index $n - 1 \equiv \mathrm{d}\ln\mathcal{P}/\mathrm{d}\ln k$ and its running $n' \equiv \mathrm{d}n/\mathrm{d}\ln k$. This traditional parametrization is motivated by simplicity and the standard slow-roll approximation.

The standard slow-roll approximation is satisfied by the simplest single-component models of inflation, but one also has to be prepared for not so simple single-component models, and it may generically not be satisfied by multi-component models of inflation. Currently there is no observational reason to disfavour multi-component models of inflation, and there are some theoretical reasons to prefer them in terms of naturalness [4]. Thus using the standard slow-roll approximation to justify a parametrization of the power spectrum is dangerous.

The truncated Taylor expansion could give a poor approximation if the range of k of our interest extends far from k_*, unless we take a sufficient number of higher derivative terms in the Taylor expansion. However, increasing the number of derivative terms to as many as one desires would not be a practical approach for parameter estimation.

In this paper, we propose an improved parametrization of the power spectrum, which is as simple as the traditional truncated Taylor expansion in that it uses the same number of parameters, reproduces the traditional truncated Taylor expansion in the standard slow-roll limit, but has a better motivated form outside of this limit. Because our parametrization has the same number of free parameters as the traditional truncated Taylor expansion, it can be straightforwardly implemented in existing numerical codes.

The layout of our paper is as follows. The truncated Taylor expansion and its drawbacks are discussed in section 2. Our proposed parametrization is presented in section 3.1, and its inflationary motivation in section 3.2. In section 4 cosmological parameter estimation via the Markov chain Monte Carlo (MCMC) method using our parametrization is compared with that using the traditional truncated Taylor expansion. The conclusion and discussion are in section 5.

2. The traditional approach

We describe the traditional approach, explain the weakness of its traditional justification, and describe some possible undesirable consequences of adopting it.

2.1. The traditional approach: a truncated Taylor expansion of the power spectrum

Traditionally, the deviation from a scale-invariant spectrum is quantified by the tilt $n-1 \equiv \mathrm{d}\ln\mathcal{P}/\mathrm{d}\ln k$ and its running $n' \equiv \mathrm{d}n/\mathrm{d}\ln k$ by Taylor expanding in log–log space around a pivot point $\ln k_*$:

$$\ln\mathcal{P}(k) = \ln\mathcal{P}_* + (n_*-1)\ln\left(\frac{k}{k_*}\right) + \frac{1}{2}n'_*\ln^2\left(\frac{k}{k_*}\right). \qquad (2.1)$$

This traditional truncated Taylor expansion approach assumes that the second and higher derivatives of n are negligible, which is not a trivial assumption, especially for $|\ln(k/k_*)| \gtrsim 1$. Generally, one might expect this to be a good assumption if $|n'_*\ln(k/k_*)| \ll |n_*-1|$, but a bad one if $|n'_*\ln(k/k_*)| \gtrsim |n_*-1|$. We shall show in section 4 that current observations are consistent with $|n'_*\ln(k/k_*)| \sim |n_*-1|$, and hence that there is no observational justification for this assumption.

2.2. The traditional justification of the traditional approach: the standard slow-roll approximation

The standard slow-roll approximation assumes that the inflationary slow-roll parameters are both small *and* slowly varying. In terms of observable quantities, smallness of the relevant parameters translates to

$$|n-1| \ll 1 \qquad (2.2)$$

which is required by observations, while the slowly varying condition translates to the hierarchy

$$|n-1| \gg |n'| \gg |n''| \gg \cdots. \qquad (2.3)$$

Thus we see that the validity of the traditional truncated Taylor series approach is equivalent to the assumption of slowly varying slow-roll parameters. Thus if we assume the standard slow-roll approximation, as is often done, neglect of the higher order derivatives in equation (2.1) follows.

In terms of inflationary model building, for example to get enough inflation, there is no need for the slow-roll parameters to be either small or slowly varying. However, as noted above, smallness of the relevant parameters is required for approximate scale invariance of the spectrum. In many simple single-component models of inflation, the requirement of small slow-roll parameters forces the slow-roll parameters to be slowly varying, but there is no general requirement of this. Also, there is no need to restrict to a single-component inflaton. From the particle theory viewpoint, all scalar fields are complex in supersymmetry, and there are many scalar fields. From the inflationary model building point of view, it allows extra freedom to build more natural models; see for example [4]. In these multi-component models, the relevant slow-roll parameters[6] are small but, without fine-tuning, most of the irrelevant slow-roll parameters will not be small. The non-small irrelevant slow-roll parameters then tend to cause the relevant slow-roll parameters not to vary slowly. Thus, although standard slow-roll could be considered the generic, though not exclusive, case in single-component models, non-slowly varying slow-roll parameters may be the more generic case in multi-component models. Thus, if one wants to cover all reasonable models and hence be able to distinguish amongst them, one should relax the assumption of slowly varying slow-roll parameters.

The general slow-roll approximation [5, 6] drops this extra assumption of slowly varying slow-roll parameters, covering the cases of

$$|n-1| \gtrsim |n'| \gtrsim |n''| \gtrsim \cdots. \tag{2.4}$$

Hence the general slow-roll approximation includes equation (2.3) as a special case, so we can test the assumption of the standard slow-roll approximation, rather than assuming it *a priori*.

2.3. Possible undesirable consequences of the traditional approach

If $|n'_* \ln(k/k_*)| \gtrsim |n_* - 1|$, then the truncated Taylor series gives a very unnatural form for the spectrum which is not motivated by any model of inflation, and which can give misleading parameter estimations. In general, the running can become appreciable for high k even if it is negligible for small k due to the possible k dependence of the running. Thus, if we ignore the running of the running, as is done in the traditional truncated Taylor expansion, the running of the spectrum at high k may be too biased by the data at low k because the running at high k is forced to be same as that at low k. In those cases, we should take account of the running of the running and more generally all the higher order terms. Considering infinitely many terms or an infinite number of parameters would, however, be impractical in actual parameter estimations, and we shall suggest a more appropriate way of parametrizing the power spectrum in section 3. Thus taking only the first few terms in the Taylor expansion could give a poor representation of the power spectrum and lead us to incorrect parameter estimations.

3. A better approach

We suggest a different way to parametrize the spectrum, which does not require any extra parameters compared with the traditional truncated Taylor expansion parametrization.

[6] In multi-component models, there are many slow-roll parameters. The relevant slow-roll parameters are the ones that directly affect the spectrum; see [5].

In the standard slow-roll limit it reduces to the usual truncated Taylor series form, but has a more sensible extension beyond the standard slow-roll limit.

3.1. Improved parametrization

Instead of the truncated Taylor expansion, we parametrize the spectrum as

$$\ln \mathcal{P}(k) = \ln \mathcal{P}_* + \frac{(n_*-1)^2}{n'_*}\left[\left(\frac{k}{k_*}\right)^{n'_*/(n_*-1)} - 1\right] \tag{3.1}$$

and hence the spectral index as

$$n - 1 = (n_* - 1)\left(\frac{k}{k_*}\right)^{n'_*/(n_*-1)} \tag{3.2}$$

and the running as

$$n' = n'_*\left(\frac{k}{k_*}\right)^{n'_*/(n_*-1)} \tag{3.3}$$

where, as before, k_* is an arbitrary reference point. Note that our parametrization has a simple form and uses the same number of parameters as the truncated Taylor series of equation (2.1). Expanding equation (3.1) in the limit $|n'_* \ln(k/k_*)| \ll |n_* - 1|$, we see that our parametrization reproduces the standard slow-roll Taylor series

$$\ln \mathcal{P} = \ln \mathcal{P}_* + (n_* - 1)\ln\left(\frac{k}{k_*}\right) + \frac{1}{2}n'_* \ln^2\left(\frac{k}{k_*}\right) + \cdots \tag{3.4}$$

of equation (2.1), but with a more sensible extension to the domain $|n'_* \ln(k/k_*)| \gtrsim |n_* - 1|$.

3.2. Inflationary motivation of our parametrization

The truncated Taylor series and our parametrization are equivalent for $|n'_* \ln(k/k_*)| \ll |n_* - 1|$, but our parametrization is also well motivated for $|n'_* \ln(k/k_*)| \gtrsim |n_* - 1|$.

Specifically, in the general slow-roll approximation the spectrum for multi-component inflation models is given by [5][7]

$$\ln \mathcal{P} = \int_0^\infty \frac{\mathrm{d}\xi}{\xi}\left[-k\xi\, W'(k\xi)\right]\left[\ln \Pi^2 + \frac{2}{3}\frac{\Pi'}{\Pi}\right] \tag{3.5}$$

where $\Pi = \Pi(\ln\xi)$, $\Pi' \equiv \mathrm{d}\Pi/\mathrm{d}\ln\xi$, and ξ is minus the conformal time: $\xi = -\int \mathrm{d}t/a \simeq 1/aH$ where a is the scale factor and H is the Hubble parameter. The window function $-x\, W'(x)$ is given in [5, 6] and has the properties

$$\int_0^\infty \frac{\mathrm{d}x}{x}\left[-xW'(x)\right] = 1 \tag{3.6}$$

and

$$\lim_{x\to 0}\left[-x\, W'(x)\right] = \mathcal{O}\left(x^2\right). \tag{3.7}$$

[7] Note that in some cases there may be extra terms. See [5] for the full story.

Π represents the relevant inflationary parameters that directly affect the spectrum and is defined in [5].

At zeroth order, Π^2 can be regarded as constant and the normalization property of the window function leads to

$$\ln \mathcal{P} = \ln \Pi^2. \tag{3.8}$$

Our parametrization of equation (3.1) arises from a Π^2 of the form

$$\ln \Pi^2 = \ln \Pi_\infty^2 - B\xi^{-\nu} \tag{3.9}$$

where Π_∞^2 is a constant and the second term with the constant coefficient B is assumed to be small, or

$$\frac{\Pi'}{\Pi} = \frac{1}{2}\nu B\xi^{-\nu}. \tag{3.10}$$

For $\nu < 2$, substituting into equation (3.5) gives

$$\ln \mathcal{P} = \ln \Pi_\infty^2 - Ak^\nu \tag{3.11}$$

where A is a constant depending linearly on B and non-trivially on ν. For $\nu \geq 2$, the late time part of the integral dominates and $n-1$ becomes proportional to k^2. See [4] for a more detailed discussion. Comparing with equation (3.1), we have

$$\frac{n_*'}{n_* - 1} = \begin{cases} \nu & \text{for } \nu < 2 \\ 2 & \text{for } \nu \geq 2 \end{cases} \tag{3.12}$$

with standard slow-roll corresponding to $\nu \ll 1$. Note that simple single-component inflation models tend to satisfy $|n'| \sim |n-1|^2 \ll |n-1|$. A concrete example of a particle theory motivated inflationary model which gives a spectrum of the form of equation (3.12) is given in [4].

Of course, the general slow-roll approximation can also accommodate cases where Π'/Π cannot be expressed as a power of ξ. In other words, this power law case is still a special case of the more general class of inflation models which the general slow-roll approximation can handle.

4. Likelihood analysis

We perform an estimation of cosmological parameters using different parametrizations of the primordial perturbation spectrum: (1) our spectral parametrization given by equation (3.2); (2) the truncated Taylor expansion given by equation (2.1); and (3) the case of constant spectral index. We model a flat universe with the cosmological parameters $\Omega_b h^2$, $\Omega_c h^2$, Θ_s, $\ln A$ and τ. Here, A is related to the amplitude of curvature perturbations at horizon crossing, $|\Delta_R|^2 = 2.95 \times 10^{-9} A$ at the scale $k_* = 0.05h$ Mpc^{-1}. The angular acoustic peak scale Θ_s is the ratio of the sound horizon at last scattering to that of the angular diameter distance to the surface of last scattering, and is a useful proxy for the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc [7].

We use a Markov chain Monte Carlo (MCMC) technique with a modified form of CosmoMC [8]. We use the CMB data from first-year WMAP [9], ACBAR [10], CBI [11] and VSA [12], the galaxy power spectrum from SDSS [13] and Lyman-α [14]. Each MCMC

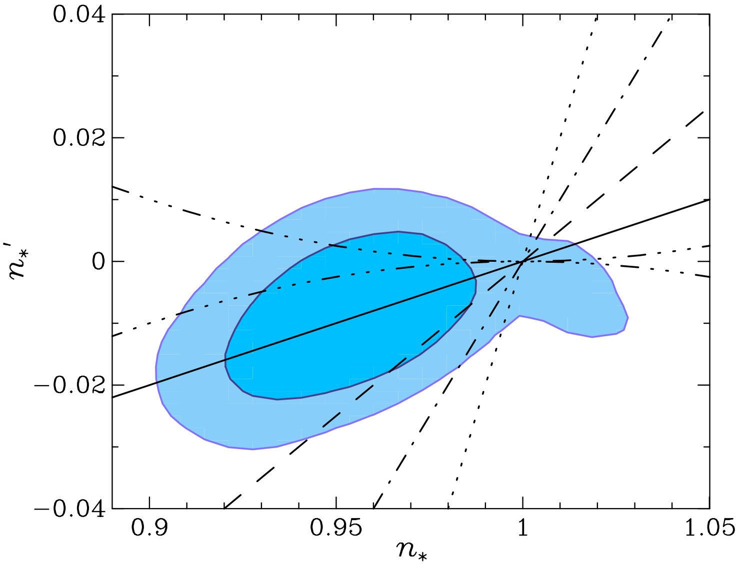

Figure 1. Shown are the likelihood contours of 68.3% and 95.4% for (n_*, n_*') in our parametrization, using observations of the CMB, the SDSS 3D power spectrum of galaxies, and the matter power spectrum inferred from the SDSS Lyman-α forest. Also shown are the lines of $n_*'/(n_* - 1) = 0.2, 0.5, 1, 2$ as solid, dashed, dot–dashed and dotted, respectively. The curves $n_*'/(n_* - 1)^2 = \pm 1$ are triple-dot–dashed.

analysis included approximately 10^5 points, with an acceptance efficiency of approximately 45%. The parameter space was sampled from flat priors:

$$\begin{aligned}
&0.005 \le \Omega_b h^2 \le 0.1\\
&0.01 \le \Omega_c h^2 \le 0.99\\
&0.005 \le \Theta_s \le 0.1\\
&-0.68 \le \ln A \le 0.62\\
&0.01 \le \tau \le 0.3\ (0.8)\\
&0.5 \le n_* \le 1.5\\
&-0.5 \le n_*' \le 0.5
\end{aligned} \tag{4.1}$$

which are all well outside of the regions of high probability, except for the requirement of $\tau \le 0.3$ for our parametrization case (1), which we include to exclude unphysically high optical depths otherwise allowable by the data and this form of the power spectrum. We use $\tau \le 0.8$, outside appreciable levels of the PDF for cases (2) and (3). Since currently there are only upper limits on the contribution of tensor perturbations, we do not include tensor perturbations in our analysis.

The 2D likelihood of the parameters n_*' and n_* is shown in figure 1, where we also show lines of constant $n_*'/(n_* - 1)$ and the curves for $n_*'/(n_* - 1)^2 = \pm 1$. Note that in simple single-component inflation models one often gets $|n'| \sim |n-1|^2 \ll |n-1|$, and our parametrization indicates the current data to be consistent with a range well beyond that where this relation holds. In figure 2, we show the cosmological parameter estimation using the three different parametrizations, and in figure 3, we show the estimations of n_*

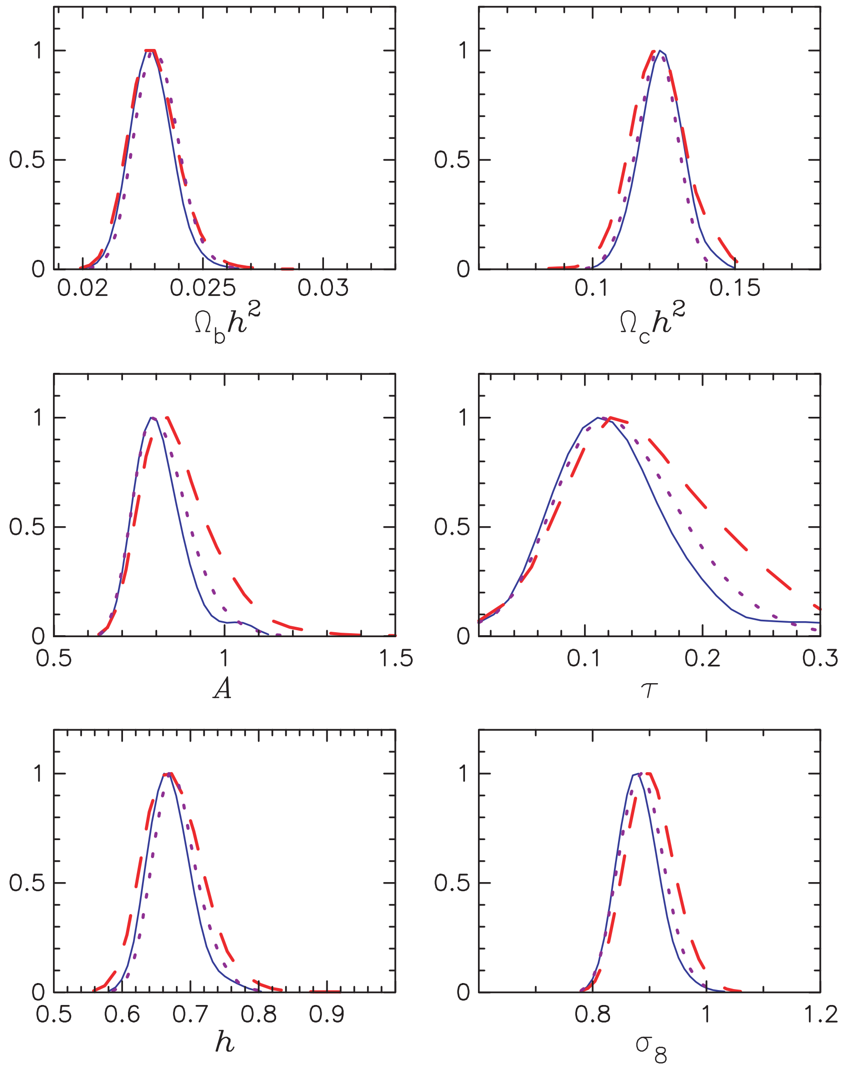

Figure 2. Plotted are the marginalized one-dimensional probability distribution functions for cosmological parameter estimation using our parametrization (solid), the truncated Taylor expansion (dashed) and constant spectral index (dotted).

and $n'_*/(n_* - 1)$. The central values for the models' parameters, their uncertainty and χ^2 are given in table 1.

In figure 1, the 95.4% CL contour region has a stretched region towards positive $n_* - 1$ and negative n'_* compared with the elliptic shaped 68.3% CL contour. This arises due to the combination of the fit trying to satisfy the low ℓ multipoles simultaneously with the high optical depth allowed.

The data used in our analysis cover a k range of $\Delta \ln k \sim 10$. This means the central values in table 1 giving $|n'_*| \sim 0.2\,|n_* - 1|$ for our parametrization, or $|n'_*| \sim 0.4\,|n_* - 1|$ for the truncated Taylor series, are inconsistent with the range of validity of the truncated Taylor series, as they give $|n'_* \ln(k/k_*)| \sim |n_* - 1|$. Note that the central values of n'_* differ because, for $n'_*/(n_* - 1) > 0$, our parametrization alters the power spectrum more

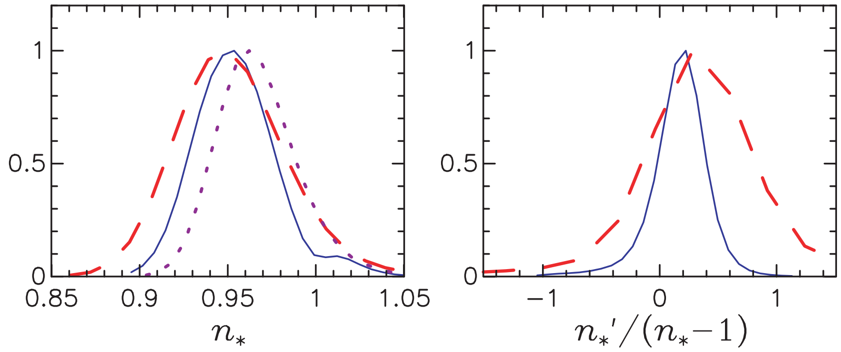

Figure 3. In the left panel, we show the marginalized one-dimensional probability distribution functions for n_* for our parametrization (solid), for the truncated Taylor expansion (dashed) and for constant spectral index (dotted). The right panel is that for $n_*'/(n_*-1)$.

Table 1. Central values, 68.3% uncertainties and χ^2 for the different models of the primordial scalar spectrum.

	$n'=0$	$n''=0$	$n''=n'^2/(n-1)$
$\Omega_b h^2$	$0.02310^{+0.00087}_{-0.00088}$	$0.02298^{+0.00093}_{-0.00094}$	$0.02320^{+0.00081}_{-0.00080}$
$\Omega_c h^2$	$0.1223^{+0.0070}_{-0.0072}$	$0.1233^{+0.0092}_{-0.0092}$	$0.1220^{+0.0068}_{-0.0070}$
Θ_s	$1.0416^{+0.0041}_{-0.0041}$	$1.0420^{+0.0040}_{-0.0040}$	$1.0420^{+0.0040}_{-0.0040}$
τ	$0.134^{+0.021}_{-0.029}$	$0.157^{+0.024}_{-0.038}$	$0.138^{+0.024}_{-0.025}$
A	$0.814^{+0.080}_{-0.029}$	$0.854^{+0.104}_{-0.090}$	$0.803^{+0.068}_{-0.065}$
n_*	$0.967^{+0.021}_{-0.022}$	$0.953^{+0.029}_{-0.029}$	$0.955^{+0.021}_{-0.022}$
n_*'	—	$-0.019^{+0.014}_{-0.014}$	$-0.0087^{+0.0084}_{-0.0083}$
Ω_m	$0.318^{+0.042}_{-0.043}$	$0.324^{+0.056}_{-0.055}$	$0.330^{+0.044}_{-0.044}$
h	$0.680^{+0.032}_{-0.032}$	$0.678^{+0.039}_{-0.040}$	$0.671^{+0.030}_{-0.031}$
σ_8	$0.887^{+0.038}_{-0.038}$	$0.902^{+0.041}_{-0.041}$	$0.882^{+0.035}_{-0.036}$
χ^2	1664.7	1663.1	1664.1

strongly for $k > k_*$ than the truncated Taylor expansion. Despite the lack of robust data at k even larger than the SDSS Lyman-α data, this result is a good illustration of the danger of ignoring the higher derivative terms in the Taylor series when we deal with more precise cosmological data covering a wide range of scales in the future.

5. Conclusion and discussion

We discussed the possible bias and inconsistency in the cosmological parameter estimation induced by presuming a truncated Taylor series form for the power spectrum. We proposed an improved form for the power spectrum which is motivated from actual perturbation calculations applicable to a wide class of well motivated inflation models. The standard slow-roll truncated Taylor series form is just a special limit of our parametrization of the power spectrum. Our proposed form requires no additional free parameters compared with

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the traditional truncated Taylor expansion, and can be straightforwardly implemented in existing codes. Our form for the spectrum is simple:

$$\ln \mathcal{P} = \ln \mathcal{P}_0 - Ak^{\nu} \tag{5.1}$$

$$n - 1 = -\nu A k^{\nu} \tag{5.2}$$

with only three constant free parameters, $\mathcal{P}_0$, A and ν. There exist several possible ways to parametrize this form. The parametrization used in this paper, equation (3.1), was motivated by clarity in the comparison with the truncated Taylor expansion. The standard slow-roll cases correspond to $\nu = n'_*/(n_* - 1) \ll 1$. In performing the likelihood analysis via MCMC, we found that the central values for $n'_*/(n_* - 1)$ were inconsistent with $|n'_* \ln(k/k_*)| \ll |n_* - 1|$, the basic assumption behind the truncated Taylor series.

Further to the technical discussion on the form of the power spectrum using the general slow-roll formula in section 3.2, let us point out that the form we presented in this paper should be regarded as an asymptotic form for small Ak^{ν}. This asymptotic form is sufficient for the range of our analysis in this paper, i.e. the data up to $k \sim 6h$ Mpc^{-1} from Lyman-α. If we can include data at higher k, well beyond the Lyman-α range, covering a change in $\ln \mathcal{P}$ of order unity, we may need to use the original form of [4] without asymptotic approximation. Otherwise $\mathcal{P}$ would decrease too rapidly for higher k (assuming $\nu > 0$). Alternatively we may soften the form of our parametrization, for example as

$$\ln \mathcal{P} = \ln \mathcal{P}_0 - \ln\left(1 + Ak^{\nu}\right) \tag{5.3}$$

$$n - 1 = -\frac{A\nu k^{\nu}}{1 + Ak^{\nu}}, \tag{5.4}$$

so that it gives equations (5.1) and (5.2) for small Ak^{ν}, and

$$\ln \mathcal{P} = \ln \mathcal{P}_* - \nu \ln\left(\frac{k}{k_*}\right) \tag{5.5}$$

$$n - 1 = -\nu \tag{5.6}$$

for large Ak^{ν}, or ideally add more parameters.

Simple single-component inflation models require the standard slow-roll approximation to produce a flat spectrum, but this is not the case for multi-component inflation models. In this sense, our parametrization would be of great interest for multi-component inflation models, such as that given in [4].

Our parametrization, the traditional truncated Taylor expansion and the case of constant spectral index led to quite similar, though not identical, results for the cosmological parameter estimations using the currently available data dominated by large scale observations such as CMB and galaxy surveys. This indicates that currently the running is not crucial to the cosmological parameter estimations, but this may change with reduced error bars in the near future. Application of a parametrization such as ours would also be important for modelling very small scale structure, such as the first objects in the universe [15] which could extend the range up to $k \sim 10^6 h$ Mpc^{-1}.

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