단일 링크 유연성 팔의 강인한 제어

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Robust Control of a One-Link Flexible Arm

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Abstract

Lightweight robot manipulators have considerable structural flexibility. Hence, the elastic behavior of the robot arms must be considered in control system design. Owing to the complexity of a dynamic model of flexible robots, it is desirable to design a controller simple. Furthermore, since the robot must handle a wide variety of payloads, the robustness of the control system becomes very important. In this paper, a simple and robust control system is proposed. The closed-loop system is shown to be stable in the sense of Lyapunov and robust to uncertainty in system parameters. The simulation results are presented to show the robustness of the simple controller against payload variations.

1 Introduction

Robot manipulators have been widely used both in dangerous circumstances and for industrial applications. They have been traditionally modeled as a chain of rigid links with colocated actuators and sensors, to ensure stable and reliable control. Moreover, the robot arms are generally made large and massive in order to remain its rigidity while carrying an assigned payload. Present generation manipulators are limited to a load-carrying capacity of typically 5–10% of their own weight by the need of structural rigidity.

Today, there is an increasing requirement for manipulators with high speed, precision and payload-handling capabilities as a result of demand for higher productivity. These qualities cannot be achieved with existing massive and heavy robot manipulators. For higher operating speeds, the manipulators should be made lightweight, but lighter members are more likely to deform elastically. Therefore, it is necessary to include dynamic effects of the distributed link flexibility in the model of the manipulators.

Such flexible manipulators provide diverse advantages as follows: higher speed, smaller actuators, lower energy consumption, lower overall cost, safer operation due to reduced inertia, less bulky design, lighter overall mass to be transported, and so on. For the flexible manipulators, however, the equations describing the manipulator dynamics become more complex; besides, they complicate the control system design which focuses primarily on the compensation for bending effects. Furthermore, since the robot must handle a wide variety of payloads, the robustness of the control system becomes very important.

Recently, numerous investigators have analyzed several modeling methods of a flexible manipulator. One of them is a modal analysis method. Dominant eigen-modes are identified by experiment, approximated modes are obtained through solving the governing partial differential equations and assumed modes can be chosen from approximation of the actual system dynamics. Finite element method (FEM) has also been used. This method discretizes the actual system into a number of elements, whose elastic and inertial properties are obtained from the actual system. These methods give the approximated static and dynamic properties of the actual system. Because the dynamics of mechanical systems with distributed flexibility are described by infinite-dimensional mathematical models, and a finite-dimensional model of the system is needed for the design of a finite-dimensional controller.

There are also various schemes proposed for the design of controllers. A technique for end-point control corroborated by experimental work has been introduced by Cannon and Schmitz [1]. By using the Linear Quadratic Gaussian (LQG) controller design method, they successfully implement an noncolocated controller for the robot. However, since the controller is sensitive to parameter variations, its performance will be degraded when payload or typical parameters of the robot are varying with time. Rovner and Cannon [2] used Recursive Least Square (RLS) algorithm to identify the system transfer function with unknown payload on the tip. The scheme requires a learning period which takes about two seconds with a sampling frequency of 50Hz. The experiments have shown good results, but the robustness of the algorithm still depends upon the number of coefficients of the transfer function to be identified. Other approaches are as follows: finite element approach, modal control, Model Reference Adaptive Control (MRAC), conventional control, feedforward control, and combined state space and frequency domain techniques, acceleration feedback control [3]–[5].

For the design of a flexible manipulator controller, there are two facts to consider: first, the simplicity; second, the robustness. Owing to the complexity of dynamic model of flexible manipulators, it is desirable to design a controller simple. Furthermore, robustness of the controller is also very important for the robot to handle a wide range of payload variations. In general, control systems with state observer have been used. The control systems, however, require heavy computational load. Moreover, observer must be varied in operation to cope with payload variations. Hence, load forecast is needed.

In this paper, a simple and robust controller is proposed. For the simplicity, the control system only uses measurable data such as: hub ratio, tip position and tip ratio. Sliding mode control method, one of robust control methods, is introduced for the robustness. On account of robustness, load forecast is not necessary.

The remainder of this paper is organized as follows: The mathematical model of a single-link flexible arm is described in Section 2. On the basis of the mathematical model, a simple and robust controller is designed in Section 3. In Section 4, simulation results on position control with payload variation are presented. The conclusions are given in Section 5.

2 Dynamic model of a flexible robot arm

Consider a uniform, slender beam connected via a rigid hub to the armature of an electric motor and having a payload as shown in Figure 1. Where x, L, ρ, E, I and M_p are distance along the length of the beam, length of the beam, mass per unit length, Young's modulus of elasticity and cross-sectional area moment of inertia and payload mass. I_h is the hub inertia and I_b is the inertia of the beam about the motor armature ($= \frac{1}{3}\rho L^3$).

The following assumptions are made:

- The deflection w is small ($w \ll L$).
- Euler-Bernoulli beam assumptions are used.
- Beam inertia and flexibility are uniformly distributed over the link length.
- The motion occurs only in the horizontal plane.

Because of the last assumption, the effects of gravity are not considered.

The displacement of points along the deformed profile of the beam is described in terms of radial and circumferential coordinates x and y. From Figure 1, it is apparent that y is related to the angle of rotation of rigid mode, q_o, and the flexural displacement of the beam, w, as follows:

$$y(x,t) = w(x,t) + x \cdot q_o(t).$$

In terms of this variable a fourth-order partial differential equation of motion for a single-link flexible arm is given in the form [1]

$$EI\frac{\partial^4 y}{\partial x^4} + \rho\frac{\partial^2 y}{\partial t^2} = 0 \tag{1}$$

with four boundary conditions:

$$y(0,t) = 0, \tag{2}$$

$$EIy''(0,t) + \tau - I_h\ddot{\theta}_h = 0, \tag{3}$$

$$EIy''(L,t) = 0, \tag{4}$$

$$EIy'''(L,t) - M_p\ddot{y}(L,t) = 0. \tag{5}$$

Then the following equation can be obtained.

$$\ddot{q}_i + \omega_i^2 q_i = \frac{\phi_i'(0)}{I_T}\tau, \qquad \text{for } i = 0, 1, 2, \cdots.$$

The model can be reconfigured in the state-space form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\tau, \\ \mathbf{y} &= \mathbf{C}\mathbf{x}, \end{aligned} \tag{6}$$

where

$$\mathbf{x}^t = [\, q_0 \;\; \dot{q}_0 \;\; q_1 \;\; \dot{q}_1 \;\; \cdots \;\; q_n \;\; \dot{q}_n \,],$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & & & & & \\ 0 & 0 & & & & & \\ & & 0 & 1 & & & \\ & & -\omega_1^2 & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & 1 \\ & & & & & -\omega_n^2 & 0 \end{bmatrix},$$

$$\mathbf{B}^t = \frac{1}{I_T}[\, 0 \;\; 1 \;\; 0 \;\; \phi_1'(0) \;\; \cdots \;\; 0 \;\; \phi_n'(0) \,],$$

$$\mathbf{C} = \begin{bmatrix} L & 0 & \phi_1(L) & 0 & \cdots & \phi_n(L) & 0 \\ 0 & L & 0 & \phi_1(L) & \cdots & 0 & \phi_n(L) \\ 0 & 1 & 0 & \phi_1'(0) & \cdots & 0 & \phi_n'(0) \end{bmatrix},$$

where ^{t} denotes transpose and the three rows of C are the tip-position sensor, tip-rate sensor and the hub-rate sensor measurement vectors.

Practically, a model based on as few as four modes may be suitable for simulation and control design. In this paper, the model based on three modes is considered.

3 Design of a controller

In this paper, one of the control objectives is to introduce additional damping into the flexible motion. This has been done by using additional sensors to measure the flexible link vibrations and feed measurements of sensors back to the controller. Generally, strain gauges have been used in order to observe the states. This method, however, leads to heavy computational load. Hence, a simple control system using only directly measurable outputs is presented in this section.

Theoretically, any flexible system has an infinite number of elastic modes. Due to physical limitations, a limited number of sensors and actuators can be applied, thus restricting the controller design to a few critical modes in case of state utilization. Note that outputs of sensors would contain information about the unmodeled as well as the modeled modes. This is referred to as observation spillover. Similarly, the control action would affect both the modeled and unmodeled modes leading to the control spillover. Based on the work done by Balas [3], the higher unmodeled modes may have a harmful or destabilizing effect on the system response. To avoid the observation spillover, low pass filtering of the sensor outputs can be used. Control spillover can also be avoided by inserting the Low Pass Filter (LPF) in the controller output stage. In this paper, the controller is designed based on the model equation of (6).

For the system (6), consider the following switching surface

$$s_{rigid} = \dot{e}_{rigid} + \lambda_1 e_t. \tag{7}$$

In the sliding mode, $\dot{s}_{rigid} = 0$. Applying this condition to equation (7) leads to the following equation:

$$\begin{aligned} \dot{s}_{rigid} &= \ddot{e}_{rigid} + \lambda_1 \dot{e}_t \\ &= -\ddot{q}_0 + \lambda_1 \dot{e}_t \\ &= -\frac{1}{I_T}\tau + \lambda_1 \dot{e}_t \\ &= 0. \end{aligned}$$

Therefore, equivalent iuput torque is

$$\tau_{eq} = \hat{I}_T \lambda_1 \dot{e}_t.$$

Now, consider the following control law

$$\tau = \hat{I}_T\left(\lambda_1 \dot{e}_t + \lambda_2 \dot{e}_h + K sgn(s)\right), \tag{8}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$, and

$$\begin{aligned} s &= \dot{e}_t + \lambda_1 e_t, \\ sgn(s) &= \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0, \end{cases} \\ K &= |(\beta - 1)\lambda_1 \dot{e}_t - \lambda_2 \dot{e}_h| + \eta, \\ \eta &> 0. \end{aligned}$$

With this controller, following lemma is obtained.

lemma 1 *For the system (6) and the proposed controller (8), the sliding mode exists provided that*

$$\sup_t \left| \sum_{i=1}^{\infty} \frac{\phi_i(L)}{L}\ddot{q}_i(t) \right| < \frac{\eta}{\beta}.$$

proof
It is sufficient to show that $s\dot{s} < 0$, if $s \neq 0$.

$$\begin{aligned} s\dot{s} &= s\left[\ddot{e}_t + \lambda_1 \dot{e}_t\right] \\ &= s\left[-\ddot{q}_0 - \sum_{i=1}^{\infty}\frac{\phi_i(L)}{L}\ddot{q}_i(t) + \lambda_1\dot{e}_t\right] \\ &= s\left[\lambda_1\dot{e}_t - \sum_{i=1}^{\infty}\frac{\phi_i(L)}{L}\ddot{q}_i(t) - \frac{1}{I_T}\tau\right] \\ &= s\left[\lambda_1\dot{e}_t - \sum_{i=1}^{\infty}\frac{\phi_i(L)}{L}\ddot{q}_i(t) - \frac{\hat{I}_T}{I_T}\left(\lambda_1\dot{e}_t + \lambda_2\dot{e}_h + K sgn(s)\right)\right] \\ &= s\left[\left(1 - \frac{\hat{I}_T}{I_T}\right)\lambda_1\dot{e}_t - \sum_{i=1}^{\infty}\frac{\phi_i(L)}{L}\ddot{q}_i(t) - \frac{\hat{I}_T}{I_T}\left(\lambda_2\dot{e}_h + K sgn(s)\right)\right] \\ &= s\frac{\hat{I}_T}{I_T}\left[\left(\frac{I_T}{\hat{I}_T} - 1\right)\lambda_1\dot{e}_t - \frac{I_T}{\hat{I}_T}\sum_{i=1}^{\infty}\frac{\phi_i(L)}{L}\ddot{q}_i(t) - \lambda_2\dot{e}_h - K sgn(s)\right] \\ &< 0, \qquad \text{if } s \neq 0. \end{aligned}$$

□

Furthermore, following lemma is also obtained for the stability.

theorem 1 *For the system (6) and the proposed controller (8), the overall system is stable in the sliding mode.*

Proof
Let us choose the Lyapunov function candidate as

$$V = \frac{1}{2}\left[\sum_{i=0}^{n}\left(\omega_i^2 q_i^2 + \dot{q}_i^2\right) + \frac{\lambda_1^3}{4\lambda_2}\frac{\hat{I}_T}{I_T}e_t^2\right].$$

Then V is a p.d.f. And in the sliding mode,

$$s = \dot{e}_t + \lambda_1 e_t = 0.$$

Therefore,

$$\begin{aligned} \dot{V} &= \sum_{i=0}^{n}\left(\omega_i^2 q_i \dot{q}_i + \dot{q}_i\left(-\omega_i^2 q_i + \frac{\hat{I}_T}{I_T}\phi_i'(0)\tau\right)\right) + \frac{\lambda_1^3}{4\lambda_2}\frac{\hat{I}_T}{I_T}e_t\dot{e}_t \\ &= \frac{\hat{I}_T}{I_T}\sum_{i=0}^{n}\phi_i'(0)\dot{q}_i\left(\lambda_1\dot{e}_t + \lambda_2\dot{e}_h\right) + \frac{\lambda_1^3}{4\lambda_2}\frac{\hat{I}_T}{I_T}\left(-\frac{\dot{e}_t}{\lambda_1}\right)\dot{e}_t \\ &= -\frac{\hat{I}_T}{I_T}\dot{e}_h\left(\lambda_1\dot{e}_t + \lambda_2\dot{e}_h\right) - \frac{\lambda_1^2}{4\lambda_2}\frac{\hat{I}_T}{I_T}\dot{e}_t^2 \\ &= -\frac{1}{4\lambda_2}\frac{\hat{I}_T}{I_T}\left(4\lambda_2^2\dot{e}_h^2 + 4\lambda_1\lambda_2\dot{e}_h\dot{e}_t + \lambda_1^2\dot{e}_t^2\right) \\ &= -\frac{1}{4\lambda_2}\frac{\hat{I}_T}{I_T}\left(2\lambda_2\dot{e}_h + \lambda_1\dot{e}_t\right)^2 \\ &\leq 0. \end{aligned}$$

□

Now, consider the meaning of the term $\lambda_2\dot{e}_h$ (= hub error rate) in the controller (8). Assume that the exact value of I_T is known and K is zero for the simplicity of analysis. Then the control law is rewritten in the form:

$$\tau = I_T\left(\lambda_1\dot{e}_t + \lambda_2\dot{e}_h\right). \tag{9}$$

And the system (6) is rewritten as follows:

$$\ddot{q}_i + \omega_i^2 q_i = \frac{\phi_i'(0)}{I_T}\tau.$$

Inserting equation (9) into this equation gives:

$$\ddot{q}_i + \left[\phi_i'(0)\left(\lambda_1\frac{\phi_i(L)}{L} + \lambda_2\phi_i'(0)\right)\right]\dot{q}_i + \left[\omega_i^2 + \phi_i'(0)K_i\frac{\phi_i(L)}{L}\right]q_i$$
$$= \phi_i'(0)\left(\lambda_1\left(\dot{e}_t + \frac{\phi_i(L)}{L}\dot{q}_i\right) + \lambda_2\left(\dot{e}_h + \phi_i'(0)\dot{q}_i\right)\right).$$

In this equation, the right-hand side does not contain the ith mode and the sign of $\phi_i'(0)\phi_i(L)$ are not same for all modes. Therefore, it is desirable to use the $\dot{e}_h$ factor in the feedback mechanism. Since the factor $\lambda_2\dot{e}_h$ leads to the term $\lambda_2\left(\phi_i'(0)\right)^2\dot{q}_i$, additional damping is introduced. Because dominant vibrational mode is generally the first one and there is an uncertainty in payload, design of λ_2 value must be based on these factors.

4 Simulation results

In order to demonstrate the effectiveness of the controller developed in the previous section, numerical simulation has been performed on a single-link three-mode flexible manipulator. Numerical paramters for the manipulator are given in Table 1. The model parameters for the first three flexible modes with various payloads are given in Table 2, 3 and 4.

In the Table 2, 3 and 4, mode number zero means the rigid mode. For the rigid mode, $\phi_0(x)$ is defined as follows:

$$\phi_0(x) = x.$$

From these tables, it is obvious that the system parameters are largely affected by payload variations.

It is assumed that the flexible manipulator is initially at rest. The controller in equation (21) is discontinuous and it is well known that synthesis of such a controller gives rise to chattering of trajectory about sliding surface $s = 0$. In order to avoid the chattering phenomenon, the function $sgn(s)$ in the controller (21) has been replaced by $sat(s)$. The function $sat(s)$ is defined as follows

$$sat(s) = \frac{s}{|s| + \delta},$$

where $\delta > 0$.

Figs. 2~7 show the performance of the proposed controller. These figures demonstrate that the tip angular position θ_t is successfully regulated for various payloads. Rise time and settling times are given in Table 5 and 6. In these tables, the rise time is defined as the time required to rise from 10 percent to 90 percent of its final value, and the settling time is defined as the time to decrease and stay within 5 percent or 1 percent of its final value. Figure 3 shows that there is no chattering in the control torque τ.

In practice, gripper of the flexible manipulator may drop payload in the course of motion. In this case, the payload is changed abruptly. Figs. 4~7 show the performance of such a case. In Figure 4 and 5, gripper drop the $0.4kg$ payload at $0.7sec$. Similarly, in Figure 6 and 7, dropping of payload is occurred at $0.9sec$.

5 Conclusions

A simple and robust control system of a single-link flexible arm is proposed. This controller do not need to estimate the modal functions of the system. It only uses output measurement such as: tip position, tip position rate and hub rate. Conventional control system uses vibrational modes of the system as states. Hence, it requires heavy computational load because of many matrix and vector operation. In addition, because the conversion matrix that is used in state estimation is affected by payload variation, conventional control system may be sensitive to payload variation and parameter uncertainty. In the proposed controller, these drawbacks are eliminated by using robust control law: variable structure controller. It is clear that the controller proposed in section 3 has a simple form. In order to verify the robustness of the controller, the simulation results are given in section 4.

References

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Nomenclature

q_i	$e^{j\omega_i t}$, ith modal function.
ϕ_i	$\phi_i(x)$, ith mode shape, $0 \le x \le L$.
θ_t	$\left(= \sum_{i=0}^{\infty} \frac{\phi_i(L)}{L} q_i(t)\right)$, tip angular position (rad).
θ_h	$\left(= \sum_{i=0}^{\infty} \phi_i'(0) q_i(t)\right)$, hub angle (rad).
θ_d	desired tip angular position (rad).
e_t	$(= \theta_d - \theta_t)$, error of tip angular position (rad).
e_h	$(= \theta_d - \theta_h)$, error of hub angle (rad).
e_{rigid}	$(= \theta_d - q_0(t))$, error of rigid mode (rad).
I_T	$(= I_h + I_b + M_p L^2)$, total inertia, $I_{T_{min}} \le I_T \le I_{T_{max}}$.
$\hat{I}_T$	$\left(= \sqrt{I_{T_{max}} I_{T_{min}}}\right)$, estimated total inertia.
$I_{T_{min}}$	minimum value of total inertia, $I_{T_{min}} > 0$.
$I_{T_{max}}$	maximum value of total inertia.
β	$\left(= \sqrt{\frac{I_{T_{max}}}{I_{T_{min}}}}\right)$, margin of I_T.

Table 1: Parameters for the flexible manipulator

Parameters	Symbol	Numerical Value
Modulus of elasticity	E	$6.9 \times 10^{10}\ N/m^2$
Cross-sectional area moment of inertia	I	$8.31934 \times 10^{-11}\ m^4$
Length	L	$1.0\ m$
Linear density	ρ	$0.233172\ kg/m$
Inertia of the beam	I_b	$7.7724 \times 10^{-12}\ kgm^2$
Hub inertia	I_h	$5.176 \times 10^{-3}\ kgm^2$
Mass of gripper		$0.3\ kg$
Mass of payload		$[0,\ 0.4]\ kg$
Mass of total payload	$\hat{M}_p$	$[0.3,\ 0.7]\ kg$
Maximum total inertia	$I_{T_{max}}$	$0.7829\ kgm^2$
Minimum total inertia	$I_{T_{min}}$	$0.3829\ kgm^2$
Estimated total inertia	$\hat{I}_T$	$0.5475\ kgm^2$
Margin of I_T $(= \sqrt{I_{T_{max}}/I_{T_{min}}})$	β	1.4299

Table 2: Modal parameters for the first three modes with no payload

Mode Number i	Natural frequency ω_i (rad/sec)	$\phi_i'(0)$	$\phi_i(L)$
0	0	1	1
1	42.0	5.478	-0.359
2	108.2	5.929	0.217
3	261.0	2.387	-0.178

Table 3: Modal parameters for the first three modes with 0.2 kg payload

Mode Number i	Natural frequency ω_i (rad/sec)	$\phi_i'(0)$	$\phi_i(L)$
0	0	1	1
1	41.1	6.731	-0.286
2	107.4	7.367	0.167
3	259.7	2.965	-0.135

Table 4: Modal parameters for the first three modes with 0.4 kg payload

Mode Number i	Natural frequency ω_i (rad/sec)	$\phi_i'(0)$	$\phi_i(L)$
0	0	1	1
1	40.6	7.788	-0.244
2	107.1	8.565	0.140
3	259.1	3.446	-0.113

Table 5: Rise time and Settling times for various payloads

Payload	$0.0kg$	$0.2kg$	$0.4kg$
Rise Time (sec)	0.594	0.640	0.610
Settling time (5%)	0.966	1.016	1.004
Settling time (1%)	1.354	1.390	1.348

Table 6: Rise time and Settling times

Payload dropping time	No dropping	$0.7sec$	$0.9sec$
Rise Time (sec)	0.610	0.660	0.610
Settling time (5%)	1.004	1.044	1.024
Settling time (1%)	1.348	1.460	1.414

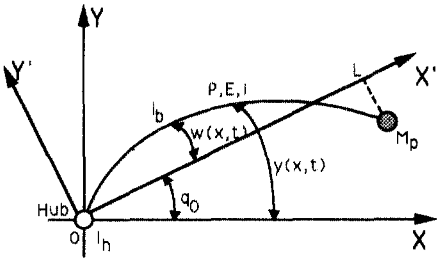

Figure 1: Geometry of the flexible arm

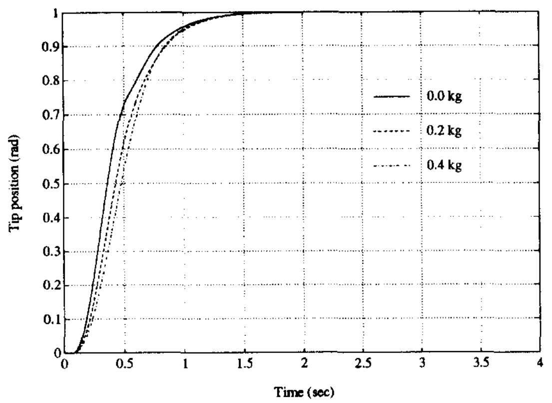

Figure 2: Tip position on regulation for 0.0 *kg*, 0.2 *kg* and 0.4 *kg* payloads

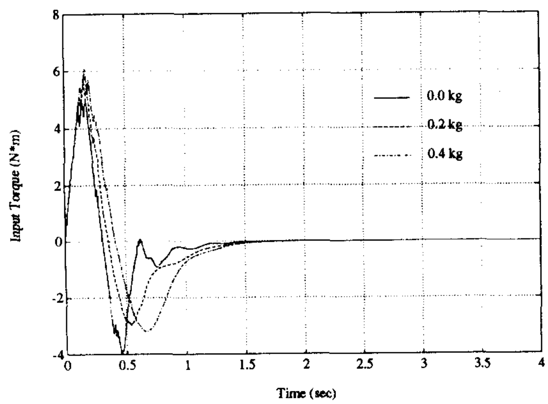

Figure 3: Control torque on regulation for 0.0 *kg*, 0.2 *kg* and 0.4 *kg* payloads

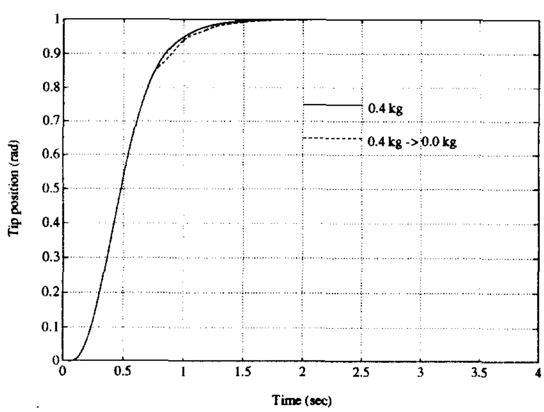

Figure 4: Tip position – payload is varied from 0.4 *kg* to 0.0 *kg* at 0.7 *second*

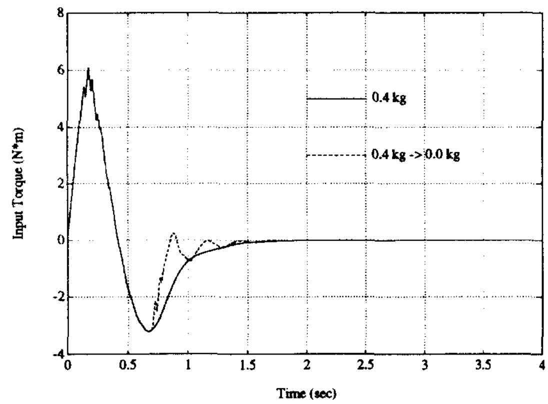

Figure 5: Control torque – payload is varied from 0.4 *kg* to 0.0 *kg* at 0.7 *second*

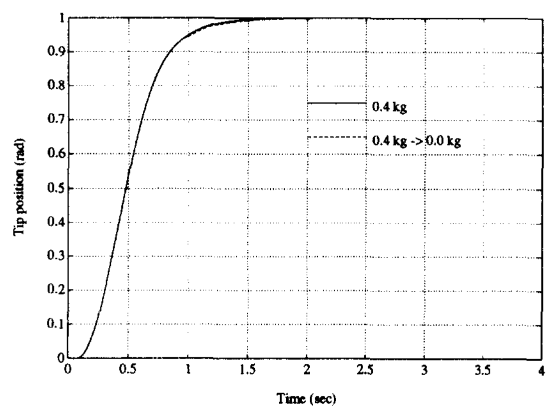

Figure 6: Tip position – payload is varied from 0.4 *kg* to 0.0 *kg* at 0.9 *second*

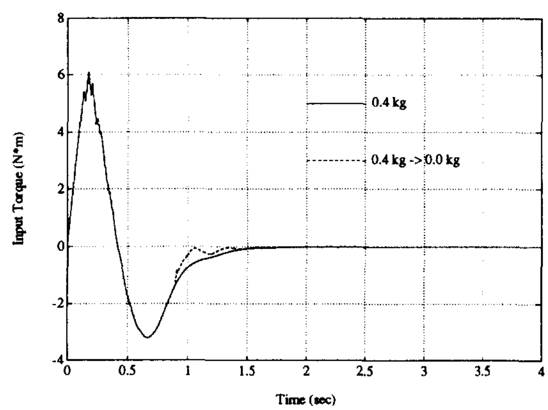

Figure 7: Control torque – payload is varied from 0.4 *kg* to 0.0 *kg* at 0.9 *second*