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Adaptive Control of Flexible Joint Robot Manipulators

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Abstract

This paper presents an adaptive control scheme for flexible joint robot manipulators. This control scheme is based on the Lyapunov direct method with the arm energy-based Lyapunov function. The proposed adaptive control scheme uses only the position and velocity feedback of link and motor shaft. The adaptive control system of flexible joint robots is asymptotically stable regardless of the joint flexibility value. Therefore, the assumption of weak joint elasticity is not needed. Also, joint flexibility value is unknown. Simulation results are presented to show the feasibility of the proposed adaptive control scheme.

1 Introduction

Almost all industrial robots exhibit joint flexibility due to mechanical compliance of their gear boxes such as harmonic drives. Especially, in case of the direct-drive robot, the actuators are directly connected links and the robot operates at high speed. Due to these effects, the robot introduces elastic deformations in the joints. Therefore, joint flexibility should be taken into account in the modeling and control if high performances are to be achieved. When we assume that the joint elasticity may be modeled as a linear torsional spring [1], the links are not directly actuated by the external forces/torques due to this spring. Therefore, the design of control algorithms is more difficult than that of the rigid robot.

When the manipulator parameters are supposed to be exactly known, many approaches have been proposed and the stabilization and tracking problem is solved very well. These methods are based on feedback linearization [2][3][4], singular perturbation theory [5] and on the idea of integral manifold [6].

When the parameters are unkown, the design of control scheme is more difficult. In the case of feedback linearization, this technique can provide robustness to parameteric uncertainty only if the link position, velocity, acceleration and jerk are available for feedback. The computational burden of this approach is greater than the computed torque method for rigid robots. In order to overcome the measurement problem of the link acceleration and jerk, the concept of integral manifold has been proposed [6][7]. But the chief drawback to this approach is its lack of robustness to parameteric uncertainty. On the other hand, an adaptive control algorithm based on singular perturbation technique [5] is simpler than other approaches. But in this case, the assumption of weak joint elasticity is required. Elsewhere, adaptive control algorithms have been presented in [8][9].

In this paper, by properly selecting the Lyapunov function, we presents an adaptive control scheme to be robust to parameteric uncertainty. This control scheme doesn't require the measurement of link acceleration and jerk, that is, uses only the position and velocity feedback of link and motor shaft. Here, the joint flexibility is unkown and is not assume to be weak value. Also, Asymptotic stability is guaranteed regardless of the joint flexibility value. Simulation results are shown to verify the validity of the proposed control scheme. Conclusions and further study are presented to obtain better results.

2 Control of Flexible Joint Robots

The dynamic equation of flexible joint manipulators is as follows.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q - q_m) = 0 \tag{1}$$

$$D_m\ddot{q}_m + B_m - K(q - q_m) = \tau \tag{2}$$

where an n-link manipulator becomes a $2n$ degrees of freedom system.

$q \in \Re^n$: displacement vector of link joint angles

$q_m \in \Re^n$: displacement vector of motor rotor angles

$D(q) \in \Re^{n\times n}$: inertia matrix of link

$C(q,\dot{q}) \in \Re^{n\times n}$: Centrifugal and Coriolis terms matrix of link

$G(q) \in \Re^n$: vector of gravity

$D_m \in \Re^{n\times n}$: constant inertia matrix of motor

$B_m \in \Re^{n\times n}$: constant damping and friction matrix of motor

$K \in \Re^{n\times n}$: diagonal positive definite joint stiffness matrix of the rotor shafts

$\tau \in \Re^n$: vector of input torque

From the above dynamic model, we can find the following properties which is needed to design a control algorithm.

P_1) The link inertia matrix $D(q)$ is symmetric and positive definite, always invertible.

P_2) The matrices D and C are not independent. by a proper definition of $C(q,\dot{q})$, the matrix $\dot{D}-2C$ is *skew-symmetric* [10].

P_3) The individual terms of equation (1) can be represented by a linear relationship between a properly selected set of unknown manipulator parameters (i.e. link masses, moments of inertia, etc) and known functions of the generalized coordinates. In other words,

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(\ddot{q},\dot{q},q)P \tag{3}$$

where $Y(\ddot{q},\dot{q},q) \in \Re^{n\times r}$ is called the regressor matrix of known functions, and $P \in \Re^r$ is a vector of unknown constant parameters.

Now, let us derive the desired motor trajectory from the equation (2). Let $q_d(t) \in C^4$ denote the desired link trajectory. Here, $q_d, \dot{q}_d, \ddot{q}_d, q_d^{(3)}, q_d^{(4)}$ are all bounded and continuously differentiable. The desired motor trajectory may now be computed as follows:

$$\begin{aligned} q_{md}(t) &= K^{-1}Y(\ddot{q}_d,\dot{q}_d,q_d)P + q_d(t) && (4)\\ \dot{q}_{md}(t) &= K^{-1}\dot{Y}(\ddot{q}_d,\dot{q}_d,q_d)P + \dot{q}_d(t) && (5)\\ \ddot{q}_{md}(t) &= K^{-1}\ddot{Y}(\ddot{q}_d,\dot{q}_d,q_d)P + \ddot{q}_d(t) && (6) \end{aligned}$$

When the manipulator parameters are exactly known, we now design a controller based on Lyapunov stability. First of all, let us define the several useful error signals.

The link position error is

$$e = q_d - q \quad \in \Re^n \tag{7}$$

The motor rotor position error is

$$e_m = q_{md} - q_m \quad \in \Re^n \tag{8}$$

Here,

$$e_a = \begin{pmatrix} e \\ e_m \end{pmatrix} \quad \in \Re^{2n} \tag{9}$$

We define a new extended error as follows:

$$s_a = \begin{pmatrix} s \\ s_m \end{pmatrix} = \dot{e}_a + k_v e_a \quad \in \Re^{2n} \tag{10}$$

Where $k_v = diag(k_{v1}, k_{v2}) \in \Re^{2n\times 2n}$ is constant gain matrix, i.e., $k_{v1} > 0 \in \Re^{n\times n}, k_{v2} > 0 \in \Re^{n\times n}$.

Thus, the extended link error is

$$s = \dot{e} + k_{v1}e \quad \in \Re^n \tag{11}$$

The extended motor rotor error is

$$s_m = \dot{e}_m + k_{v2}e_m \quad \in \Re^n \tag{12}$$

The modified velocity signal and acceleration signal are defined as

$$\begin{aligned} \eta_a &= \begin{pmatrix} \eta \\ \eta_m \end{pmatrix} = \begin{pmatrix} s+\dot{q} \\ s_m+\dot{q}_m \end{pmatrix} = \begin{pmatrix} \dot{q}_d + k_{v1}e \\ \dot{q}_{md} + k_{v2}e_m \end{pmatrix} && (13)\\ \dot{\eta}_a &= \begin{pmatrix} \dot{\eta} \\ \dot{\eta}_m \end{pmatrix} = \begin{pmatrix} \ddot{q}_d + k_{v1}\dot{e} \\ \ddot{q}_{md} + k_{v2}\dot{e}_m \end{pmatrix} && (14) \end{aligned}$$

Now, consider the following Lyapunov function candidate, which may be so called the extended error energy, similar to the energy of the trajectory deviating from the desired trajectory

$$V = \frac{1}{2}s^T D(q)s + \frac{1}{2}s_m^T D_m s_m + \frac{1}{2}\int_0^t (s-s_m)^T dt K \int_0^t (s-s_m)dt \tag{15}$$

We will derive the controller to make $\dot{V}(t) < 0$.

First, let's find the time derivative of V. Here, simply rewrite $D(q), C(q,\dot{q})$ and $G(q)$ as D, C and G respectively.

$$\dot{V} = s^T D\dot{s} + \frac{1}{2}s^T\dot{D}s + s_m^T D_m\dot{s}_m + (s-s_m)^T K\int_0^t (s-s_m)dt \tag{16}$$

by the property that $(\dot{D}-2C)$ is skew-symmetric,

$$\begin{aligned} \dot{V} &= s^T D(\dot{\eta}-\ddot{q}) + s^T Cs + s_m^T D_m(\dot{\eta}_m - \ddot{q}_m)\\ &\quad + s^T K\int_0^t (s-s_m)dt - s_m^T K\int_0^t (s-s_m)dt && (17) \end{aligned}$$

by the dynamic model (1) and (2) and some manipulations,

$$\begin{aligned} \dot{V} &= s^T\{D\dot{\eta} + C\eta + G + K[(q-q_m) + \int_0^t (s-s_m)dt]\}\\ &\quad + s_m^T\{D_m\dot{\eta}_m + B_m\dot{q}_m\\ &\quad + K[(q_m-q) - \int_0^t (s-s_m)dt] - \tau\} && (18) \end{aligned}$$

We now define a following control input

$$\begin{aligned} \tau &= D_m\dot{\eta}_m + B_m\dot{q}_m + K[(q_m-q) - \int_0^t (s-s_m)dt]\\ &\quad + k_{d2}s_m + \frac{s_m s^T}{\|s_m\|^2}(Y_a P_a + k_{d1}s) && (19) \end{aligned}$$

where

$$Y_a(\ddot{q}_d,\dot{q}_d,q_d,\dot{q},q,q_m)P_a =$$

$$D(q)\dot{\eta} + C(q,\dot{q})\eta + G(q) + K[(q-q_m) + \int_0^t (s-s_m)dt] \tag{20}$$

P_a is the parameters vector of the manipulator. $k_{d1} > 0 \in \Re^{n\times n}$ and $k_{d2} > 0 \in \Re^{n\times n}$ are constant diagonal positive definite matrices, respectively. We assume that $\|s_m\|^2 > \epsilon > 0$, where ϵ is a suitably small positive number determined to guarantee the numerical stability of the simulation.

Now, substituting the equation (19) into (18), $\dot{V}$ is finally as follows

$$\dot{V} = -s^T k_{d1} s - s_m^T k_{d2} s_m = -s_a^T k_d s_a \tag{21}$$

where $k_d = diag(k_{d1}, k_{d2}) \in \Re^{2n\times 2n}$ is constant positive definite matrix.

Therefore, $\dot{V}$ is negative definite and thus s, s_m converge to zero asymptotically as time goes to infinity. Hence, $e, \dot{e}, e_m$ and $\dot{e}_m$ converge to zero as time increases to infinity. The above controller is useful only if $\|s_m\|^2 > \epsilon > 0$ because τ diverges to infinity as $\|s_m\|^2$ goes to zero.

Now, let's consider the case when $\|s_m\|^2 \le \epsilon$.

Notice as $\|s_m\|^2$ goes to zero, the structure of the system is reduced and thus the Lyapunov function $V(t)$ resembles that of rigid robots. We design the second stage controller using this property. The dynamic equations (1) and (2) can be rewrited as the following equation:

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + D_m\ddot{q}_m + B_m\dot{q}_m \tag{22}$$

Before the design of the second stage controller, let us define the following region where $\ddot{e}_{mi} = \ddot{q}_{mdi} - \ddot{q}_{mi}$ as

$$\mu_{min}[i] \le \ddot{e}_{mi} \le \mu_{max}[i] \quad \text{for} \quad i = 1, 2, \ldots, n \tag{23}$$

where n is the number of links and $\mu_{min}[i], \mu_{max}[i]$ are real scalars. We also define vector $\lambda = (\lambda_1, \ldots, \lambda_n)^T \in \Re^n$ as

$$\lambda_i = \frac{1}{2} D_{mi}\{sgn(s_i)(\mu_{min}[i] - \mu_{max}[i]) + \mu_{min}[i] + \mu_{max}[i]\} \tag{24}$$
$$\text{for} \quad i = 1, 2, \ldots, n$$

where

$$sgn(s_i) = \begin{cases} +1 & \text{if } s_i > 0 \\ -1 & \text{if } s_i < 0 \\ 0 & \text{if } s_i = 0 \end{cases} \tag{25}$$

We can write this relation, i.e., $D(q)\dot{\eta} + C(q,\dot{q})\eta + G(q) = \bar{Y}(\ddot{q}_d, \dot{q}_d, q_d, \dot{q}, q)P$.

By the same procedures as above, we will design the second stage controller. Let us now consider the following Lyapunov function candidate

$$\bar{V} = \frac{1}{2} s^T D(q) s \tag{26}$$

Hence,

$$\begin{aligned} \dot{\bar{V}} &= s^T D\dot{s} + \frac{1}{2} s^T \dot{D} s && (27)\\ &= s^T D(\dot{\eta} - \ddot{q}) + s^T C s && (28)\\ &= s^T(D\dot{\eta} + C\eta + G + D_m\ddot{q}_m + B_m\dot{q}_m - \tau) && (29) \end{aligned}$$

We define the second controller as:

$$\tau = \bar{Y}P + D_m\ddot{q}_{md} + B_m\dot{q}_m + k_D s - \lambda \tag{30}$$

where $k_D \in \Re^{n\times n}$ is constant diagonal positive definite matrix.

Substituting this controller into equation (29),

$$\begin{aligned} \dot{\bar{V}} &= s^T(-D_m\ddot{e}_m + \lambda - k_D s) && (31)\\ &= -s^T k_D s - s^T(D_m\ddot{e}_m - \lambda) && (32)\\ &= -s^T k_D s - \sum_{i=1}^{n} s_i(D_{mi}\ddot{e}_{mi} - \lambda_i) \le 0 && (33) \end{aligned}$$

$\dot{\bar{V}}$ is negative definite. Thus, s converges to zero with time going to infinity, consequently, this implies that both $\dot{e}(t)$ and $e(t)$ converge to zero as time goes to infinity.

We proposed a control scheme when the parameters are exactly known. This controller only requires the mesurement of $\dot{q}, q, \dot{q}_m$ and q_m. The structure of the proposed controller is composed of two stage controllers as magnitude of $\|s_m\|^2$. Namely, when $\|s_m\|^2 > \epsilon > 0$, we used the first stage controller (19). On the other hand, when $\|s_m\|^2 \le \epsilon$, we should select the second stage controller (30).

When the parameters are unknown, the design of an adaptive controller will be addressed in the next section.

3 Adaptive Control of Flexible Joint Robots

As was presented in the previous section, we proposed the two stage controller. The approach of the previous section can be also used to design an adaptive controller as the dynamic parameters of the manipulator are unknown.

Here, because the parameters are unknown, we cannot directly compute the desired motor trajectory using equation (4)(5)(6). Therefore, based on the estimated value of the parameters, we may obtain an estimate of the desired motor position as $\hat{q}_{md}$ using equation (4). That is,

$$\hat{q}_{md}(t) = \hat{K}^{-1} Y(\ddot{q}_d, \dot{q}_d, q_d)\hat{P} + q_d(t). \tag{34}$$

where $\hat{P}$ is the estimate of the parameters.

Here, the definition of the motor rotor position error should be modified as the estimate of motor rotor position error. Namely,

$$\hat{e}_m(t) = \hat{q}_{md}(t) - q_m(t) \quad \in \Re^n \tag{35}$$

Using the approach of the previous section, the following theorem can be stated.

Theorem 1

The flexible joint robot system given by the dynamic model (1) and (2), with the following two stage control and adaptation laws, is asymptotically stable and thus the tracking errors converge to zero as time goes to infinity.

1.The first stage controller : when $\|s_m\|^2 > \epsilon > 0$

- Control law : when $\|s_m\|^2 > \epsilon > 0$ for a scalar ϵ

$$\begin{aligned} \tau = \; & D_m\dot{\eta}_m + B_m\dot{q}_m + \hat{K}_m[(q_m - q) - \int_0^t (s - s_m)dt] \\ & + k_{d2}s_m + \frac{s_m s^T}{\|s_m\|^2}\{\bar{Y}\hat{P} + \hat{K}_l[(q - q_m) + \int_0^t (s - s_m)dt] \end{aligned}$$

$$+k_{d1}s\} \tag{36}$$

where

$$\begin{aligned}\bar{Y}\hat{P} &= \bar{Y}(\ddot{q}_d, \dot{q}_d, q_d, \dot{q}, q)\hat{P} \\ &= \hat{D}(q)\dot{\eta} + \hat{C}(q,\dot{q})\eta + \hat{G}(q) \end{aligned} \tag{37}$$

where $\hat{D}, \hat{C}$ and $\hat{G}$ are the estimates of D, C and G, respectively. Also, $\hat{K}_m$ and $\hat{K}_l$ are the estimates of K corresponding to the different update laws.

- Adaptation law

$$\dot{\phi}_E = -\Gamma Y_E^T s_a \tag{38}$$

where $\phi_E = P_E - \hat{P}_E$ is extended parameters deviation vector, Γ is constant diagonal positive definite matrix, $s_a = \begin{pmatrix} s \\ s_m \end{pmatrix}$, and

if $(D - \hat{D})\dot{\eta} + (C - \hat{C})\eta + (G - \hat{G}) = \bar{Y}(P - \hat{P})$

and $\psi = P - \hat{P}$, then

$$Y_E\dot{\phi}_E = \begin{bmatrix} \bar{Y}\psi + (K - \hat{K}_l)(q - q_m + \int_0^t (s - s_m)dt) \\ (K - \hat{K}_m)(q_m - q - \int_0^t (s - s_m)dt) \end{bmatrix}. \tag{39}$$

2. <u>The second stage controller</u> : when $\|s_m\|^2 \le \epsilon$

- Control law

$$\tau = \bar{Y}\hat{P} + D_m\ddot{\bar{q}}_{md} + B_m\dot{q}_m + k_D s - \lambda \tag{40}$$

where λ is given in equation (24).

- Adaptation law

$$\dot{\psi} = -\Lambda\bar{Y}^T s \tag{41}$$

where Λ is constant diagonal positive definite matrix.

Proof of Theorem 1

In order to prove the Theorem 1, consider the following Lyapunov function candidate based on equation (15):

$$\begin{aligned} V_a &= \frac{1}{2}s^T D(q)s + \frac{1}{2}s_m^T D_m s_m \\ &+\frac{1}{2}\int_0^t (s - s_m)^T dt K \int_0^t (s - s_m)dt \\ &+\frac{1}{2}\phi_E^T\Gamma^{-1}\phi_E \end{aligned} \tag{42}$$

The time derivative of V is computed according to equation (18).

$$\begin{aligned} \dot{V}_a &= s^T[D\dot{\eta} + C\dot{\eta} + G + K(q - q_m) + K\int_0^t (s - s_m)dt] \\ &+s_m^T[D_m\dot{\eta}_m + B_m\dot{\eta}_m - B_m s_m \\ &+K(q_m - q) - K\int_0^t (s - s_m)dt - \tau] \\ &+\dot{\phi}_E^T\Gamma^{-1}\phi_E \end{aligned} \tag{43}$$

Substituting the proposed first control input (36) into equation (43),

$$\begin{aligned} \dot{V}_a &= s^T[(D - \hat{D})\dot{\eta} + (C - \hat{C})\eta + (G - \hat{G}) \\ &+(K - \hat{K}_l)(q - q_m + \int_0^t (s - s_m)dt)] \\ &+s_m^T[(K - \hat{K}_m)(q_m - q - \int_0^t (s - s_m)dt)] \\ &-s^T k_{d1}s - s_m^T k_{d2}s_m + \dot{\phi}_E^T\Gamma^{-1}\phi_E \end{aligned} \tag{44}$$

Let

$$\begin{aligned} Y_l\phi_l &= (D - \hat{D})\dot{\eta} + (C - \hat{C})\eta + (G - \hat{G}) \\ &+(K - \hat{K}_l)[(q - q_m) + \int_0^t (s - s_m)dt] \end{aligned} \tag{45}$$

$$Y_m\phi_m = (K - \hat{K}_m)[(q_m - q) - \int_0^t (s - s_m)dt] \tag{46}$$

Thus,

$$\begin{aligned} \dot{V}_a &= s^T Y_l\phi_l + s_m^T Y_m\phi_m - s^T k_{d1}s - s_m^T k_{d2}s_m \\ &+\dot{\phi}_E^T\Gamma^{-1}\phi_E \\ &= s_a^T Y_E\phi_E - s^T k_{d1}s - s_m^T k_{d2}s_m + \dot{\phi}_E^T\Gamma^{-1}\phi_E \end{aligned} \tag{47}$$

Here, let

$$Y_E\phi_E = \begin{pmatrix} Y_l\phi_l \\ Y_m\phi_m \end{pmatrix} \tag{48}$$

Now, the first stage adaptation law (38) is substituted into (47). In the end,

$$\dot{V}_a = -s^T k_{d1}s - s_m^T k_{d2}s_m \tag{49}$$

If $\|s_m\|^2 > \epsilon > 0$, we use the first stage control and adaptation law (36)(38). Then, $\dot{V}_a$ is negative semi-definite and The system given by equation (1) and (2) is asymptotically stable with all the signals in the system remaining bounded. Here, Asymptotic stability is guaranteed regardless of the joint flexibility value.

Next, as $\|s_m\|^2 \longrightarrow 0$, the reduced dynamic model is given in equation (22). In order to verify the second control and adaptation law, the following Lyapunov function candidate based on equation (26) is presented as:

$$\bar{V}_a = \frac{1}{2}s^T D(q)s + \frac{1}{2}\psi^T\Lambda^{-1}\psi \tag{50}$$

The time derivative of $\bar{V}_a$ is

$$\dot{\bar{V}}_a = s^T D\dot{s} + \frac{1}{2}s^T\dot{D}s + \dot{\psi}^T\Lambda^{-1}\psi \tag{51}$$

$$\begin{aligned} &= s^T(D\dot{\eta} + C\eta + G + D_m\ddot{q}_m + B_m\dot{q}_m - \tau) \\ &+\dot{\psi}^T\Lambda^{-1}\psi \end{aligned} \tag{52}$$

Here, we substitute the second control law (40) into equation (52).

$$\dot{\bar{V}}_a = s^T\bar{Y}\psi - s^T D_m\ddot{\bar{e}}_m - s^T k_D s + s^T\lambda + \dot{\psi}^T\Lambda^{-1}\psi \tag{53}$$

Now, let us apply the second stage adaptation law (41).

$$\dot{\bar{V}}_a = s^T(-D_m\ddot{\bar{e}}_m + \lambda - k_D s) \tag{54}$$

$$= -s^T k_D s - s^T(D_m\ddot{\bar{e}}_m - \lambda) \tag{55}$$

$$= -s^T k_D s - \sum_{i=1}^{n} s_i(D_{mi}\ddot{\bar{e}}_{mi} - \lambda_i) \le 0 \tag{56}$$

$\dot{\bar{V}}_a$ is negative semi-definite. Thus, s converges to zero with time going to infinity, consequently, this implies that both $\dot{e}(t)$

and $e(t)$ converge to zero as time goes to infinity. If $||s_m||^2 \leq \epsilon$, we should switch the control mode from the first stage to the second stage. The second controller make the steady state error still remaining in the end of the first control stage converge to zero.

Therefore, proof of Theorem 1 is complete and also shows the feasibility of the two stage adaptive control. □

4 Simulation Results

In order to verify the validity of the two stage adaptive controller proposed in the previous section, computer simulation has been performed on a single link flexible joint robot manipulator with one revolute joint. The dynamic model of a single-link flexible joint robot is as follows:

$$J_l\ddot{q} + C_l\dot{q} + h\sin(q) + K(q - q_m) = 0 \quad (57)$$

$$J_m\ddot{q}_m + B_m\dot{q}_m - K(q - q_m) = \tau \quad (58)$$

where $h = \frac{1}{2}mgL$.

The simulation results are composed of two cases, that is, regulation and tracking control problem. Numerical parameters for the manipulator are given in Table 1.

Table 1: Parameters for a single-link flexible joint manipulator

Parameters	Symbol	Value
Stiffness	K	15
Inertia of link	J_l	0.2
Inertia of motor rotor	J_m	0.05
Gravity	g	9.8
Length of link	L	0.5
Mass of link	m	2.5
Co. and Cen. terms of link	C_l	0.5
Friction term of motor	B_m	0.05

In the Table 1, all the values have SI units. 'Co.' represents 'Coriolis' and 'Cen.' represents 'Centrifugal'.

In the simulation, unknown parameters are K, J_l, C_l, h, and known parameters is J_m, B_m. It is assumed that the flexible joint manipulator is initially at rest. That is, $q(0) = q_m(0) = 0, \dot{q}(0) = \dot{q}_m(0) = 0, \ddot{q}(0) = \ddot{q}_m(0) = 0$. The sampling time is 0.01 seconds. ϵ is 0.1, μ_{min} is -1.0 and μ_{max} is 1.0.

The results for the regulation problem are given in Figure 1.1 and 1.2. In this regulation control, the desired link position is $q_d = 1$(radian). On the other hand, the simulation results for the tracking control problem are shown in Figure 2.1 ~ 2.4. In this tracking control, the desired link trajectory is $q_d(t) = \cos(\frac{\pi}{2}t)$ (radian).

5 Conclusions

We proposed two stage adaptive control scheme by properly selecting the Lyapunov function similar to total energy of the overall system. During the course of proof of Lyapunov stability, the terms divided by $||s_m||^2$ appeared. Hence, the control scheme was separated into two and we presented two stage control alogorithms as the magnitude of $||s_m||^2$. From the proposed control scheme, acceleration an jerk measurements of link and motor are not needed and only position and velocity feedback signals are required to update the control input. Also, Asymptotic stability is ensured regardless of the joint flexibility value. The wind-up phenomenon due to the integrator in the proposed controller should be taken into account in the practical implementation of this controller.

Now, the ideas that merge two stage control algorithms into only one control algorithm and the develoments of the exact estimation processes are problems which may be chiefly handled in the further study.

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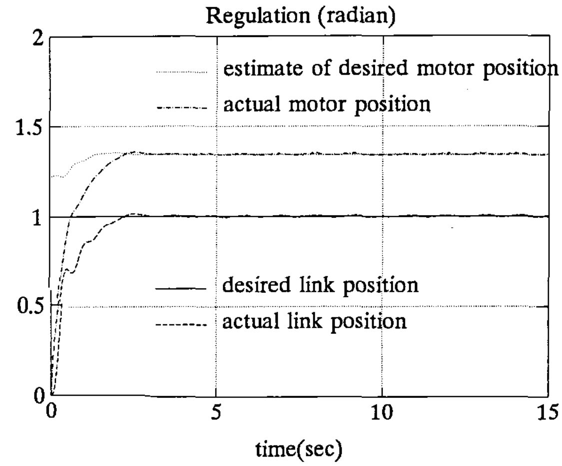

Figure 1.1 Regulation : The desired and actual link position, The estimate of desired motor position and actual link position.

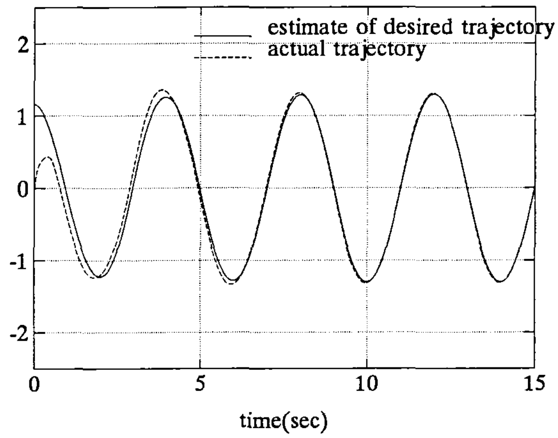

Figure 2.2 The estimate of desired motor rotor trajectory and the actual trajectory.

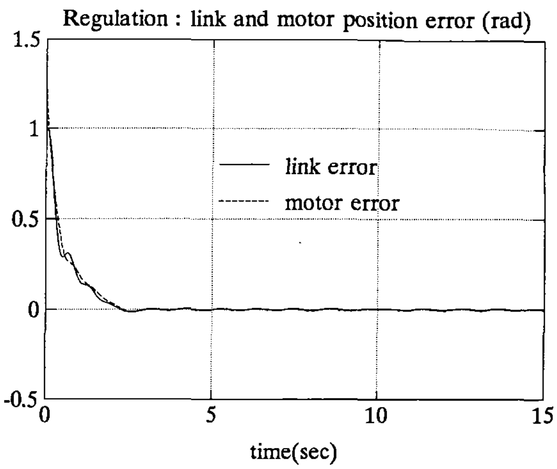

Figure 1.2 The link and motor rotor position error.

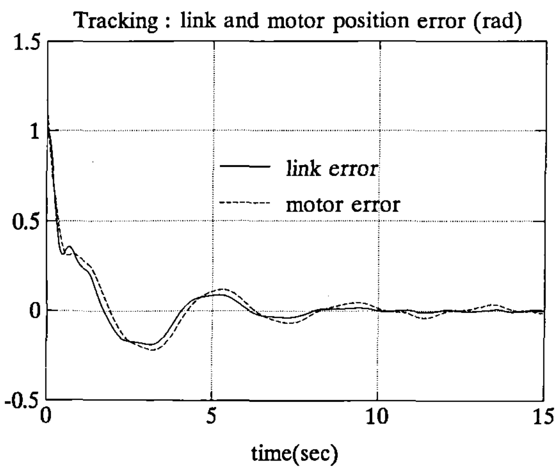

Figure 2.3 The link and motor rotor position error.

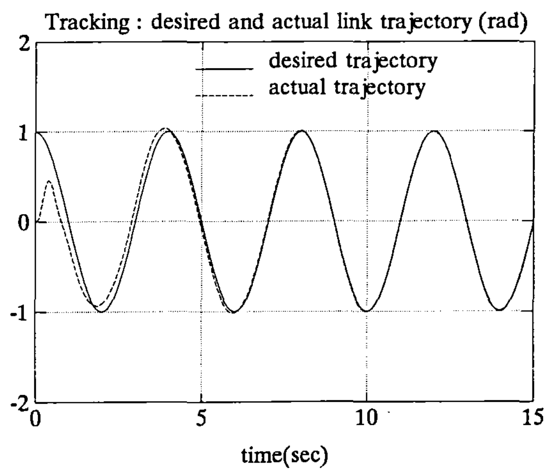

Figure 2.1 Tracking : The desired and actual link trajectory.

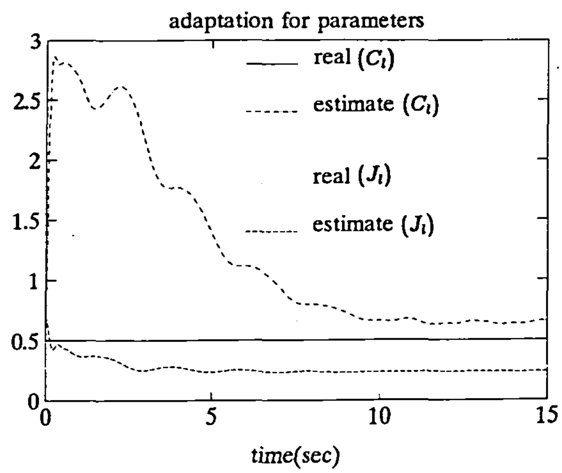

Figure 2.4 The adaptation process for J_l and C_l.