Local Motion Planner for Unicycle-like Vehicle : Guaranteeing Collision Avoidance Even in Unknown Environment

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Abstract

This paper deals with the motion planning problem for a unicycle-like vehicle whose motion is confined by its own nonholonomic constraints. Generally, it is known that it is difficult to develop the feedback control scheme of nonholonomic systems due to the fact that nonholonomic system cannot be stabilized to a given any configuration based on a smooth time-invariant state feedback law. In this paper, we exploit a simple feedback scheme for unicycle model, based on an approximation of the desired configuration given by local holonomic planner that ignores motion constraints. To prevent the approximation error from causing the collision with obstacles, we propose an inequality constraint, based on the analysis of vehicle's motion , which is assumed to be governed by the constant control input during the sensor's sampling time. Consequently, we formulate our problem as the constrained optimization problem and the feedback scheme based on local sensor information is established by simply solving this problem. Through some simulations, we confirm the validity and effectiveness of our algorithm.

1 Introduction

This paper deals with the problem of planning constrained motions where the constraints are nonholonomic in nature. Specifically, we focus on the motion planning of the unicycle-like vehicle using the feedback control scheme.

Generally, it is difficult for the nonholonomic systems to exploit the feedback control law due to the fact that nonholonomic system cannot be stabilized to a given any configuration based on the continuous and smooth time-invariant state feedback law [1]. This motivated some complex feedback laws, for example, the time-periodic function based law [2] and so on. On the other hand, these previous methods do not take into account the presence of the unknown obstacles and require a sequence of feasible targets to complete a point-to-point motion such as a car parking problem.

In contrast to these nonholonomic planners, the local holonomic planners furnish the simple and powerful feedback scheme for the mobile robot navigating through the partially known environment. Using these merits, A. D. Luca and his collegues [3] have proposed a feasible projection strategy to modify the output of local holonomic planner. Their scheme provides a very simple and powerful feedback motion planner in a local sense. Instead, their method has a serious drawback that it cannot guarantee the collision avoidance with obstacles in spite of the collision avoidance being a basic and intrinsic demand for motion planning.

In this paper, we focus only on the goal position reaching problem. The basic idea of this paper is on the extension of A. D. Luca *et al.*'s work [3], in a sense that our proposed algorithm approximates the desired configuration given by the local holonomic planner. We propose the inequality constraints to prevent the approximation error from causing the collision with obstacles. Note that this is very different from A. D. Luca *et al.*'s work [3]. For this purpose, we analyze the local motion of vehicle, which is assumed to be governed by the constant control input during the sensor's sampling time. Finally, we formulate our problem as the constrained optimization problem under the assumption of the lower-bounded curvature due to the mechanical restrictions. Through the computer simulations, we will show the validity and effectiveness of our algorithm.

2 Motion Planner for Unicycle Model

Consider a unicycle-like vehicle positioned on the plane $\Re^2$ with respect to the base frame $< b >$, whose motion is governed by the combined action of both the angular velocity w and the linear velocity vector v. A linear velocity vector v is assumed to be always directed as x axis of its attached frame $< a >$ where its origin is located at the center of vehicle, as depicted in Fig. 1. Then, the kinematic model of the unicycle-like vehicle, which involves the vehicle's Cartesian position x, y and its own orientation θ, is known as follows [3]:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \tag{1}$$

where v is the linear velocity and w is the angular velocity. The values of x, y, θ are all measured with respect to the base frame $< b >$. This model takes into account the nonholonomic constraint

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{2}$$

that specifies the tangent direction along any feasible path for the vehicle.

As stated earlier, A. D. Luca *et al.* [3] have presented a direct projection strategy to modify the output of local holonomic planner in the on-line manner. Their approach generates the velocity level control inputs that implement the desired motion, which is updated by a local holonomic planner, in the least-square sense. For any desired motion $\dot{\mathbf{x}}_{\mathbf{d}}$, their solution is given by the pseudo-inversion, as follows:

$$\begin{aligned} \mathbf{u} &= \mathbf{G}^{\#}(\mathbf{x})\dot{\mathbf{x}}_{\mathbf{d}} \\ &= [\mathbf{G}^T(\mathbf{x})\mathbf{G}(\mathbf{x})]^{-1}\mathbf{G}^T(\mathbf{x})\dot{\mathbf{x}}_{\mathbf{d}} \end{aligned} \tag{3}$$

where $\mathbf{x} \in \Re^n$ is a configuration vector of the generalized coordinate, $\mathbf{u} \in \Re^{n-m} (n > m)$ is an admissible control input vector, and $\mathbf{G}(\mathbf{x})$ is a uncycle model. Although their simple and efficient local planner furnishes a feasible motion in most situations where including even the unknown environment, no guarantee for the collision avoidance can cause some serious problems. This is because the least-square error $[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]^T[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]$ can drive the vehicle into the obstacle region when the desired motion $\dot{\mathbf{x}}_{\mathbf{d}}$ does not hold the nonholonomic constraints.

Thus, now we require to analyze the vehicle's motion, which is assumed to be governed by the constant control input vector $\mathbf{u} = (v, w)^T$ during the time interval of the command updating. This will be helpful to derive the constraints for preventing the approximation error from causing the collision. For simplicity of analysis, we fix the attached frame $< a >$ shown in Fig. 1 at the command updating instant. And then, with respect to this frame, we can obtain the vehicle's motion according to the analysis of the following equation. Note that we denote this fixed frame as $< f, a >$ through this paper.

$$\begin{gathered} \begin{pmatrix} \dot{x}_{f,a} \\ \dot{y}_{f,a} \\ \dot{\theta}_{f,a} \end{pmatrix} = \begin{pmatrix} \cos\theta_{f,a} & 0 \\ \sin\theta_{f,a} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_{f,a} \\ w_{f,a} \end{pmatrix} \\ (x_{f,a}(0), y_{f,a}(0), \theta_{f,a}(0))^T = (0,0,0)^T, 0 \le t \le \delta t_c \end{gathered} \tag{4}$$

where $v_{f,a}$ and $w_{f,a}$ have the constant values respectively and δt_c is the time interval for the control input updating, that is, the sensor's sampling time. The subscript "f, a" denotes that the values of $x_{f,a}, y_{f,a}, \theta_{f,a}$ are all measured with respect to the attached frame which is fixed at the instant for the control input updating (that is, $t = 0$ in a local view). As shown in Fig. 2, a constant control input $\mathbf{u}_{\mathbf{f,a}} = (v_{f,a}, w_{f,a})^T$ yields a circular path with a turning radius ρ during δt_c. Note that the value of $v_{f,a}$ is not negative at all because a linear velocity vector $v_{f,a}$ is assumed to be always directed as x axis of the attached frame.

Integrating the equation for the main direction of vehicle yields

$$\int_0^t \dot{\theta}_{f,a} d\tau = \int_0^t w_{f,a} d\tau \tag{5}$$

$$\theta_{f,a}(t) = w_{f,a} t \qquad (\theta_{f,a}(0) = 0) \tag{6}$$

where $0 \le t \le \delta t_c$. Then, the value of turning radius ρ can be computed as follows:

$$\rho|\theta_{f,a}(t)| = \int_0^t \sqrt{\left(\frac{dx_{f,a}}{d\tau}\right)^2 + \left(\frac{dy_{f,a}}{d\tau}\right)^2} d\tau \tag{7}$$

$$\rho = \left|\frac{v_{f,a}}{w_{f,a}}\right| \tag{8}$$

Thus, for the given $v_{f,a}$ and $w_{f,a}$, the vehicle tracks the following trajectory during δt_c.

$$\begin{aligned} x_{f,a}(t) &= sgn(w_{f,a})\rho\sin\theta_{f,a}(t) \\ y_{f,a}(t) &= sgn(w_{f,a})\rho(1 - \cos\theta_{f,a}(t)) \end{aligned} \tag{9}$$

where the function of $sgn(w_{f,a})$ returns the sign value.

To prevent the collision with obstacles, we should choose the control input $\mathbf{u}_{\mathbf{f,a}} = (v_{f,a}, w_{f,a})^T$ so that the predicted locus in the equation (9) will not intersect any edges of obstacles. For this purpose, we assume that the function of $d(\psi_{f,a})$ represents the model of vehicle's surroundings and it returns a proximity measure in the direction of $\psi_{f,a}$ with respect to the fixed frame $< f, a >$. This model is updated per every δt_c. During δt_c, the following constraint should be satisfied to guarantee the collsion avoidance for the given constant control input $\mathbf{u_{f,a}} = (v_{f,a}, w_{f,a})^T$.

$$d(\psi_{f,a}(t)) > \sqrt{x_{f,a}^2(t) + y_{f,a}^2(t)} \quad (10)$$

$$= 2\rho|\sin(\frac{\theta_{f,a}(t)}{2})| \quad (11)$$

$$= 2\rho|\sin(\psi_{f,a}(t))| \quad (12)$$

where $\psi_{f,a}(t) = \frac{w_{f,a}t}{2}$, $0 \le t \le \delta t_c$ and $\rho = |\frac{v_{f,a}}{w_{f,a}}|$. Fig. 2 will be helpful for the comprehensive understanding of the derivation for the above equations.

Based on the derived results so far, we can formulate our algorithm as follows:

$$\begin{gathered}\text{Minimize } [\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}]^T[\dot{\mathbf{x}}_{\mathbf{d}} - \mathbf{G}(\mathbf{x})\mathbf{u}] \\ \text{subject to} \\ d(\psi_{f,a}(t)) > 2\rho|\sin(\psi_{f,a}(t))|\end{gathered} \quad (13)$$

$$\rho = \left|\frac{v_a}{w_a}\right| = \left|\frac{v}{w}\right| \quad (14)$$

$$\psi_{f,a}(t) = \frac{w_{f,a}t}{2} = \frac{wt}{2} \quad (15)$$

where $0 \le t \le \delta t_c$. The solution of the equation (13) can be found based on the nonlinear programming technique. The relationship between the base frame $< b >$ and the fixed frame $< f, a >$ can be easily implemented through a simple transformation matrix.

3 Simulation Results

We demonstrated the proposed motion planner for a unicycle model in the presence with the circular obstacles, based on the potential field approach as the local holonomic motion planner.

Using the potential field approach, the desired velocity vector $\dot{\mathbf{q}}_{\mathbf{d}} = (\dot{x}_d, \dot{y}_d)$ can be obtained from the following equation at every sampling time.

$$\dot{\mathbf{q}}_{\mathbf{d}} = -\nabla_{\mathbf{q}}(U_a(\mathbf{q}) + U_r(\mathbf{q})) \quad (16)$$

where $U_a(\cdot)$ is an attractive potential function and $U_r(\cdot)$ is a repulsive potential function [4]. To complete finding the desired configuration, we require to assign the desired rotation of $\dot{\theta}_d$. For simplicity, we choose the desired steering input as follows:

$$\dot{\theta}_d = tan^{-1}\frac{\dot{y}_d}{\dot{x}_d} - \theta \quad (17)$$

where the function of $tan^{-1}(\cdot)$ returns the radian value from 0 to 2π. Therefore, we can compute the desired configuration $\dot{\mathbf{x}}_{\mathbf{d}}$ for the equation (13).

For the uncertain environment cluttered with circular osbtacles, we perform the simulation for the proposed motion planner. All the parameter values used in the simulation are given in Table 1. To solve the problem in equation (13), we use the exhaustive search technique because the serach domain is relatively small and its method is very simple for the computer analysis. We believe that another nonlinear programming techniques can also solve this problem in a short time.

In Fig. 3, we can show the successful result of the local holonomic planner. The nonholonomic motions obtained for the initial $\theta_0 = 0$ are shown in Fig. 4, with the associated desired configuration. In Fig. 4, we can see that the constructed path by the proposed algorithm is different from the holonomic one. This is because our algorithm approximates the holonomic path in local sense. Anyway, we confirm that the position error goes to zero in the terminal phase. We have applied our method to several other situations, and the satisfactory results, which guarantee the collision avoidance, were always obtained.

4 Conclusions

We have presented a efficient motion planner for a unicycle-like vehicle in the presence of unknown obstacles. Our feedback scheme can be utilized in a real time, to prevent the vehicle from colliding with obstacles. This makes our method be useful in a real application.

The proposed scheme has been applied to the unicycle-like vehicle. The potential field approach has been used as the local holonomic motion planner. The simulation results confirmed that the obtained configuration approximates the desired motion in a least-square and local sense and the proposed planner provides the satisfactory results. Since the proposed planner is based on the local strategy, even in the unknown environment the vehicle system can reach the target successfully.

Future research directions include a real application of our algorithm in a real world.

References

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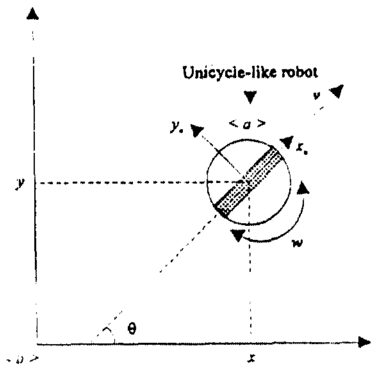

Fig. 1. A unicycle-like robot *w.r.t* the base frame

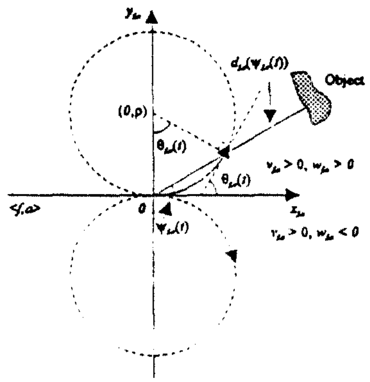

Fig. 2. A motion analysis during the time interval δt_c

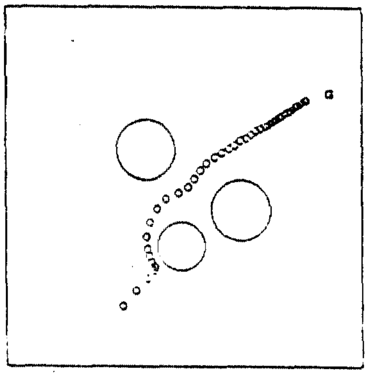

Fig. 3. A holonomic motion with potential field approach

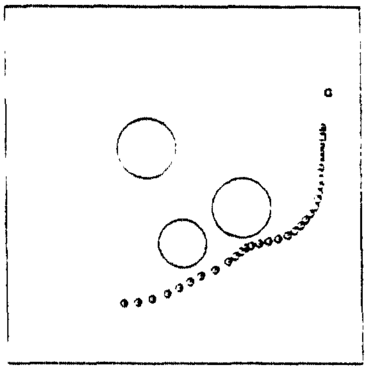

Fig. 4. A nonholonomic motion with potential field approach

Table 1. All the parameter values used in the simulations

Environment size	$300m \times 300m$
Radius of circular robot	$3m$
The number of sensors	36
The time interval for the command updating δt_c	$0.5sec$
Maximum speed v_{max}	$2.0m/sec$
Sensor range	$20.0m$