

# A Study on the Inverse Kinematics of Free-Floating Space Robot Including the Dynamics of the Spacecraft

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## Abstract

Future robotic manipulator systems will be required to perform important and complex space missions tasks such as retrieving, repairing, and servicing space station and satellites in space. In this paper some important kinematics, dynamics, and control problems unique to space robotic systems are discussed. In this paper reviews an analytical modeling method for space manipulators called the Virtual Manipulator(VM), which has a fixed base in inertial space at a point called a Virtual Ground, and presents a solution to the inverse kinematics problem using the VM concept, which is very important and difficult problem in free-floating space robotic system.

## 1 Introduction

Space robotics is a special technological field. In future space development, robotization and automation will be key technology and contribute much to the success of space projects, reducing the workload and danger of astronauts and increasing operational efficiency. A major characteristics of these robots which are clearly distinguished from ground-based robots is the lack of a fixed base in space environment. There are a lot of past work to control space robots. Among them, Vafa and Dubowsky proposed "Virtual Manipulator" concept in order to describe geometry of free-floating mechanical links. They applied it to analyze workspaces of space manipulators and to solve the kinematics and inverse kinematics problems through a full comprehension both of position and attitude change of the satellite by the manipulator reaction( [8], [9], [3] [5]). Space Robotic manipulator systems are generally classified as three categories: free-flying manipulator system, the attitude controlled space robotic systems, a

free-floating system. In free-flying case, the position and attitude of the system's spacecraft is controlled actively by reaction jets(thrusters) during the motions of its manipulator. In the attitude controlled space robotic system, only spacecraft's attitude is controlled by the reaction wheel systems and its position is not controlled. In this case, the velocity of each joint of the manipulator must be confined to prevent the reaction wheel system from being saturated. In the free-floating case, the spacecraft will move freely in response to the dynamic disturbances caused by the motions of the manipulator because position and attitude of the spacecraft are not actively controlled to conserve the attitude control fuel during the motions of manipulator. Once free-floating system is constructed and is available, it is very useful and attractive because it does not consume control fuel. The control problem for this kind of space/manipulator can be simplified using the VM concept ([8]). However this kind of system has another problem, the existence of dynamic singularity. That is, free-floating space manipulator systems have dynamic singularities where the spacecraft moves in response to manipulator motions without compensation from its attitude control system. Thus dynamic singularities must be considered in the design, planning, and control of free-floating space manipulator systems. In this paper, great attention is paid to the inverse kinematics problem in free-floating space robotic system. In this system, if the VM concept is applied the workspace analysis and kinematics problem can be solved easily but the inverse kinematics still remains as a problem to be solved. First, we briefly review the analytical modeling method, Virtual Manipulator concept and present a solution to the inverse kinematics problem considering the dynamics of the free-floating space robot using the VM concept. Therefore, we try to compen-

sate the spacecraft's motions into calculating the desired joint angles in joint space from the desired trajectory in inertia space and control the system using the computed torque method(CTM).

## 2 Modeling

A manipulator with a spacecraft in space is considered to be a space robotic system in the non-gravitational environment. The system is modeled as a set of  $(n + 1)$  rigid bodies connected by  $n$  joints.

The total kinetic energy of the whole system is defined by summing up the translational energy and rotational energy of each body ([7]).

$$T \equiv \frac{1}{2} \sum_{i=0}^n (\Omega_i^T I_i^B + m_i V_i^T V_i) \quad (1)$$

In case of free-floating, the linear and angular momenta are conserved at zero. Substituting momentum conservation equations into equation , we obtain the reduced form([7])

$$T = \frac{1}{2} \dot{q}^T \hat{H}(q) \dot{q}, \quad (2)$$

In above equations, concrete meaning of parameters is referred to ref([7]).

It is easy to prove the reduced inertia matrix  $\hat{H}(q)$  to be symmetric and positive definite. Therefore, the equation of motion for the reduced form can be derived in the same way as conventional robotics. If the Lagrangian formulation is applied, the general form of dynamics equation can be obtained as follows:

$$\hat{H}(q)\ddot{q} + B(q, \dot{q})\dot{q} = \tau \quad (3)$$

## 3 The Virtual Manipulator Method

Now, consider the Virtual Manipulator concept ([8], [9]). As mentioned above, the motions of space manipulator will disturb the attitude and position of its spacecraft. Therefore, due to the reaction force caused by the manipulator motions, space robot system is free-floating. Due to the lack of a fixed base, the planning and controlling of a spacecraft/manipulator is very difficult. So, the Virtual Manipulator concept was introduced to overcome this base floating effect.

First, we take some basic assumptions fundamental to the VM concept: The manipulators are composed of rigid bodies; No external forces and torques are acted; All geometric and mass properties are exactly known.

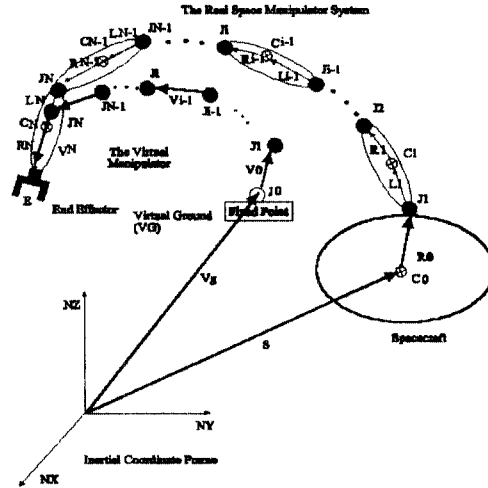


Figure 1: The Real and Virtual Manipulator System

The Virtual Manipulator(VM) is an ideal massless kinematic chain connecting its base, Virtual Ground(VG), to any point on a space robot. The Virtual Ground(VG) is an imaginary point fixed in inertial space to which the VM's base is attached, which is the center of mass of the complete system. The VM has some very useful properties as follow : ;The VM link lengths remain constant as the manipulator moves ;The joint between the first virtual link and the VG is spherical ;The axis of each virtual joint is always parallel to the axis of real manipulator system joint ;The amount of rotation of the  $i_{th}$  virtual revolute joint is equal to the rotation of the  $i_{th}$  revolute joint of the real system.

Consider the  $(N+1)$  link open chain rigid bodies formed by a spatial manipulator composed of  $N$  revolute joints and its supporting vehicle. A VM can be constructed to any point in the real manipulator. The concrete description is represented in ([8], [9]).

The first VM link represents the vehicle's orientation. This link is attached to the VG by a spherical joint which permit the three vehicle rotations w.r.t. the inertial space The properties of the VM

remain the same as long as the mass property of the system does not change. As an application of the VM approach, we consider the workspace analysis and the kinematics and inverse kinematics. First consider the workspace analysis where the free-floating and attitude-controlled cases will be taken into consideration. A VM is constructed for these cases to the end-effector of the real manipulator. The joint limits of the real manipulator are transformed into VM joint limits. The workspace of the VM is found using conventional

workspace analysis methods. Next, consider the inverse kinematics problem. However, it is difficult even to solve the kinematics problem need- less to say the inverse kinematics problem for space manipulator system because manipulator motions will result in spacecraft motions, which will change the end-effector location in inertial space. Thus, it is not possible to apply the con- ventional standard algorithms to the space robot.

#### 4 The Inverse Kinematics for the free-floating space robot using the VM

In general, it is very difficult to solve the in- verse kinematics problem for the nonredundant free-floating space robot. Past works for the in- verse kinematics of free-floating space robot have been limited to the redundant case. For exam- ple, Vafa and Dubowsky solved the inverse kinematics problem using VM concept in redundant free-floating space robot where the redundancy of space robot is used to prevent the movement of spacecraft attitude. So, the inverse kinematics problem for free-floating space robot is tranformed into the inverse kinematics problem for fixed- attitude space robot because the attitude of space- craft is fixed, if the virtual maipulator concept is used. Therefore, the inverse kinematics problem can be solved easily using the virtual manipu- lator concept because the inverse kinematics prob- lem is tranformed into that of the ground fixed industrial robot. However, the inverse kinematics problem for the nonredundant free-floating space robot can not be easily solved using VM approach by applying simple conventional inverse kinemat- ics techniques because the motions of spacecraft can not be controlled independently. So, we propose one solution of the inverse kinematics problem for nonredundant space robot in this paper. The inverse kinematics problem is difficult to solve because of the motion of spacecraft. So, if the motion of spacecraft is eliminated in the inverse kinematics problems, the inverse kinematics problem can be solved easily. In this paper, the proposed inverse kinematics technique is called the mod- ified inverse kinematics technique. The modified inverse kinematics technique is as follows. We consider two spaces in this paper; Real Space(RS) and Virtual Space(VS). The RS is a space where real manipulator is controlled and the VS is a space where the VM works. We assume that the angu- lar velocity of spacecraft can be available. Now, we propose the inverse kinematics solution. The proposed method is divided into two steps : The

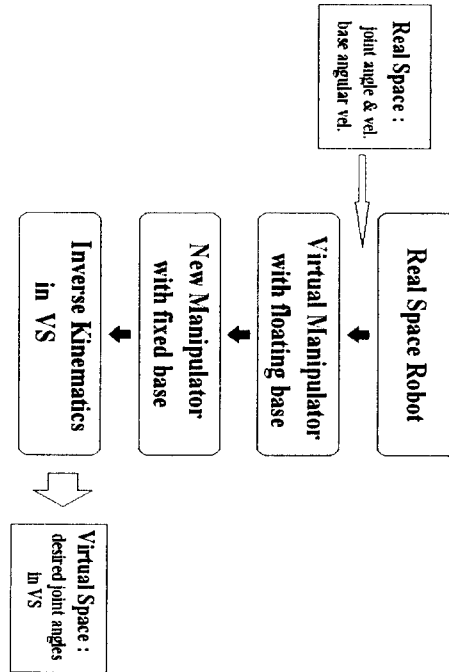


Figure 2: The total procedure in inverse kinemat- ics

first step is that the joint angles and velocities are measured and that the angular velocities of spacecraft are measured or calculated in the RS. The information obtained in first step is given into the second step where concrete inverse kinematics method is applied to the VM in the VS. In second step where all the operations are performed in the Virtual Space(VS) first if the attitude of the spacecraft in the RS is applied to the VM in the VS, the end point of the first link of the VM corresponding to the spacecraft in the RS can be obtained. All procedure is shown in Fig.2. Thus the first link of the VM, which can not be controlled independently, can be eliminated in solving the inverse kinematics in the VS. Therefore, the inverse kinematics problem for all the space robot system in the RS is transformed into the inverse kinemat- ics problem for a new manipulator corresponding to the VM's all links excluding the first link as shown in Fig.3 . Now if the conventional inverse kinematics method is applied to the transformed manipulator, that is, a new manipulator in the VS, the inverse kinematics problem can be easily solved in the VS. As the obtained solutions in the VS are the same as the solution to the real ma- nipulator, the inverse kinematics problem will be solved and we have only to control the space robot using the conventional control approaches for the ground fixed base robot.

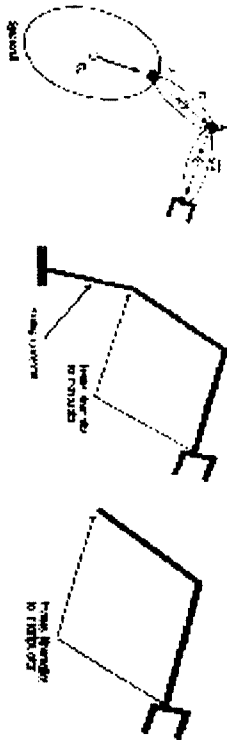


Figure 3: The transformation of manipulator

## 5 Simulation

The model of simulation is two link space manipulator system, shown in Fig.3. We perform the simulation for the inverse kinematics and then show the validity of the proposed approach through the tracking control. In tracking control the conventional computed torque method will be used. First it is kept in mind that the inverse kinematics is solved in the Virtual Space using the virtual manipulator concept. That is, the inverse kinematics solution obtained from the virtual manipulator is applied to the real manipulator system. Fig.4 shows the inverse kinematics solution obtained using the proposed inverse kinematics technique. The simulation result shows that the solution is exact, though the obtained joint angles have some unsmooth parts. The unsmooth parts represent the large spacecraft motions. Although the unsmooth parts is undesirable, they are unavoidable because the motion of spacecraft can not be controlled independantly. To diminish the unsmoothness of inverse kinematics solution, we must find the path which does not cause large base motion. This problem will be left as further study. And Fig.5 shows the tracking error in inertial space and Fig.6 shows the tracking result in inertial space. As shown above the tracking error will not converge exactly to zero because we use the actual base angular velocity as desired ve-

locity but the tracking error will not be so large and will be bounded to some confined value corresponding to the error between the actual angular velocity and the desired angular velocity. If the proposed inverse kinematics approach is applied to the space robot, almost all the conventional control algorithms can be used. As an example we showed the validity through the computed-torque method among them.

## 6 Conclusion

In this paper, we considered the inverse kinematics and motion control of space manipulator system using VM concept. In particular, the inverse kinematics technique for the free-floating space manipulator systems were mainly considered. The motion control problem will be solved simply if the VM can be constructed. But in the free-floating case, there remain some problems in the inverse kinematics because the spacecraft is not controlled independently. So we proposed the modified inverse kinematics technique including the spacecraft motions caused by the manipulator motions.

If the proposed inverse kinematics approach is applied, almost all the conventional control algorithms can be used and therefore the control problem of free-floating space robot is very simplified. We showed the validity of above fact through the tracking control using conventional computed-torque method.

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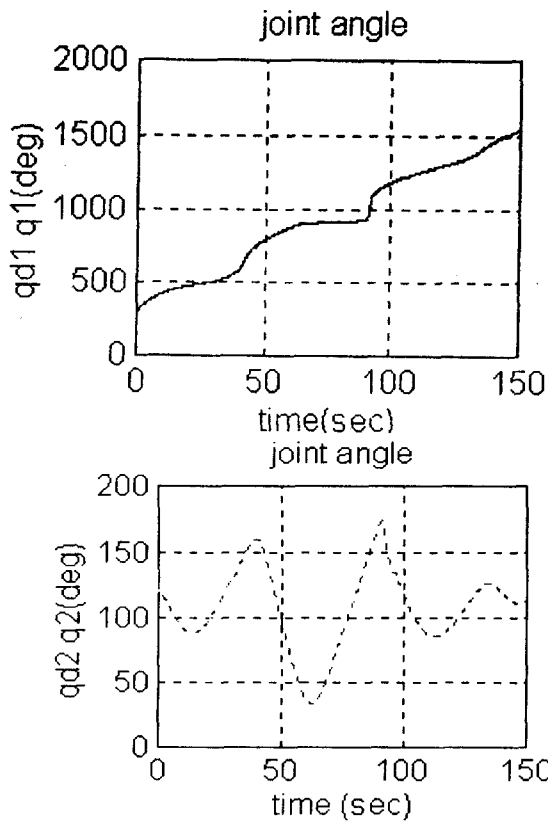
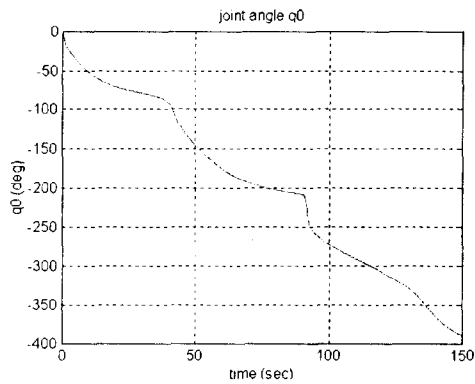


Figure 4: The Inverse Kinematics Solution

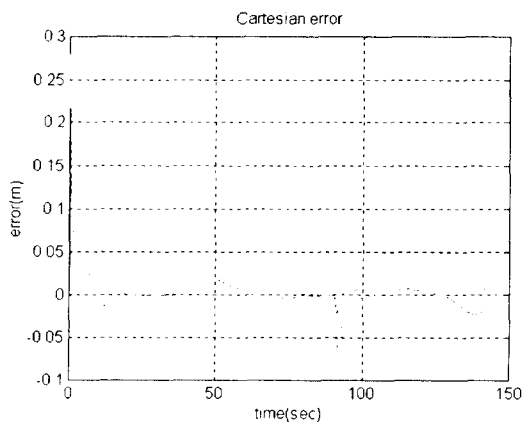


Figure 5: The Tracking Error

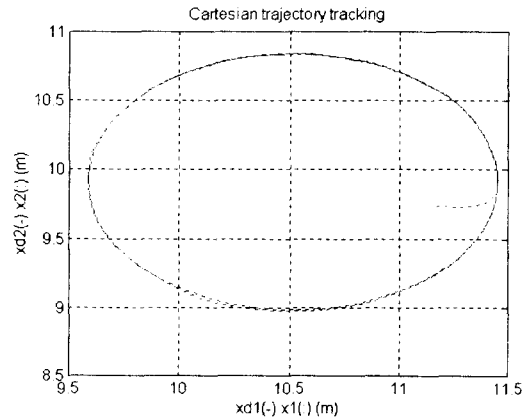


Figure 6: The Tracking Result

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