Adaptive Control Based on Speed Gradient Algorithm for Robot Manipulators

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Abstract— **In this short note, we present three types of adaptive parameter update law for the rigid body robot manipulator based on the speed gradient algorithm which is introduced by Fradkov. In the derivation of the new adaptive parameter update laws, we refer the dynamics of rigid body robot manipulators and it's error dynamics which were derived by Slotine. It is shown that the parameter update law which was derived by Slotine belongs to one of three types of adaptive parameter update law proposed in this paper. The three types of adaptive parameter update laws proposed in this paper can ensure the global stability under some conditions such as attainability and convexity in the error dynamics. Computer simulation shows that proposed control algorithm can be used for the tracking problem of rigid body robot manipulators.**

I. Introduction

Robot manipulators are multi-dimensional, flexible geometry, progarmmable mechanical systems which are ideally suited for applications in a flexible automation environment and may be used in hazardous or in unsafe environment such as in space, in deep water and in radioactive environments. However, their dynamics are inherently nonlinear and time-varying; e.g. their inertia characteristics of the manipulator depend on its configuration as well as its time-varying payload. As a consquence, standard linear feedback controllers are not able to realize the manipulator's full potential for speed and accuracy. Responding the need to overcome the problem mentioned above, a number of techniques have been proposed in recent years. Among these, adaptive control approaches have been receiving an increasing attention because these can adapt to the partially unknown and time-varying parameters in the robot dynamics, such as the load and link parameters. The earliest work on model reference adaptive control for robot manipulators was based on linear decoupled model and steepest descent parameter adaptation [1]. The first work on adaptive controls of mechanical manipulator based on the stability analysis was proposed by Horowitz [2]. Also Arimoto proposed adaptive control approach based on Lyapunov stability theory [3]. In these works the unknown parameters in the manipulator dynamic equations are assumed to be constants in the stability analysis even though they depend on the configuration of manipulator. Craig *et al.* proposed an adaptive version of the computed torque method for the control of robot manipulator [4]. In these method, they suggested a parameter update law for the stability of the over all systems in the Lyapunov sense by using the properties of positive real transfer function. However, in their method, the inverse of the inertia metrix $D(q)$ must be estimated and calculated, even though they showed the stability of their controller rigorously. Spong and Ortega presented an alternative formulation of adaptive inverse dynamic control which overcome the boundedness of the inverse of the estimated inertia matrix and the subsequent need to modify the parameter update law. This scheme requires the joint acceleration in the expression for the joint torque as well as in parameter update law [5]. Slotine *et al.* proposed the adaptive control algorithms which consist of a PD feedback part and a full dynamic feedforward compensation part, with the unknown link and payload parameters being estimated on line [6, 7, 8, 9].

Their algorithm was computationally simple because of an effective exploitation of the physical properties of manipulator dynamics, *i.e.*, they made use of the fact that the matrix $dD(q)/dt - 2C(q, \dot{q})$ is skew symmetric. They also designed the dynamic structure of the robot linearly in terms of a suitably selected set of link and payload parameters and used the variable structure system theory in the parameter adaptation for the robustness of their control algorithms. Ham proposed an adaptive control algorithm based on the Lyapunov stability theory and showed the global stability [11]. He also suggested the method how to derive the dynamic model of robot manipulator which can be expressed linearly in terms of the parameters of links and payload and showed that the dynamic model is suitable not only for the pure identification of the manipulator but also for implementation of adaptive control.

In this paper, we introduce the speed gradient algorithm which was suggested by Fradkov based on the convexity and attainability [13, 14]. Three types of parameter update law are proposed based on the adaptive control scheme and error dynamics derived by Slotine *et al.*. It is also demonstrated that the proposed adaptive control algorithms have the global stability.

The paper is organized as follows. In section 2, dynamic equations of robot manipulator and control aim are presented. In section 3, control law and error dynamics are derived based on the control scheme proposed by Slotine *et al.*. In section 4, speed gradient algorithm is introduced and three types of parameter update law are suggested based on the control law and error dynamics derived by Slotine *et al.*. Computer simulation results demonstrating the validness of the proposed adaptive control algorithm for the two-link robot manipulator are presented in section 5. Finally, section 6 contains the concluding remarks.

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II. Dynamic Equations of Robot Manipulators

The dynamic equations of the n -link robot manipulator are given as;

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(q,\dot{q}) \tag{2.1}$$

where

$M(q)$: link inertia matrix $\in R^{n\times n}$
$C(q,\dot{q})$: centrifugal and Coriolis terms matrix $\in R^{n\times n}$
$g(q)$: gravitational vector term $\in R^n$
$f(q,\dot{q})$: frictional vector term $\in R^n$
τ: generalized applied torque vector $\in R^n$
q: vector of joint angles $\in R^n$

A relation exists between matrices $D(q)$ and $C(q,\dot{q})$ [11].

$$c_{kj} = \sum_{i=1}^{n}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i \tag{2.2}$$

where c_{ij} is the (i,j) element of matrix $C(q,\dot{q})$, and d_{ij} is the (i,j) element of matrix $D(q)$. Therefore the matrix $\dot{D}(q,\dot{q}) - 2C(q,\dot{q})$ is skew symmetric. But in this paper, we don't use the above physical property and define the following term $h(q,\dot{q})$ which contains $C(q,\dot{q})\dot{q}$, $g(q)$ and $f(q,\dot{q})$.

$$h(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q) + f(q,\dot{q}) \tag{2.3}$$

Then dynamic equations of n-link robot manipulator can be expressed as

$$\tau = M(q)\ddot{q} + h(q,\dot{q}) \tag{2.4}$$

Assume the desired trajectory $\ddot{q}_d$, $\dot{q}_d$ and q_d to be bounded. The tracking error vector $\tilde{q}$ is defined as

$$\tilde{q} = q - q_d \tag{2.5}$$

The controller problem is to derive a control law for the generalized forces/torques τ such that the tracking error vectors $\tilde{q}$ and $\dot{\tilde{q}}$ converge to zero vectors as fast as possible.

III. Error Dynamics and Control Law

In this section, error dynamics and control law which were derived by Slotine *et al.* are briefly discussed. A vector s which is used as a measure of tracking is defiened as

$$s = \dot{\tilde{q}} + \Lambda\tilde{q} \tag{3.1}$$

where Λ is arbitrary $n\times n$ stable matrix. It is convenient to rewrite (3.1) as follows:

$$s = \dot{q} - \dot{q}_r \tag{3.2}$$

where $\dot{q}_r$ is defiened as

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}. \tag{3.3}$$

We use the control law which is similar to the control law derived by Slotine *et al.* as follows:

$$\begin{aligned}\tau &= M(q)\ddot{q}_r + h(q,\dot{q}) - \frac{1}{2}\dot{M}(q,\dot{q})s - Ks \\ &= W(q,\dot{q},\ddot{q}_r,s)\theta - Ks\end{aligned} \tag{3.4}$$

where

$$W(q,\dot{q},\ddot{q}_r,s)\theta = M(q)\ddot{q}_r + h(q,\dot{q}) - \frac{1}{2}\dot{M}(q,\dot{q})s \tag{3.5}$$

and K is arbitrary $n\times n$ positive definite matrix. Above control law requires that the parameters of robot manipulator must be known. However, in practice, the parameters of a robot manipulator are partially known or unknown. Hence we must replace the controller parameter vector θ of a robot manipulator with the estimate of θ and find a parameter updation law which can guarantee the stability of the over all system. We shall discuss this parameter updation law in the next section by using the speed gradient algorithm. Under the assumption that θ is partially known or unknown, we use the following control law where $\hat{\theta}$ is an estimate of θ,

$$\begin{aligned}\tau &= \hat{M}(q)\ddot{q}_r + \hat{h}(q,\dot{q}) - \frac{1}{2}\hat{\dot{M}}(q,\dot{q})s - Ks \\ &= W(q,\dot{q},\ddot{q}_r,s)\hat{\theta} - Ks\end{aligned} \tag{3.6}$$

where $\hat{M}(q)$, $\hat{h}(q,\dot{q})$ and $\hat{\dot{M}}(q,\dot{q})$ are estimates of $M(q)$, $h(q,\dot{q})$ and $\dot{M}(q,\dot{q})$ respectively.
If we apply the above control law to (2.4), we can obtain the following error dynamics.

$$M(q)\dot{s} = \tilde{M}(q)\ddot{q}_r + \tilde{h}(q,\dot{q}) - \frac{1}{2}\tilde{\dot{M}}(q,\dot{q})s - Ks \tag{3.7}$$

where

$$\tilde{M}(q) = \hat{M}(q) - M(q)$$
$$\tilde{h}(q,\dot{q}) = \hat{h}(q,\dot{q}) - h(q,\dot{q}).$$

We shall use the above error dynamics in the derivation of parameter update laws based on the speed gradient algorithm.

IV. Speed Gradient Algorithm

In this section, we shall introduce speed gradient algorithm and discuss three types of parameter update law with the stability analysis by using Lyapunov stability theory. Then we shall derive the new three types of parameter update laws based on the error dynamics derived in the previous section. First, we shall introduce the definition of convex function and the well known theorem concerning convex function briefly. Its contents play important role in the stability analysis of speed gradient algorithm.

Definition 4.1 *Let S be a convex set in R^n and let $f: S \to R^1$ be a real-valued function. We say that f is a* convex function *on S if and only if $f[\lambda x_1 + (1-\lambda)x_2] \le \lambda f(x_1) + (1-\lambda)f(x_2)$ for all $x_1, x_2 \in S$ and for all λ such that $0 \le \lambda \le 1$.*

Note that convex functions are not defined if the domain is not a convex set.

Theorem 4.1 *Let S be a convex set in R^n and suppose that $f: S \to R^1$ is convex. Let x^0 be an interior point of S.*
(a) Then there are real numbers $a_1, a_2, \cdots, a_n$ such that

$$f(x) \ge f(x_0) + \sum_{i=1}^{n} a_i(x_i - x_i^0)\ ,\ \text{where } x \in S. \tag{4.1}$$

(b) If $f \in C^1$,i.e., first derivative of function f is continuous, on $S^{(0)}$, where $S^{(0)}$ denotes the set of interior points of S, then

$$a_i = \left.\frac{\partial f}{\partial x_i}\right|_{x=x^0}. \qquad i = 1,\cdot,n. \tag{4.2}$$

proof: See [15]

In general error dynamics of adaptive control system is a nonlinear differential equation and can be expressed as

$$\dot{x}(t) = F(x, \phi, t), \qquad t \geq 0. \tag{4.3}$$

Where $x(t) \in R^n$ is an error state vector, $\phi(t) \in R^m$ is a parameter estimation error vector ($\phi(t) = \hat{\theta}(t) - \theta^*$), $F(\cdot) : R^{n+m+1} \rightarrow R^n$ is a continuously differentiable vector function in x, θ . The control problem is to find the parameter update law

$$\hat{\theta}(t) = \Theta(x_0^t, \hat{\theta}_0^t, t) \tag{4.4}$$

according to some criterion of "good" functioning of the system, where notation x_0^t and $\hat{\theta}_0^t$ mean the set $\{ x(s), 0 \leq s \leq t \}$,$\{ \hat{\theta}(s), 0 \leq s \leq t \}$ respectively. Suppose this criterion requires to provide low values of some aim functional $Q_t = Q(x_0^t, \hat{\theta}_0^t, t)$. Typically $Q_t$may be local form such as $Q_t = Q(x(t), t)$, where $Q(x(t), t) \geq 0$ is a scalar smooth aim functional. Let us define a function $\omega(x, \hat{\theta}, t)$ as time derivative of Q_t (the speed of Q_t which changes along the trajectory of system). Then

$$\omega(x, \hat{\theta}, t) = (\nabla_x Q)^T F(x, \hat{\theta}, t) + \nabla_t Q \tag{4.5}$$

where $\nabla_x Q$, and $\nabla_t Q$ denote the gradients of Q in x and t respectively.

With the above definition, we will introduce three types of parameter update law proposed by Fradkov.

Algorithm 4.1 .

(differential type)

$$\dot{\hat{\theta}}(t) = -\Gamma \nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t) \tag{4.6}$$

(integral type)

$$\hat{\theta}(t) = -\psi(x, \hat{\theta}, t) - \Gamma \int_0^t \nabla_{\hat{\theta}} \omega(x, \hat{\theta}, s) ds \tag{4.7}$$

(finite type)

$$\hat{\theta}(t) = \theta^{\circ}(x, t) - \gamma(x, t) \psi(x, \hat{\theta}, t) \tag{4.8}$$

where Γ is a symmetric, positive definite matrix, $\psi(\cdot)$ satisfies pseudo gradientity condition, *i.e.*, $\psi^T \nabla_{\hat{\theta}} \omega \geq 0$, where $\nabla_{\hat{\theta}} \omega$ denotes the gradient of ω in $\hat{\theta}$ and $\gamma(x, t) > 0$ is a scalar.

Theorem 4.2 *[14] Let system (4.3),(4.7) have unique solution for any initial conditions $x(0)$,$\hat{\theta}(0)$, and functions $F(x, \hat{\theta}, t)$, $\nabla_x Q(x, t)$, $\psi(x, t)$, $\nabla \omega(x, \hat{\theta}, t)$ be locally bounded in t (bounded in some region $\{(x, \hat{\theta}, t) : \|x\| + \|\hat{\theta}\| \leq \beta \leq \infty, \text{for } t \geq 0\}$) and following conditions beheld:*

(a) Growth condition: $\inf_t Q(x, t) \rightarrow \infty$ as $\|x\| \rightarrow \infty$.

(b) Convexity condition: function $\omega(x, \hat{\theta}, t)$ is convex in $\hat{\theta}$.

(c) Attainability condition: vector $\theta^* \in R^m$ and a function $\rho(Q)$ exists such that $\rho(Q) > 0$ when $Q > 0$ and

$$\omega(x, \theta^*, t) \leq -\rho(Q). \tag{4.9}$$

Then all solutions of system (4.3),(4.7) are bounded and $Q_t \rightarrow 0$ as $t \rightarrow \infty$.

Proof: The proof is based on the Lyapunov-like function

$$V_t = Q_t + \frac{1}{2}(\hat{\theta}(t) - \theta^* + \psi(x, \hat{\theta}, t))^T \Gamma^{-1} (\hat{\theta}(t) - \theta^* + \psi(x, \hat{\theta}, t)) \tag{4.10}$$

By convexity and attainability condition, the following inequalities can be drived:

$$\omega(x, \hat{\theta}, t) - (\hat{\theta}(t) - \theta^*)^T \nabla_\theta \omega \leq \omega(x, \theta^*, t) \leq -\rho(Q). \tag{4.11}$$

The time derivative of V_t along a trajectory of the system is given by

$$\dot{V}_t = \dot{Q}_t - (\hat{\theta}(t) - \theta^* + \psi(x, \hat{\theta}, t))^T \nabla_{\hat{\theta}} \omega(x, \hat{\theta}^*, t) \tag{4.12}$$

From the pseudo gradientity condition $\psi^T \nabla_{\hat{\theta}} \omega \geq 0$,

$$\dot{V}_t \leq \omega(x, \hat{\theta}, t) - (\hat{\theta}(t) - \theta^*)^T \nabla_{\hat{\theta}} \omega(x, \hat{\theta}^*, t) \tag{4.13}$$

From (4.9),(4.11), $\dot{V}_t$ can be expressed as

$$\dot{V}_t \leq \omega(x, \theta^*, t) \leq -\rho(Q) < 0. \tag{4.14}$$

Therefore $Q_t \rightarrow 0$ as $t \rightarrow \infty$. ∎

Theorem 4.3 *Let conditions of theorem 4.2 are fulfilled with $\rho(Q) \equiv 0$ in (4.9). Then all solutions of system (4.3),(4.6) are bounded.*

Proof: The proof is similar to Theorem 4.2 but in this case,

$$\dot{V}_t \leq \omega(x, \theta^*, t) \leq -\rho(Q) \leq 0. \tag{4.15}$$

Therefore we can assure that Q_t is bounded. ∎

Theorem 4.4 *Let conditions of theorem 4.2 are fulfileed as well as strong pseudo gradientity condition*

$$\psi(x, t)^T \nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t) \geq \kappa \|\nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t)\|^\delta \tag{4.16}$$

for some $\kappa > 0$ and $\delta \geq 1$ and inequality

$$\kappa \gamma(x, t) \|\nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t)\|^{\delta - 1} \geq \|\theta^{\circ} - \theta^*\| \tag{4.17}$$

then all the solutions of system (4.3),(4.8) are bounded and $Q_t \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Let $V_t = Q_t$. The time derivative of V_t along the system trjectory is given by

$$\dot{V}_t = \dot{Q}_t = \omega(x, \hat{\theta}, t) \tag{4.18}$$

By convexity condition,

$$\begin{aligned}
\dot{V}_t &= \omega(x, \hat{\theta}, t) \leq \omega(x, \theta^*, t) + (\hat{\theta} - \theta^*)^T \nabla_{\hat{\theta}} \omega \\
&= \omega(x, \theta^*, t) + (\hat{\theta} - \theta^{\circ})^T \nabla_{\hat{\theta}} \omega + (\theta^{\circ} - \theta^*)^T \nabla_{\hat{\theta}} \omega \\
&= \omega(x, \theta^*, t) - \gamma(x, t) \psi(x, \hat{\theta}, t)^T \nabla_{\hat{\theta}} \omega \\
&+ (\theta^{\circ} - \theta^*)^T \nabla_{\hat{\theta}} \omega
\end{aligned} \tag{4.19}$$

From (4.16),

$$\begin{aligned}
\dot{V}_t &\leq \omega(x, \theta^*, t) - \kappa \gamma(x, t) \|\nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t)\|^\delta \\
&+ (\theta^{\circ} - \theta^*)^T \nabla_{\hat{\theta}} \omega \\
&\leq \omega(x, \theta^*, t) - \kappa \gamma(x, t) \|\nabla_{\hat{\theta}} \omega(x, \hat{\theta}, t)\|^\delta \\
&+ \|\theta^{\circ} - \theta^*\| \|\nabla_{\hat{\theta}} \omega\|
\end{aligned} \tag{4.20}$$

From (4.17),

$$\dot{V}_t \leq \omega(x, \theta^*, t) \leq -\rho(Q) < 0. \tag{4.21}$$

Therefore $Q_t \rightarrow 0$ as $t \rightarrow \infty$. ∎

Now we shall derive new types of parameter update law based on the error dynamics of robot manipulator discussed in section III by using the above speed gradient algorithm. First we choose the aim functional Q_t as follows;

$$Q_t = \frac{1}{2} s^T M(q) s. \tag{4.22}$$

By using (3.7), we can find that $\omega(s, \hat{\theta}, t)$ which corresponds to $\omega(x, \hat{\theta}, t)$ defined in speed gradient algorithm can be expressed as

$$\begin{aligned} \omega(s, \hat{\theta}, t) &= \dot{Q}_t = s^T M(q)\dot{s} + \frac{1}{2} s^T \dot{M}(q, \dot{q}) s \\ &= s^T(\tilde{M}(q)\ddot{q}_r + \tilde{h}(q, \dot{q}) - \frac{1}{2}\hat{\dot{M}}(q, \dot{q})s - Ks) \\ &+ \frac{1}{2} s^T \dot{M}(q, \dot{q}) s \\ &= s^T(\tilde{M}(q)\ddot{q}_r + \tilde{h}(q, \dot{q}) - \frac{1}{2}\tilde{\dot{M}}(q, \dot{q})s) - s^T K s \\ &= s^T W(q, \dot{q}, \ddot{q}_r, s)(\hat{\theta} - \theta^*) - s^T K s \end{aligned} \tag{4.23}$$

From the above equation, we can see that $\omega(s, \hat{\theta}, t)$ is linear in terms of $\hat{\theta}$ and that $\omega(s, \hat{\theta}, t)$ is convex function in $\hat{\theta}$. Now we choose $\psi(s, t)^T$ which corresponds to $\psi(x, \hat{\theta}, t)^T$ defined in speed gradient algorithm as follow such that it satisfies pseudo gradientity condition, *i.e.*, $\psi^T \nabla_{\hat{\theta}} \omega \geq 0$.

$$\psi(s, t)^T = W(q, \dot{q}, \ddot{q}_r, s)^T s \tag{4.24}$$

From the above discussion, we can see that all the conditions of theorem 4.2 (growth condition, convexity condition and attainability condition) are satisfied. Therefore we can propose the following new three types of parameter update law for the adaptive control of robot manipulator which guarantee the stability of the over all system.

Algorithm 4.2 .
(differential type)

$$\dot{\hat{\theta}}(t) = -\Gamma W(q, \dot{q}, \ddot{q}_r, s)^T s \tag{4.25}$$

(integral type)

$$\hat{\theta}(t) = -\Gamma W(q, \dot{q}, \ddot{q}_r, s)^T s - \Gamma \int_0^t W(q, \dot{q}, \ddot{q}_r, s)^T s dt \tag{4.26}$$

(finite type)

$$\hat{\theta}(t) = \theta^o(s, t) - \gamma(s, t) W(q, \dot{q}, \ddot{q}_r, s)^T s \tag{4.27}$$

In the above parameter update laws, differential type is the same as the one which was proposed by Slotine *et al.*. Even though Ham proposed a parameter update law similar to the integral type for the model reference adaptive control [12], integral type and finite type has not been proposed yet by other researchers.

V. Computer Simulations

Computer simulations are conducted to verify and demonstrate the effectiveness and performance of the proposed control algorithms. A two-link robot manipulator which is composed of two-point mass (see Fig.1) is used for computer simulations. The Runge-Kutta fourth-order intergartion method is used for integrating the dynamics, and the parameters of the two-link robot manipulator are set as in Table 1.

Table 1. Physical parameters of two-link robot mainpulator

parameters	values	units
m_1	3	kg
m_2	$2 \rightarrow 3$	kg
l_1	1.0	m
l_2	0.8	m
f_1	3	N-m-sec/rad
f_2	3	N-m-sec/rad
g	9.8	N/kg

The dynamic equations of two-link robot manipualtor are expressed as

$$\tau = M(q)\ddot{q} + h(q, \dot{q}) \tag{5.1}$$

where

$$M(q) = \begin{bmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 C_2 + (m_1 + m_2) l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 C_2 \\ m_2 l_2^2 + 2m_2 l_1 l_2 C_2 & m_2 l_2^2 \end{bmatrix}$$

$$h(q, \dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 S_2 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) + m_2 l_2 g C_{12} + (m_1 + m_2) l_1 g C_1 \\ m_2 l_1 l_2 S_2 \dot{q}_1 + m_2 l_2 g C_{12} \end{bmatrix}$$

Henceforth, whenever convenient we use the shorthand notation $C_i = cos(q_i), S_i = sin(q_i), C_{ij} = cos(q_i + q_j), S_{ij} = sin(q_i + q_j)$ for trigonometric functions. We assume that m_1 and m_2 are unknown, *i.e.*, θ^* is defined as

$$\theta^* = [\, m_1 \quad m_2 \,]^T. \tag{5.2}$$

Then $W(q, \dot{q}, \dot{q}_r, s)$ is expressed as

$$W(q, \dot{q}, \dot{q}_r, s) = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \tag{5.3}$$

where

$$\begin{aligned} w_{11} &= l_1^2 \ddot{q}_{r1} + l_1 g C_1 \\ w_{12} &= l_1^2 \ddot{q}_{r1} + l_2^2(\ddot{q}_{r1} + \ddot{q}_{r2}) + l_1 l_2 C_2 (2\ddot{q}_{r1} + \ddot{q}_{r2}) \\ &= l_1 l_2 S_2 \dot{q}_2 (s_1 + \frac{1}{2} s_2 - 2\dot{q}_1 - \dot{q}_2) + g(l_2 C_{12} + l_1 C_1) \\ w_{21} &= 0 \\ w_{22} &= (l_2^2 + l_1 i_2 C_2)\ddot{q}_{r1} + l_2^2 \ddot{q}_{r2} + l_1 l_2 S_2 (\frac{1}{2}\dot{q}_2 s_1 + \dot{q}_1^2) \\ &+ l_2 g C_{12} \end{aligned}$$

and

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}}_1 \\ \dot{\tilde{q}}_2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{bmatrix}$$

$$q_r = \begin{bmatrix} q_{r1} \\ q_{r2} \end{bmatrix}$$

The desired joint trajectory q_d is given as

$$q_d = \begin{bmatrix} q_{d1} \\ q_{d2} \end{bmatrix} = \begin{bmatrix} 0.5 + 0.5sin(0.5t) \\ -0.5 - 0.5sin(0.4t) \end{bmatrix} \tag{5.4}$$

We set the initial value $\hat{\theta}(0)$ as $[2.5 \quad 1.5]^T$. But after 8 second, mass m_2 is changed from 2 kg to 3 kg. Matrices Γ and Λ are set to diagonal matrices $0.5I$ and $4I$ and γ is set to constant 0.5. Figure 2, 3 and 4 show the angular displacements and parameter estimates when we are using differental type, integral type and finite type parameter update law respectively. As we can see from the figures, the actual joint angles converge to their desired values within about 2 seconds. Even though the joint tracking errors converge to zero, the proposed algorithm does not guarantee that the estimates of parameter converge to its true values. The reason is due to the fact that the speed gradient algorithm is originally designed such that the Lyapunov candidate function is not a Lyapunov function with respect to the parameter estimation error vector even though it is a Lyapunov function with respect to stste error vector. However, as we can see from the figures, the parameter estimates converge to thier true values because the sufficient rich conditions are satisfied.

IV. Conclusion

In this paper, we propose new parameter update laws for the adaptive control of robot manipulator based on the speed gradient algorithm. It is also shown that the parameter update law which was considered by Slotine et al is a special type among the three types of parameter update law proposed in this paper. The proposed adaptive control algorithm is simulated for the two-link robot manipulator under the assumption that we have no prior knowledge about its link masses. Even though the joint tracking errors converge to zero, the proposed algorithm does not guarantee that the estimates of parameter converge to its true values. The reason is due to the fact that the speed gradient algorithm is originally designed such that the Lyapunov candidate function is not a Lyapunov function with respect to the parameter estimation error vector even though it is a Lyapunov function with respect to stste error vector. However, the parameter estimates will converge to their true values rapidly under the assumption that the sufficient rich condition is satisfied.

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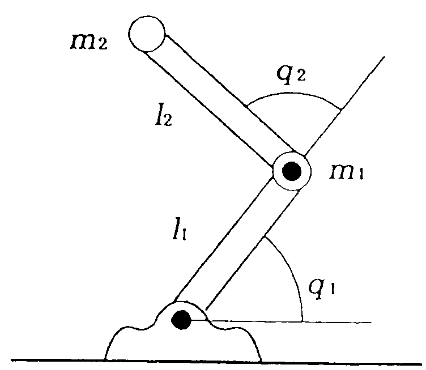

Fig. 1. 2-link robot manipulator.

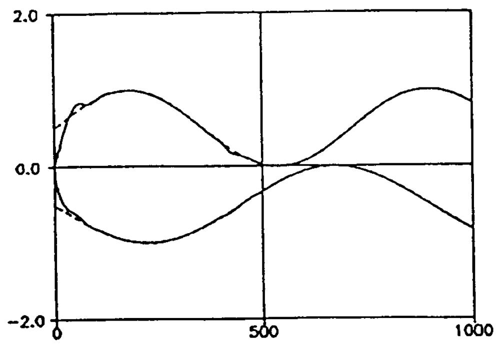

Fig. 2.a Angular displacements of joint (differential type, T=20*msec*).

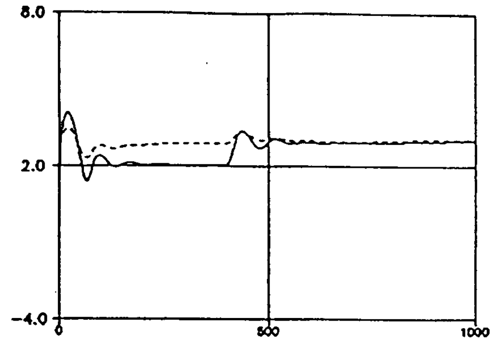

Fig. 2.b Parameter estimates (differential type, T=20*msec*).

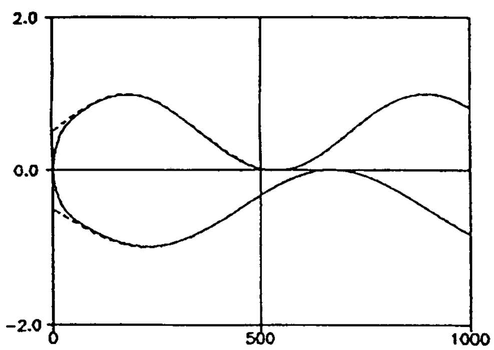

Fig. 3.a Angular displacements of joint (integral type, T=20*msec*).

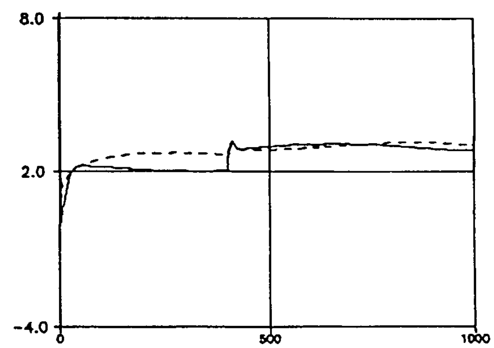

Fig. 3.b Parameter estimates (integral type, T=20*msec*).

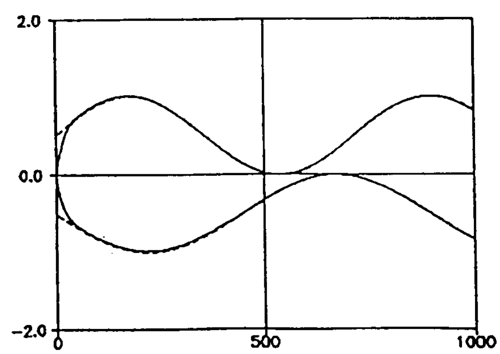

Fig. 4.a Angular displacements of joint (finite type, T=20*msec*).

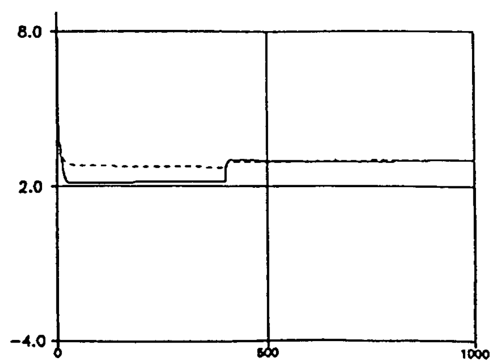

Fig. 4.b Parameter estimates (finite type,T=20*msec*).