AIP Applied Physics Letters

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Citation: Appl. Phys. Lett. **81**, 1341 (2002); doi: 10.1063/1.1499520
View online: http://dx.doi.org/10.1063/1.1499520
View Table of Contents: http://apl.aip.org/resource/1/APPLAB/v81/i7
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Propagation of water waves through finite periodic arrays of vertical cylinders

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(Received 2 January 2002; accepted for publication 17 June 2002)

We study the transmission of water waves propagating in finite-size two-dimensional periodic structures which consist of bottom-mounted cylinders using the multiple-scattering method. Complete band gaps exist between the first and the second bands in square and triangular periodic structures, as well as one modeled on a graphite atomic lattice. We investigate the dependence of the band edges on the filling fraction. The graphite-type structure shows the band gap at a lower filling fraction than the others. Therefore, the graphite-type structure may be more suitable for practical coastal protection. We also calculate the first-order force on cylinders located along the symmetry direction. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1499520]

Special attention has been given to structures which consist of a large number of different elements situated in periodic arrays.[1–4] For example, two-dimensional (2D) photonic crystals constructed with periodic dielectric composites have photonic band gaps (PBGs). Multiple coherent transmissions and reflections add up to prevent electromagnetic waves from propagating in the crystals over a wide range of frequencies.[1–3] Some photonic crystals can have complete PBGs where waves cannot propagate along any direction. By analogy, we can deduce that also for water waves periodic structures may exhibit a certain frequency range where the propagation of wave is forbidden, i.e., water wave band gaps (WWBGs).[5,6] The propagation of water waves over submerged bars has been widely studied due to its practical importance in coastal protection, since they show WWBGs which result from the Bragg reflection.[7,8] However, for better protection of coasts, it is desirable to design a superior periodic structure with a wider band gap and a smaller lattice constant, which can fit even into a small coastal area.

In this letter, the WWBGs of finite 2D lattices having square, triangular, and graphite-type structures composed of circular cylinders as a function of filling fraction. Our attention in the present work will be restricted to a simple, but not unrealistic, geometry of N vertical circular cylinders spanning the whole depth of water, i.e., the cylinders stand from the bottom to the air above the water surface. The cylinders need not be filled. Shell-type cylinders may be enough as long as shells are strong and thick.

In order to investigate the WWBGs, we calculate the transmission spectra of water waves in 2D periodic arrays of cylinders. For the calculation we use the multiple-scattering method in which diffracted waves are linked to the incoming one and represented by Fourier–Bessel expansions.[8,9] From the translation properties of Bessel functions, the scattering problem is reduced to a linear one.

Under the usual assumption of linear water theory there exists a velocity potential $\Phi(x,y,z,t)$, where x and y are the coordinates of the horizontal plane in the mean free-surface, i.e., the plane of calm water surface without waves, $z=0$, and z is vertically upward. We assume that there are N fixed vertical cylinders, each of which extends from the bottom at $z=-h$ up through the free-surface. The depth dependence of the problem can then be factored out and if we also assume that all motion is time-harmonic with angular frequency ω, we can write

$$\Phi(x,y,z,t)=\mathrm{Re}[\phi(x,y)\cosh k(z+h)e^{-i\omega t}], \tag{1}$$

where k is the wave number. Φ satisfies Laplace's equation together with a free-surface boundary condition

$$\frac{\partial\Phi}{\partial z}+\frac{\omega^2}{g}\Phi=0 \quad \text{on} \quad z=0, \tag{2}$$

where g is the gravitational acceleration.[5,8] While the dispersion relation between ω and k for electromagnetic waves is linear, it is $\omega^2=gk\tanh kh$ for water waves. The elevation η of the water wave from the mean free-surface is then given by

$$\eta=\mathrm{Re}\left[-\frac{i\omega}{g}\phi(x,y)e^{-i\omega t}\right]. \tag{3}$$

The rest of the calculation employs the multiple-scattering approach, which uses Fourier–Bessel expansion. The details of this method are given in Ref. 8.

Transmittance of water waves is calculated along the symmetry directions, because the complete WWBG (CWWBG), where incident waves cannot propagate along any direction, is determined by the symmetry points in the first Brillouin zone of the periodic structures.[10] Physically significant solutions of Laplace's equation in periodic structures can be assigned to the wave vectors in the first Brillouin zone. The filling fraction f of these structures is defined as the fractional area occupied by the cylinders and the trans-

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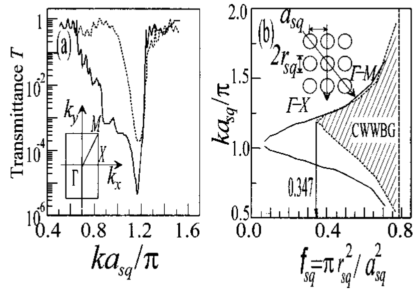

FIG. 1. (a) Transmission spectra of a square structure along the symmetry directions at the filling fraction $f_{sq}=0.385$, and $r_{sq}/a_{sq}=0.35$ where r_{sq} is the radius of the cylinder and a_{sq} the lattice constant. The inset shows the symmetry points of the first Brillouin zone. The $\Gamma-X$ ($\Gamma-M$) direction is represented by a solid (dashed) line. (b) The WWBG edges of the square structure described in (a) as a function of the filling fraction. The $\Gamma-X$ ($\Gamma-M$) direction is represented by a solid (dashed) line. The arrow indicates the value of $f_{sq}=0.347$ above which a CWWBG (hatched area) exists. The vertical dashed line denotes the filling fraction $f_{sq}=\pi/4$, at which the cylinders touch. The inset shows the symmetry directions.

mission coefficient T as $|\eta_{\text{trans}}/\eta_{\text{inc}}|^2$, where η_{trans} is the amplitude of transmitted water waves and η_{inc} that of incident ones.

Figure 1(a) shows the transmission spectra of the square structure consisting of 11×11 cylinders ($N=121$) with the filling fraction $f_{sq}=0.385$. A series of numerical tests which have been performed varying the number of cylinders and the shape of the array show little change in the center frequencies of WWBGs. The solid and the dashed lines represent the transmittance of the incident wave in the $\Gamma-X$ and $\Gamma-M$ directions, respectively, where Γ, M, and X are the symmetry points in the first Brillouin zone of the square lattice. The wave number is normalized to the lattice constant a_{sq} due to the scale invariance of Laplace's equation. There exists a CWWBG between $ka_{sq}/\pi=1.133$ and 1.232, if we define the WWBG as the frequency region where the transmittance is below 10^{-2}. The solid (dashed) line defines the edges of the first WWBG along the $\Gamma-X$ ($\Gamma-M$) direction in Fig. 1(b). The WWBG of the $\Gamma-M$ direction appears at higher wave number k than that of the $\Gamma-X$ direction, since the wave number at the M point is larger than that at the X point in the Brillouin zone. The hatched CWWBG occurs when the two WWBGs along the both symmetry directions overlap at $f_{sq}\geqslant 0.347$. The WWBGs widen in k as f_{sq} increase until water waves of all wave vectors are totally reflected when the cylinders approach to touch each other.[4]

Transmittance of the 2D triangular structure, which is derived by putting cylinders at the centers of 7×7 hexagons ($N=259$) is shown in Figs. 2(a) and 2(b). Figure 2(a) is obtained for the filling fraction $f_{tr}=0.184$, and in Fig. 2(b) the first WWBG edges are shown as a function of f_{tr}. The triangular lattice exhibits a very narrow CWWBG at a relatively high filling fraction of $f_{tr}\geqslant 0.655$ (indicated by an arrow). This result is consistent with the fact that a 2D triangular lattice has a much smaller complete PBG than a 2D square lattice when they are made with perfect conducting cylinders.[11]

The symmetry reduction of atomic configuration by introducing a basis composed of two dielectric cylinders in simple 2D lattices can remarkably increase complete PBGs.[12,13] So we calculate the WWBG in a 2D structure

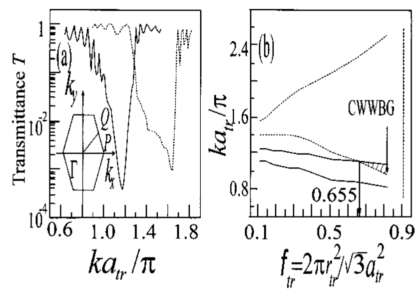

FIG. 2. (a) Transmission spectra of a triangular structure at $f_{tr}=0.184$. The inset shows the symmetry points of the first Brillouin zone. The $\Gamma-Q$ ($\Gamma-P$) direction is represented by a solid (dashed) line. (b) The WWBG edges of the triangular structure described in (a) as a function of the filling fraction. The $\Gamma-Q$ ($\Gamma-P$) direction is represented by a solid (dashed) line.

modeled on the atomic configuration of graphite, which is a triangular structure having two cylinders per unit cell as a basis.[14] In Fig. 3(a), T versus the normalized wave vector is plotted for water waves propagating in a 2D graphite-type structure consisting of 7×7 hexagons ($N=210$) at vertices of which the cylinders are centered. The spectra are calculated for the filling fraction f_{gr} of 0.151. The Brillouin zone of the structure is the same as that of the triangular structure, so the same notations are used as in the triangular structure. One can see that the shape of two transmission spectra along the $\Gamma-Q$ and the $\Gamma-P$ directions are very similar to each other; a feature not observed in the square structure case [cf. Fig. 1 (a)]. Figure 3(b) shows the first WWBG edges as a function of f_{gr}. It is noteworthy that the value of the filling fraction at which the CWWBG opens is much smaller in the graphite-type structure ($f_{gr}=0.141$) than in the square ($f_{sq}=0.347$) and in the triangular ($f_{gr}=0.655$) periodic structures. And also the width of CWWBG of the graphite-type structure is appreciably larger compared to those of the other two. These are consistent with the fact that the width of the PBG for the TE mode, whose electric field is parallel to the plane of propagation in a 2D graphite-type dielectric structure, is larger than those of square and triangular lattices of dielectric rods in the air.[14] The WWBG along the $\Gamma-P$ direction is larger than that along the $\Gamma-Q$ because the P point has the larger k than the Q point. The WWBG of the graphite-type structure also diverges as the cylinders approach to touch each other. These results can be extended to periodic structures consisting of submerged or floating cylinders. Furthermore, the cylinders may be composed of many

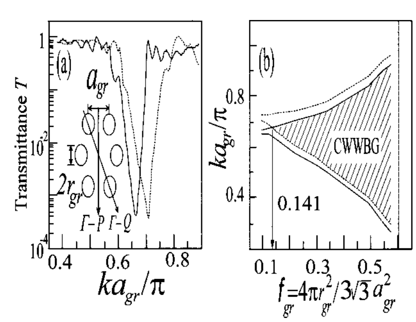

FIG. 3. (a) Transmission spectra of a graphite-type structure at $r_{gr}/a_{gr}=0.25$ and $f_{gr}=0.151$. The $\Gamma-Q$ ($\Gamma-P$) direction is represented by a solid (dashed) line. (b) The WWBGs of the graphite-type structure described in (a) as a function of the filling fraction. The $\Gamma-Q$ ($\Gamma-P$) direction is represented by a solid (dashed) line.

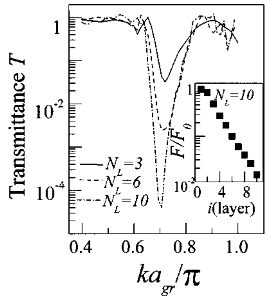

FIG. 4. Transmission spectra of a graphite-type structure varying the number of layers N_L along the $\Gamma-P$ direction at $f_{gr}=0.151$ as a function of the wave number. The inset shows the first-order force on the cylinder of the ith layer when $N_L=10$.

rods of smaller size arranged in a circle. These rods need not touch each other, as long as the distance between two adjacent rods is much less than the wavelength of water wave. Water may flow through not only the large spaces between the cylinders but also the small gaps between the rods, while the propagation of water waves over the structure is prohibited. This may significantly reduce the contamination of water along the coast as well as the construction cost of bulwarks.

Now consider a graphite-type structure which consists of N_L layers of hexagons along the propagation direction ($\Gamma-P$ direction) with the same lateral width as the previous graphite-type structure at $f_{gr}=0.151$. As shown in Fig. 4, the wave transmission decreases drastically as the thickness of the structure increases. The first-order force amplitude F acting on a cylinder can be calculated from the coefficients of Fourier–Bessel expansions of Φ.[8] The inset of Fig. 4 depicts the attenuation of F on the cylinder in the ith layer of hexagons from the incident surface for the case of $N_L=10$, where F_0 is the first-order force on an isolated cylinder. The force decays exponentially, and is reduced to about $F_0/100$ at the last layer of the structure.

In conclusion, we studied water wave propagation through 2D periodic structures of parallel cylinders in water. Employing the multiple-scattering method, the WWBGs for water waves was calculated for periodic square, triangular, and graphite-type arrays of cylinders. Among them, the graphite-type structure exhibited the largest CWWBG at the smallest value of the filling fraction, suggesting that this structure would be a good candidate for practical coastal protection. Also the change in the transmission of water waves and the first-order force exerted on cylinders by the water waves was investigated varying the size of the graphite-type structure along the direction of wave propagation.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[3] K. M. Ho, C. T. Chan, and C. M. Soukoulis, Phys. Rev. Lett. **65**, 3152 (1990).
[4] M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993); M. Torres, J. P. Adrados, and F. R. Montero de Espinosa, Nature (London) **398**, 114 (1999); Z. Ye and E. Hoskinson, Appl. Phys. Lett. **77**, 4428 (2000).
[5] A. D. Heathershaw, Nature (London) **296**, 343 (1982); J. T. Kirby and R. A. Dalrymple, J. Fluid Mech. **133**, 47 (1983); C. C. Mei, *ibid.* **152**, 315 (1985).
[6] T. Chou, J. Fluid Mech. **369**, 333 (1998); P. Mciver, *ibid.* **424**, 101 (2000).
[7] H. Kagemoto and D. K. P. Yue, J. Fluid Mech. **166**, 189 (1986); S. Kakuno and P. L.-F. Liu, J. Waterw. Port Coast. Ocean Eng. **119**, 302 (1993).
[8] C. M. Linton and D. V. Evans, J. Fluid Mech. **215**, 549 (1990).
[9] V. Twersky, J. Acoust. Soc. Am. **24**, 42 (1952); G. Tayeb and D. Maystre, J. Opt. Soc. Am. A **14**, 3323 (1997); L. M. Li, Z. Q. Zhang, and X. Zhang, Phys. Rev. B **58**, 15589 (1998).
[10] K. Sakoda, Phys. Rev. B **52**, 8992 (1995).
[11] N. A. Nicorovici, R. C. McPhedran, and L. C. Botten, Phys. Rev. E **52**, 1135 (1995); T. Suzuki and P. K. L. Yu, Phys. Rev. B **57**, 2229 (1998).
[12] C. M. Anderson and K. P. Giapis, Phys. Rev. Lett. **77**, 2949 (1996).
[13] D. Caballero, J. Sánchez-Dehesa, C. Rubio, R. Mártinez-Sala, J. V. Sánchez-Pérez, F. Meseguer, and J. Llinares, Phys. Rev. E **60**, R6316 (1999).
[14] D. Cassagne, C. Jouanin, and D. Bertho, Phys. Rev. B **53**, 7134 (1996).