An Exploratory Study on Contingency Rules to Control a Production Process subject to an Assignable Cause

April 1997

Subject Area: Contingent Process Control, Statistical Process Control

Bowon Kim
Graduate School of Management
Korea Advanced Institute of Science and Technology

Bowon Kim
Graduate School of Management at KAIST
207-43 Cheongryangri Dongdaemoon
Seoul, 130-012 Korea
T. +82 2 958 3610, F. +82 2 958 3604, bwkim@cais.kaist.ac.kr

An Exploratory Study on Contingency Rules to Control a Production Process subject to an Assignable Cause

Abstract

We present two contingency rules to control a production process that is subject to an assignable cause, i.e., the process shifts from in-control to out-of-control state when an assignable cause occurs. In this paper, we assume a 100% inspection policy as opposed to the sampling concept. However, the first rule we suggest is comparable with the traditional sampling model in that its criterion to intervene into the process is based on the number of defective products. The second rule uses more information than the first does: it triggers intervening into the process when the inter-arrival time between two consecutive defective products is smaller than an optimally derived cut-off rate. We first show how the two rules can be derived, and, as a preliminary analysis, propose conditions under which one contingency rule might be more effective than the other.

1. Introduction

Production process control is an essential part of production and operations management, and operations improvement can be facilitated through effective controlling of the production process. Effective process control enables operations managers to intervene into the production system for controlling various forms of variance occurring in the process (Jaikumar 1988). Inadequate process control causes the production system to produce defective goods. A production process can be affected by assignable causes, also referred to as 'contingent events' in this paper, that shift the process state from 'in-control' to 'out-of-control.' One of the key issues related with the process control is to determine when to intervene into the production process in order to prevent or minimize the negative effect that can be done by those contingent events.

In the literature, process control is also perceived as related to a more comprehensive function in operations management, i.e., inventory control/management. Hall (1983) suggests that the inventory level is an indication of the efficiency of process control. This relationship between inventory and efficient process control has been further investigated in Hall (1987), Jaikumar (1988), Karmarkar (1987), and more recently Klastorin, et al. (1993). Focusing on a more specific function of process control, researchers in SPC (Statistical Process Control) theory have developed several economic rules to intervene into the production process to minimize the costs due to out-of-control process and also defective products it produces (Chiu 1976, Chiu and Wetherill 1974, Chiu and Cheung 1977, Costa 1993, Duncan 1956, Gibra 1978, Goel and Wu 1973, Nelson 1993, Taylor 1968). A typical SPC model would suggest an economic rule taking into account an assignable cause, in-control *versus* out-of-control state, false alarm (e.g., a defective product from an in-control process) *versus* true alarm signaling occurrence of a contingent event, and probabilistic distributions of processing time and event time. Markov decision theory has been used in some of the economic models: based on principles of dynamic program and Bayesian theory, a Markov decision model capitalizes on a sequential updating in determining the process state (Bertsekas 1987, Dreyfus and Law 1977, Ross 1970 and 1983).

In this paper, we present two contingency rules to control a production process, assuming a complete inspection policy, i.e., 100% inspection. The process is assumed to shift from in-control to out-of-control state when an assignable cause affects it. The first rule we propose triggers intervening into the process when the number of defective products is larger than an optimally derived cut-off number. The second rule uses additional information about the inter-arrival times between two consecutive defective products: it triggers the intervention when the inter-arrival time between two consecutive defectives is smaller than an optimally derived cut-off rate.

In the next section, we describe key characteristics of the production process assumed in the current research. In section 3, two control rules, first-order contingency rule and second-order contingency rule, are defined and their analytical models are developed. A preliminary analysis, based on a simulation run, comparing the two rules' performance is presented also in the section. In the final section, section 4, we discuss the analysis outcomes and suggest a few managerial implications.

2. Contingency in Production Process Control

An assignable cause, also referred to as a contingent event in this paper, is an event that causes a production process to shift from an 'in-control' state to an 'out-of-control' state, and involves an inherent uncertainty. Whether a process is 'in-control' or 'out-of-control' is a relative concept, and this relativity makes the process control 'contingent.' For instance, if an in-control process always produces a good product, then the process control is not contingent since a decision maker (*DM*), i.e., an operations manager, can determine the state of the production process perfectly by simply observing the products produced by the process: when a defective product is produced, the *DM* can be sure that the process is in out-of-control state.

The problem becomes more complicated since P(process produces defectives | in-control) > 0 and P(process produces defectives | out-of-control) < 1. In other words, even if the process is not affected by an assignable cause, i.e., in in-control state, it still produces defective products but with much smaller probability. Likewise, a process affected by an assignable cause can still produce good products, albeit with lower probability.

Let's define,

P(process produces a defective | in-control) = α,

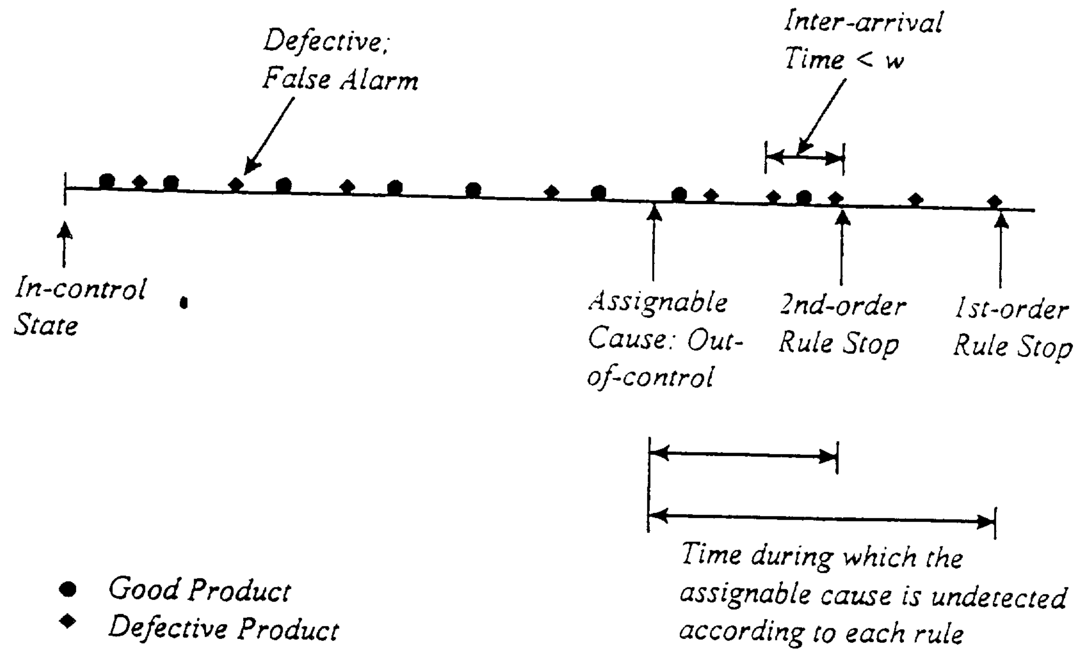
P(process produces a defective | out-of-control) = β.

Then, we assume $\alpha < \beta$.

We focus on a 100% inspection policy as opposed to a sampling plan. Considering the availability of sophisticated inspection equipment at a decreasing price, it seems not too constraining to assume a complete inspection. Other assumptions necessary for mathematical modeling will be made in the ensuing associated sections.

A general picture of production process operations is in Figure 1.1. The process starts in in-control state. As mentioned already, before an assignable cause affects the process, it can produce defective products, which are referred to as false alarms (implying that the defectives are false signals for the process' being out-of-control). Once an assignable cause occurs, the process enters an out-of-control state and produces defectives more frequently, i.e., with much higher probability. Until the managers detect the process state and intervene into it, the process remains in the out-of-control state. The two contingency rules we develop enable the managers to take an appropriate measure to control the production process optimally.

Figure 1.1 Production Process and Inspection Procedure



3. First-order versus Second-order Contingency Rule

We develop two contingency rules to control a production process subject to an assignable cause. Although we assume an 100% inspection policy different from a sampling policy, the first rule, "first-order contingency rule," is comparable with the sampling concept in that it uses the number of false alarms as a criterion to trigger the *DM* to intervene into the process. The second rule, "second-order contingency rule," is different from more traditional approaches in SPC (possibly including the first-order contingency rule) in that it utilizes an additional information about the inter-arrival times between two consecutive defectives.

In this section, we develop mathematical models for the contingency rules. As a preliminary analysis based on a limited set of parameter values, we also set up a simulation run to compare the relative performance of the two rules.

Before presenting the mathematical models, we point out basic attributes of the production process in point:

- As alluded before, the production process utilizes a 100% inspection policy.
- With the inspection, the manager (*DM*) can determine whether the product is good or defective, but can not decide if an assignable cause has affected the process.
- In order to determine whether an assignable cause (contingent event) has occurred, the manager has to stop the process and carry out a thorough investigation.

Thus, the primary objective of the contingency rules is to help the manager decide when to intervene into the process so as to minimize the total costs associated with the process stopping, based on the information related with the occurrence pattern of defective products.

3.1. First-order Contingent Process Control

Researchers focusing on an economic design of SPC strategy have extensively studied the nature of contingent process control (Chiu 1976, Duncan 1956, Nelson 1993). We call this line of research 'Economic Statistical Process Control (ESPC).' According to ESPC, the expected number of samples taken during an in-control period is (Chiu 1976):

$$N_e = \sum_{x=0}^{\infty}\left(\int_{xh}^{(x+1)h} x\lambda e^{-\lambda t}\,dt\right) = \frac{e^{-\lambda h}}{1-e^{-\lambda h}} \qquad \ldots\ldots(1)$$

where
h: a sampling interval (i.e., a sample is drawn ever h hour),
x: number of samples taken during an in-control period,
y: time for an assignable cause to occur, following an exponential distribution with a probability density function of $f_y(y) = \lambda e^{-\lambda y}$.

With P(process produces a defective | in-control) $=\alpha$ as the probability that a false alarm occurs in a sample (i.e., a defective product is produced while the process is in-control), we can obtain the expected number of false alarms during the in-control period by multiplying N_e by α.

As mentioned before, we assume a 100% inspection policy. Under the policy, we need to modify (1) accordingly. We can regard h as a processing time to produce a product. As in the literature, we model h as following an exponential distribution with a parameter μ, i.e.,

h: processing time (time necessary to produce a unit of product),
$f_h(h)$: probability density function of h, and $f_h(h) = \mu e^{-\mu h}$.

Thus, the expected number of false alarms during an in-control period under the 100% inspection policy is

$$\alpha \int_0^\infty \left(\frac{e^{-\lambda h}}{1-e^{-\lambda h}} f_h(h) \right) dh = \alpha \int_0^\infty \frac{\mu e^{-(\lambda-\mu)h}}{1-e^{-\lambda h}} dh \qquad \ldots\ldots(2)$$

For a numerical analysis, (2) can be further simplified as

$$\alpha\, n \int_0^1 \frac{(1-k)^n}{k} dk, \qquad \ldots\ldots(3)$$

where $n = \frac{\mu}{\lambda}$ and $k = 1 - e^{-\lambda h}$.

Let's define $\lceil A \rceil$ as the largest integer which is less than or equal to A. The first-order contingent process control enforces a decision rule to intervene into the process when the number of defective products hits $\left\lceil \alpha\, n \int_0^1 \frac{(1-k)^n}{k} dk \right\rceil + 1$: since $\alpha \int_0^1 \frac{(1-k)^n}{k} dk$ is the *most likely average* number of false alarms during an in-control period, the m^{th} defective product, where $m = \left\lceil \alpha\, n \int_0^1 \frac{(1-k)^n}{k} dk \right\rceil + 1$, is the first defective after the process enters an out-of-control period *on average*. Since this first-order contingency rule is consistent with a general SPC model, we expect its performance to be comparable with that of a policy based on the sampling concept.

3.2. Second-order Contingent Process Control

We can see that the first-order contingency rule does not utilize all of the possible information given. For instance, it does not use information associated with P(process produces a defective | out-of-control) = β. A critical rationale underlying the second-order contingency rule is to utilize the information by observing the difference in inter-arrival times between consecutive defectives as the process status changes.

Figure 3.1 Inter-arrival time of two consecutive defective products

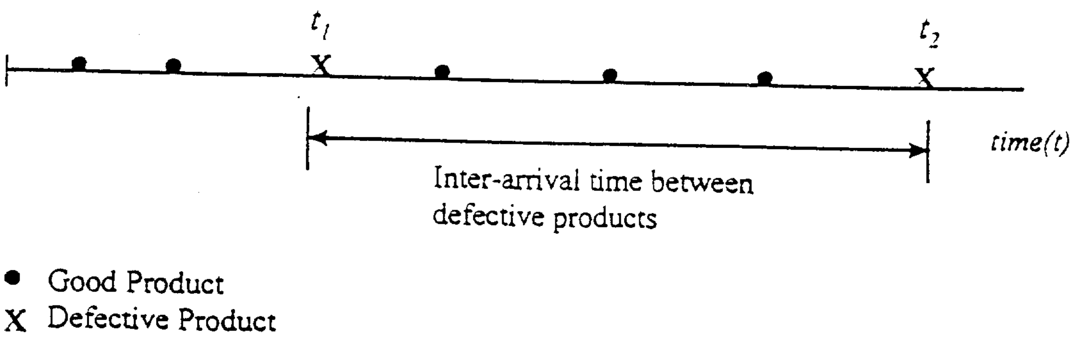


Figure 3.1 depicts the concept of an inter-arrival time. Suppose at t_1 a defective product is produced. At this point, however, the decision maker (*DM*) can not be sure about if the process is *in-control* or *out-of-control*: all the *DM* can know is that the

product is defective. Then, each time a product is produced, she has to consider the following probabilities:

$$P(defect \mid in\text{-}control)=\alpha \;,\; P(defect \mid out\text{-}of\text{-}control)=\beta \;,$$

$$P(in\text{-}control)=P(h<y)=\frac{\mu}{\mu+\lambda} \;,\; P(out\text{-}of\text{-}control)=P(h>y)=\frac{\lambda}{\mu+\lambda} \;,$$

$$P(in\text{-}control \mid defect)=\frac{\alpha\mu}{\alpha\mu+\beta\lambda} \;,\; P(out\text{-}of\text{-}control \mid defect)=\frac{\beta\lambda}{\alpha\mu+\beta\lambda} \qquad \ldots\ldots(4)$$

Suppose at t_2 another defective product is produced. Now the decision maker must decide whether the process is affected by an assignable cause and thus out-of-control. The *DM* can do so by analyzing the inter-arrival time of the two consecutive defective products, i.e., $t_2 - t_1$. In making a decision, the *DM* wants to minimize the total cost due to two types of mistake. One such mistake is called type I error that occurs when the *DM* determines that the process is out-of-control when it actually is in-control. The other, type II error, occurs when the *DM* determines that the process is in-control when in fact it is out-of-control. The *DM* must balance the costs associated with the two errors according to their relative weights.

For the second-order contingent process control, we suggest a cut-off rate of inter-arrival time, ϖ as a decision criterion. Let's define the inter-arrival time of the two latest consecutive defective products as $\tau = t_2 - t_1$. Then, the analytic rule works as follows. If $\tau \le \varpi$, then stop the production process since it is determined as 'out-of-control.' Otherwise, the process is assumed to be in-control.

In order to develop a rule using ϖ, we need to specify two additional probability distributions, one for the inter-arrival time between two consecutive defective products while the process is in-control and the other while the process is out-of-control.

Let's define,

u : inter-arrival time between two consecutive defective products while the process was in-control when the first defective was produced,
$g_u(u)$: probability density function of u ,
v : inter-arrival time between two consecutive defective products while the process was out-of-control when the first defective was produced,
$g_v(v)$: probability density function of v .

It is reasonable to assume $E(u) > E(v)$, i.e., the mean inter-arrival time of defective products while the process is out-of-control is shorter than that while the process is in-control.

Assuming independence and memoryless property (Wolff 1989), we can show that u follows an exponential distribution with a parameter,

$P(defect\ and\ in\text{-}control) \times \mu = P(defect \mid in\text{-}control) \times P(in\text{-}control) \times \mu = \left[\alpha \frac{\mu}{\mu+\lambda}\right]\mu.$

Thus, $g_u(u) = \frac{\alpha\mu^2}{\mu+\lambda} e^{-\frac{\alpha\mu^2}{\mu+\lambda}u}$. ......(5)

Applying the same logic, we can obtain

$P(defect\ and\ out\text{-}of\text{-}control) \times \mu = P(defect \mid out\text{-}of\text{-}control) \times P(out\text{-}of\text{-}control) \times \mu$

$= \left[\beta \frac{\lambda}{\mu+\lambda}\right]\mu$, and $g_v(v) = \frac{\beta\mu\lambda}{\mu+\lambda} e^{-\frac{\beta\mu\lambda}{\mu+\lambda}v}$. ......(6)

Since $E(u) > E(v)$ is assumed, from (5) and (6) we can obtain

$\frac{\alpha\mu^2}{\mu+\lambda} < \frac{\beta\mu\lambda}{\mu+\lambda}$, and therefore $\alpha\mu < \beta\lambda$. ......(7)

Thus, the objective of the current decision making problem is to minimize

Total Cost ($TC(\varpi)$)$=\Phi_I + \Phi_{II}$

$= c_1 \times P_2(\tau \leq \varpi \mid defect\ and\ in\text{-}control) \times P_1(in\text{-}control \mid defect) +$

$c_2 \times P_2(\tau > \varpi \mid defect\ and\ out\text{-}of\text{-}control) \times P_1(out\text{-}of\text{-}control \mid defect)$ ......(8)

Φ_I : cost due to type I error,
Φ_{II} : cost due to type II error,
c_1 : unit penalty cost associated with type I error,
c_2 : unit penalty cost associated with type II error,
P_i: probability associated with an event at t_i, i=1, 2 (see Figure 3.1).

In (8), we assume an additive cost function, i.e., linearly summing the two cost factors with appropriate weights. Another caveat is in order. Although the decision making process is considered dynamic, the objective function, $TC(\varpi)$, does not incorporate a dynamic decision making mechanism. Rather, it is a point decision making. The *DM* makes a decision by observing the probabilities relevant at the particular time point a defective product is made, i.e., the probabilities of mistakes she would make if ϖ is used as a cut-off rate. Thus, the objective function tries to minimize the weighted sum of the probabilities at a time point: it does not intend to minimize the total cost summed over an entire decision time horizon. In effect, the *DM* considers the past cost as sunk, and the future cost as less relevant for the current decision making. Thus, the goal of this analytic rule is to find out the most likely 'timing' an assignable cause affects the production process by observing an inter-arrival time between two consecutive defective products. Consequently, it might be possible that c_1 and c_2

depend on time, t. But if the overall production cycle is short enough to make the costs, c_1 and c_2, change little within the current decision time horizon, we can regard c_1 and c_2 as constants for the purpose of the current decision making. In this paper, we consider such an environment.

The detailed objective function is

$$TC(\varpi)=c_1\left(\int_0^{\varpi} g_u(u)du\right)\frac{\alpha\mu}{\alpha\mu+\beta\lambda}+c_2\left(\int_{\varpi}^{\infty} g_v(v)dv\right)\frac{\beta\lambda}{\alpha\mu+\beta\lambda}$$

$$=\frac{\alpha\mu c_1}{\alpha\mu+\beta\lambda}\left[1-e^{-\frac{\alpha\mu^2}{\mu+\lambda}\varpi}\right]+\frac{\beta\lambda c_2}{\alpha\mu+\beta\lambda}\left[e^{-\frac{\beta\mu\lambda}{\mu+\lambda}\varpi}\right]. \qquad \text{......(9)}$$

In order to obtain an optimal ϖ, we differentiate (9) in terms of ϖ,

$$\frac{dTC(\varpi)}{d\varpi}=\left[\frac{\alpha\mu c_1}{\alpha\mu+\beta\lambda}\right]\left[\frac{\alpha\mu^2}{\mu+\lambda}\right]e^{-\frac{\alpha\mu^2}{\mu+\lambda}\varpi}-\left[\frac{\beta\lambda c_2}{\alpha\mu+\beta\lambda}\right]\left[\frac{\beta\mu\lambda}{\mu+\lambda}\right]e^{-\frac{\beta\mu\lambda}{\mu+\lambda}\varpi}$$

$$=\theta_1 e^{-\frac{\alpha\mu^2}{\mu+\lambda}\varpi}-\theta_2 e^{-\frac{\beta\mu\lambda}{\mu+\lambda}\varpi}=0, \qquad \text{......(10)}$$

where $\theta_1\equiv\left[\frac{\alpha\mu c_1}{\alpha\mu+\beta\lambda}\right]\left[\frac{\alpha\mu^2}{\mu+\lambda}\right]$ and $\theta_2\equiv\left[\frac{\beta\lambda c_2}{\alpha\mu+\beta\lambda}\right]\left[\frac{\beta\mu\lambda}{\mu+\lambda}\right]$.

After rearranging (10), the optimal ϖ can be described as

$$\varpi^*=\left[\frac{\mu+\lambda}{\alpha\mu^2-\beta\mu\lambda}\right]ln\left[\frac{\theta_1}{\theta_2}\right]. \qquad \text{......(11)}$$

In order for (11) to be a meaningful solution, it must be satisfied that $\varpi^*\geq 0$. Since we know $\alpha\mu<\beta\lambda$, i.e., $\alpha\mu^2-\beta\mu\lambda<0$, from (7), it must be that $ln\left[\frac{\theta_1}{\theta_2}\right]\leq 0$, i.e., $\theta_1\leq\theta_2$.

By rearranging θ_1 and θ_2 in (10), we can see that $\theta_1\leq\theta_2$ implies $\frac{\alpha\mu}{\beta\lambda}\leq\sqrt{\frac{c_1}{c_2}}$. Thus, for an acceptable solution, we need to impose $\frac{\alpha\mu}{\beta\lambda}\leq\sqrt{\frac{c_1}{c_2}}$. ......(12)

Theorem 1. $\frac{\alpha\mu}{\beta\lambda}\leq\sqrt{\frac{c_1}{c_2}}$ is a necessary condition for $TC(\varpi)$ to have an admissible optimal solution. Theorem 1 is proved above.

We also know $\frac{\alpha\mu}{\beta\lambda} < 1$ from (7). Therefore, if $c_1 \le c_2$, then (12) is always satisfied because $\frac{\alpha\mu}{\beta\lambda} < 1 \le \sqrt{\frac{c_1}{c_2}}$. That $c_1 \le c_2$ implies the penalty to a type II error is at least as great as that to a type I error. It makes sense since 'determining an out-of-control process as in-control' can be more detrimental to the process control than 'determining an in-control process as out-of-control' provided the probability that the process is out-of-control is the same as the probability that the process is in-control. Even if $c_1 \le c_2$ is not valid, (12) can still be satisfied. For a nontrivial solution, we assume (12) is satisfied throughout the remaining discussion (or, we impose it as a constraint).

Theorem 2. (Sufficient Condition for Optimality) ϖ^* that solves (11) is the optimal solution for $TC(\varpi)$.

Sufficient condition for optimality calls for

$$\frac{d^2TC(\varpi)}{d\varpi^2} = -\theta_1 \left[\frac{\alpha\mu^2}{\mu+\lambda}\right] e^{-\frac{\alpha\mu^2}{\mu+\lambda}\varpi} + \theta_2 \left[\frac{\beta\mu\lambda}{\mu+\lambda}\right] e^{-\frac{\beta\mu\lambda}{\mu+\lambda}\varpi} \qquad \text{......(13)}$$

In order for (11) to be optimal, (13) with ϖ^* must be nonnegative

$$-\theta_1 \left[\frac{\alpha\mu^2}{\mu+\lambda}\right] e^{-\frac{\alpha\mu^2}{\mu+\lambda}\varpi} + \theta_2 \left[\frac{\beta\mu\lambda}{\mu+\lambda}\right] e^{-\frac{\beta\mu\lambda}{\mu+\lambda}\varpi} \ge 0 \qquad \text{......(14)}$$

$$e^{\left[\frac{\alpha\mu^2-\beta\mu\lambda}{\mu+\lambda}\right]\varpi^*} \ge \frac{\theta_1}{\theta_2}\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right]$$

$$\left[\frac{\alpha\mu^2-\beta\mu\lambda}{\mu+\lambda}\right]\varpi^* \ge ln\left[\frac{\theta_1}{\theta_2}\right] + ln\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right] \qquad \text{......(15)}$$

From (7) $\alpha\mu^2 - \beta\mu\lambda < 0$ or $\frac{\beta}{\alpha} > \frac{\mu}{\lambda}$, (15) becomes

$$\varpi^* \le \left[\frac{\mu+\lambda}{\alpha\mu^2-\beta\mu\lambda}\right] ln\left[\frac{\theta_1}{\theta_2}\right] + \left[\frac{\mu+\lambda}{\alpha\mu^2-\beta\mu\lambda}\right] ln\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right], \text{ i.e.,}$$

$$\varpi^* \le \varpi^* + \left[\frac{\mu+\lambda}{\alpha\mu^2-\beta\mu\lambda}\right] ln\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right] \qquad \text{......(16)}$$

Since $\alpha\mu^2 - \beta\mu\lambda < 0$, $ln\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right] < 0$, and thus $\left[\frac{\mu+\lambda}{\alpha\mu^2-\beta\mu\lambda}\right] ln\left[\frac{\alpha\mu^2}{\beta\mu\lambda}\right] > 0$. Therefore, (16) is supported.

Therefore, the second-order contingency rule uses ϖ^* derived above as a decision criterion. One caveat is in order. Even if the process is in-control at t_1, there is possibility that a contingent event occurs between t_1 and t_2. This may cause a slight underestimation of an assignable cause occurring time. However, for the second-order contingency rule, we specifically focus on the inter-arrival time distribution of defective products, and we need to pay attention to the defective products only, trying to pinpoint the assignable cause occurring time by observing the defective products' inter-arrival times. Note that this rule is based on u and v rather than h and y that constitute the underlying distribution of the production process.

3.3. Comparative Analysis

The primary thrust of this paper is to derive contingency rules to intervene into a production process that can be affected by an assignable cause. However, since we expect the first-order contingency rule to perform in a comparable way with a general SPC model, it can be useful to compare the two contingency rules' performance.

We designed a simulation run to obtain the following 3 performance measures (Law and Kelton 1982):

a) *%_detected*: proportion of successful process stopping, i.e., (*number of process stops when there was an assignable cause, i.e.. successful process stops*) divided by (*total number of process stops*),
b) *mean_stop*: mean process stopping time.
c) *%_undetected*: period during which an assignable cause has not been detected as % of process stopping time, i.e., *[(process stopping time - assignable cause occurring time) / process stopping time]*.

Figure 3.2 Proportion of Successful Process Stopping

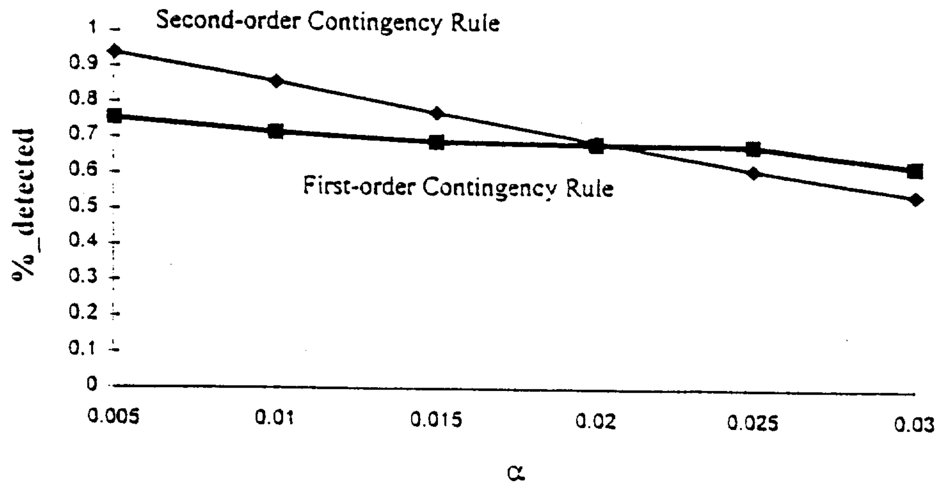


Figure 3.3 Mean Process Stopping Time

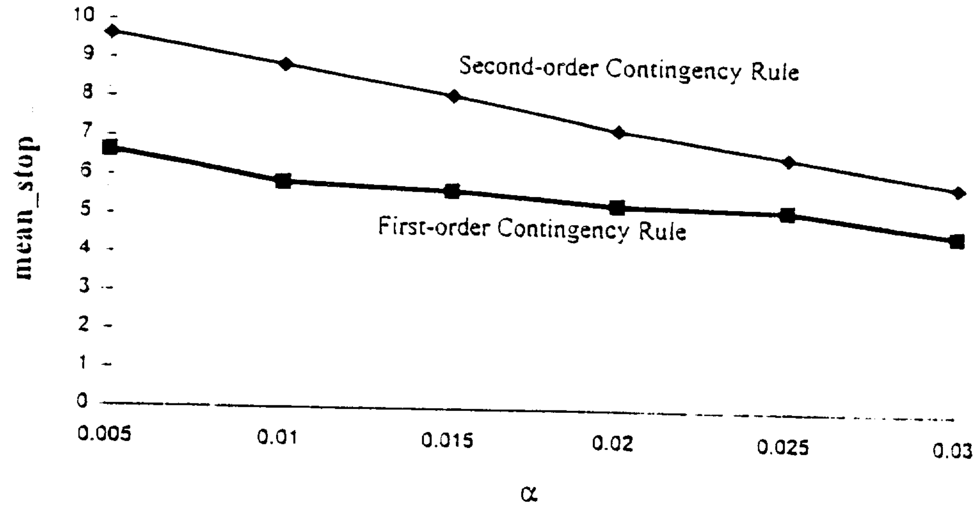
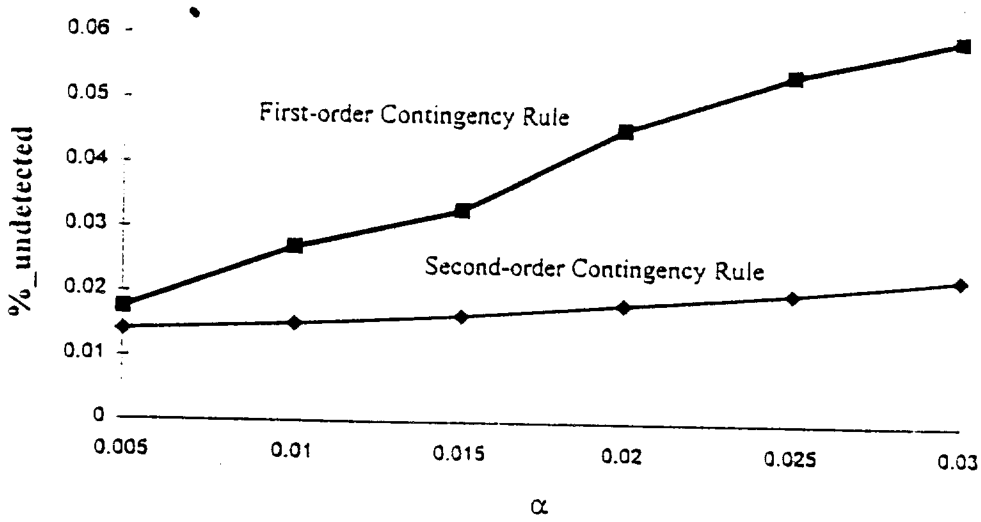


Figure 3.4 Period of Contingent Event Undetected



The simulation was run with the following parameters: λ=0.1, μ=20, β=0.5~1.0, α=0.005~0.03, and $c_1 = c_2$. The simulation results are in Figures 3.2~3.4 (*%_detected* in Figure 3.2, *mean_stop* in Figure 3.3, and *%_undetected* in Figure 3.4): in the figures, for each α, the different values by β were averaged, since it turned out that $0.5 \le \beta \le 1.0$ does not affect the solution very sensitively.

Given the simulation results, we can tentatively conclude:

a) The second-order contingency rule performs better than the first-order contingency rule in terms of mean process stopping time and period during which an assignable cause has not been detected.

b) In terms of proportion of successful process stopping, the second-order rule is better than the first-order rule up to a certain value of α, that is, once $\alpha \geq 0.02$, the second-order rule loses its advantage over the first-order rule.

An exploratory implication of these results is that the value of β as decision information might decrease as α becomes larger, i.e., P(process produces a defective | in-control) increases: as α increases and approaches β, the information β provides becomes less valuable. Therefore, for an incapable process (that produces a defective frequently even if no assignable cause is present), a traditional SPC model works reasonably. But, if the process is highly capable, the second-order contingency rule might be performing better than the traditional model, i.e., the value of additional information, β, can be significant.

4. Discussion and Implications

In this paper, we developed two contingency rules to control a production process subject to an assignable cause. A primary distinction between the two rules is whether to use an additional information about the process behavior. Based on a simulation, we showed some of the conditions under which one contingency rule might perform better than the other. That is, we tentatively concluded that for a capable process, the second-order contingency rule that takes into account information of β performs better than the first-order rule. However, we have to be very careful in extrapolating the simulation result too far: the simulation run was based on only a set of parameters, thus the result might be valid within a limited range. It seems necessary to simulate the models with more realistic and comprehensive parameter values. Another suggestion for further research is concerned about the assumptions for probabilistic distributions. For this research, we assumed exponential distributions for both processing time and contingent cause occurrence time. If a more general distribution like a normal or a uniform distribution had been used, the contingency rules might have taken different forms.

As mentioned before, the main thrust of this paper is to derive analytical rules for a production system with a particular set of attributes such as 100% inspection policy and identification of a good from a defective product on site. This can further limit the validity of the paper's conclusion. Nevertheless, the mathematical conceptualization can be a helpful guide in pursuing this line of research.

Reference

Bertsekas, D. P. (1987), *Dynamic Programming: Deterministic and Stochastic Models*, Prentice Hall.

Chiu, W. K. (1975), "Economic Design of Attribute Control Charts," *Technometrics*, 17, 1, 81-87.

Chiu, W. K. (1976), "Economic Design of *np* Charts for Processes Subject to a Multiplicity of Assignable Causes," *Management Science*, 23, 4, 404-411.

Chiu, W. K. and G. B. Wetherill (1974), "A Simplified Scheme for the Economic Design of $\bar{x}$-Charts," *Journal of Quality Technology*, 6, 2, 63-69.

Chiu, W. K. and K. C. Cheung (1977), "An Economic Study of $\bar{x}$-Charts with Warning Limits," *Journal of Quality Technology*, 9, 4, 166-171.

Costa, A. F. B. (1993). "Joint economic design of $\bar{X}$ and R control charts for processes subject to two independent assignable causes." *IIE Transactions*, 25(6), 27-33.

Dreyfus, S. B. and A. M. Law (1977), *The Art and Theory of Dynamic Programming*. Academic Press, Inc.

Duncan, A. J. (1956), "The Economic Design of $\bar{x}$-Charts Used to Maintain Current Control of a Process," *Journal of the American Statistical Association*, 51, 274, 228-242.

Duncan, A. J. (1971), "The Economic Design of $\bar{x}$-Charts When There is a Multiplicity of Assignable Causes," *Journal of the American Statistical Association*, 66, 333, 107-121.

Gibra, I. N. (1978), "Economically Optimal Determination of the Parameters of *np*-Control Charts," *Journal of Quality Technology*, 10, 1, 12-19.

Goel, A. L. and S. M. Wu (1973), "Economically Optimum Design of CUSUM Charts," *Management Science*, 19, 11, 1271-1282.

Hall, R. W. (1983), *Zero Inventories*, Dow Jones-Irwin, Homewood, Illinois.

Hall, R. W. (1987), *Attaining Manufacturing Excellence*, Dow Jones-Irwin, Homewood, Illinois.

Jaikumar, R. (1988), "Contingent Control of Synchronous Lines: A Theory of JIT," *Harvard Business School Working Paper*, 88-061.

Karmarkar, U. S. (1987), "Lot Sizes, Lead-Times and In-process Inventories," *Management Science*, 33, 3, 409-418.

Klastorin, T. D., L. Matheson, and K. Moinzadeh (1993). "On the use of buffer inventories to minimize costs with an unreliable machine." *IIE Transactions*, 25(5), 50-62.

Law, A. M. and W. D. Kelton (1982), *Simulation Modeling and Analysis*. McGraw-Hill Book Company.

Nelson, B. L. (1993). "Estimating acceptance-sampling plans for dependent production processes." *IIE Transactions*, 25(6), 11-18.

Ross, S. M. (1970), *Applied Probability Models with Optimization Applications*. San Francisco: Holden-Day.

Ross, S. M. (1983), *Introduction to Stochastic Dynamic Programming*. New York: Academic Press.

Taylor, H. M. (1968) "The Economic Design of Cumulative Sum Control Charts," *Technometrics*, 10, 3, 479-488.

Wolff, R. W. (1989). Stochastic Modeling and the Theory of Queues. Prentice-Hall.