Pricing Deposit Insurance Premium Based on Bank Default Risk

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Abstract

This article studies the pricing of deposit insurance premium based on the default risk of insured banks. We derive an exact closed-form formula for the deposit insurance premium in the Black-Scholes framework which possesses the attributes of simplicity, fairness and accuracy. Single period pricing framework is employed to express the levy practice of deposit insurance premium. This article explicitly takes into account the effects on the deposit insurance of the timing of bank default occurrence, capital forbearance policy and diverse debt issues with different maturities of a bank. These factors motivate us to make use of the pricing technique for American digital option and exchange option to derive risk-adjusted deposit insurance premium model. Finally, experimental analysis shows that our model has more explanation power for the real world than any other method reported in the literature.

(Keywords : deposit insurance, capital forbearance, default risk, American option, Digital option, Exchange option.)

1. Introduction

Deposit insurance is a guarantee that all or a limited amount of the principal and accrued interests will be paid in case that a bank becomes insolvent and thus unable to pay back depositors. Many nations have established government-owned deposit insurance agency because it plays an important role in national economic stability. The insuring agency operates the deposit insurance fund through collection of insurance premiums from insured banks to be used to pay back depositors in circumstances where the banks become insolvent or otherwise unable to pay back the deposits held by depositors. Deposit insurance system has three main objectives. First, deposit insurance is intended to provide protection of depositors in the event that a bank fails. By protecting depositors, it strives to assure depositors that their deposits are safe and to maintain the stability of the financial system. Second, deposit insurance is instrumental in the promotion of standards of sound banking and savings mobilization for banks. Third, deposit insurance supports the banking structure and maintains public confidence in the banking system. Without deposit insurance, depositors might rush in several big banks, and banks would be passive in lending money to companies.

Most of the countries that adopt deposit insurance system at present enforce fixed-rate deposit insurance premium system. In general, the deposit insurance agency levies deposit insurance premiums periodically – annually or quarterly - to the insured banks. Some other deposit insurance agencies charge initial insurance premiums to the insured banks on joining the deposit insurance and levy special or additional insurance premiums when it is necessary. The fixed-rate premium system has the merits that it costs much less maintenance fees than risk-adjusted premium system and that it is easy to calculate the premiums for each insured financial institution.

However, the fixed-rate premium system can give rise to critical problems. First, fixed-rate premium system cannot prevent risky banks from excessive risk-taking behavior seeking for higher returns. Second, the fixed-rate premium system can cause the result that sound and healthy banks financially support unwholesome and risky banks. Third, under the fixed-rate premium system, regulatory authority tends to intervene in the insured banks to maintain the soundness and safety of the deposit insurance funds. Especially, the first two problems above are related to moral hazard. It appears to be widely known that the deposit insurance premium depending on the risk level of the individual insured banks is the most fundamental solution for the moral hazard problem. In other words, risk-adjusted deposit insurance premium system induces the insured banks to carefully control the risks of their assets and to improve the capital adequacy and soundness. Consequently, it can rectify the contradiction of the income transfer or a cross subsidy effect for high-risk banks. Moreover, it can reduce the excessive regulation and supervision of the insuring agency.

But, the most important thing in adopting risk-adjusted deposit insurance premiums is how to determine the fair value of deposit insurance premium reflecting the risk level of financial institutions. Concerning risk-adjusted deposit insurance pricing, five literatures are worth while to be reviewed. Merton (1977) first suggested a deposit insurance premium pricing model using the isomorphic relationship between deposit insurance and European common stock put options. The article assumes that a bank issues only a single homogeneous debt with a specified maturity date, which implies that the default of the bank can happen only at the maturity. Since the article, most studies for the deposit insurance premium pricing have been conducted in Black-Scholes put option pricing framework. But the assumption of only a debt issue and European option characteristic is very unrealistic. Ronn and Verma (1986) proposed a risk-adjusted deposit insurance premium model working only with the market evaluated data on equity, not using data provided by bank management or FDIC audit. They revised the Merton's model to reflect more realist conditions incorporating FDIC regulatory policy or capital forbearance and dividends. Since the article, most studies take into account the forbearance effect of the insuring agency in pricing the fair deposit insurance premiums. They calculated the corporate value of the bank and its volatility through the theory of Black and Scholes (1973). They reported that the equity of a firm can be represented as a call option on the value of the assets of the firm with the same maturity as that of the debt of the firm and with a striking price equal to the maturity value of the debt. But, this brings about a crucial problem. For this to be true, the firm should never fail to be liquidated at maturity, implying that the firm closes and disappears. The mandatory liquidation contradicts the basic assumption of the continuity of a firm. Allen and Saunders (1993) modeled deposit insurance as a callable put option in the sense that the deposit insurer can force exercise of the deposit insurance put option to close the bank. Speaking in more detail, they evaluate deposit insurance as the callable perpetual American put option since the bank can operate permanently if it always passes the regularly scheduled audits. This is the first article that adopted American option pricing concepts. However, this model is not appropriate for the periodical levy system based on the change of the credit risk of the insured banks. Duan and Yu (1994, 1999) developed multiperiod deposit insurance pricing model considering the fact that the majority of defaulting banks after reorganization like purchase-and-assumption or government-assisted merger continue to operate with deposit insurance. They regarded the deposit insurance as a stream of one-period put options with occasional asset value resets by exercising put options at the points of insolvency resolution. Because early exercise of the put option might only be permitted on certain dates throughout its life, their method can be classified into a Bermudan put option model. But, they did not make it clear how long the time to expiration of the option should be and how many early exercise opportunities the option should have. Moreover, their method costs much higher than other methods because it always depends on numerical methods like Monte Carlo simulation to calculate the deposit insurance premium. Duan and Yu (1999) extended the model of Duan and Yu (1994) by incorporating capital standard and GARCH option pricing technique to describe adequately the empirical data features such as fat-tailed return distributions, volatility clustering and the leverage effect.

This article explores the analysis of the deposit insurance premium based on bank default risk in the Black-Scholes framework. In this article, capital forbearance is incorporated in a spirit similar to Ronn and Verma's (1986). It assumes that the assets and the debts of the insured bank follow lognormal processes respectively unlike the most previous researches with the assumption of a single homogeneous-term debt issue. In general, the balance sheets of banks are primarily composed of debts over 90 percents. The debt value as well as the asset value may vary in time. Thus, we assume that both of them follow the stochastic processes.

In the previous studies, the insured banks are assumed to be liquidated at the end of the coverage period (Duan and Yu (1999)). To the contrary, this article assumes that the bank does not necessarily liquidate at the end of the period. Moreover, the bank is assumed to go into bankruptcy at any time till the end of the period, which is consistent with the characteristics of American option pricing theory. If an insured bank successfully passes the periodic audit, the insuring agency levies a new deposit insurance premium to the bank. This observed practice leads us to employ a single period price framework to price the deposit insurance premium pricing.

This article is organized as follows. In the next section, the derivation of the pricing formula for risk-adjusted deposit insurance premiums is presented. This section investigates the real practice of deposit insurance in more detail to build more proper pricing model. Section 3 provides empirical experiments assessing the deposit insurance premium values under various scenarios. The results of the pricing formula of this study are compared with those of the previous studies. This article ends with summary and conclusions in the last section.

2. Pricing Risk-Adjusted Deposit Insurance Premiums

This article follows all assumptions of Ronn and Verma (1986) except that a bank only issue a single, homogeneous-term debt and that asset value can be obtained from market equity value represented as a call option on the value of the assets. Rather, this article assumes that a bank issues a lot of diverse debts with different maturities and asset value can be obtained from the audit. The debts of a bank are assumed to be deposits of the demand type. Typically, the balance sheets of commercial banks and investment banks reflect a book value, not necessarily a market value, of approximately 90-95% debt and 10-5% equity. If the asset values show a stochastic movement, it is natural for the debts to follow a stochastic process as well. This assumption is also adequate for the purpose of measuring bank default risk modeled using option pricing theory. Using the demonstration of an isomorphic correspondence between deposit insurance and put option of Merton (1977), we shall consider a deposit insurance premium pricing as a put option written on underlying assets A and debts D with expiration date T. Following Longstaff & Schwartz (1995), assume the dynamics of the values of assets and debts follow lognormal diffusion processes

$$\frac{dA}{A} = (\mu_A - q_A)dt + \sigma_A dW_A ,$$
$$\frac{dD}{D} = (\mu_D - q_D)dt + \sigma_D dW_D ,$$

where dW_A and dW_D are standard Brownian motions, μ_A and μ_D are the expected rate of return , q_A and q_D are the continuous dividend rate, and σ_A and σ_D are the volatility of asset and debt which is less than μ_A and μ_D, respectively. The correlation between the standard Brownian motions dW_A and dW_D is ω. Throughout the article, μ_A, μ_D, σ_A, σ_D, q_A, q_D, and T are all taken to be constant and greater than or equal to 0, unless otherwise noted. the time to expiration, T, is viewed as the length of time until the next audit of the bank, as in Merton (1977).

From the basic assumption of No-arbitrage principle of option pricing theory, we can derive the following Black-Scholes-like partial differential equation for the deposit insurance premium, denoted by F :

$$\begin{aligned}&\frac{1}{2}\frac{\partial^2 F}{\partial A^2}\sigma_A{}^2 A^2 + \frac{1}{2}\frac{\partial^2 F}{\partial D^2}\sigma_D{}^2 D^2 + \frac{\partial^2 F}{\partial A \partial D}\sigma_A \sigma_D \omega A D \\ &+ \frac{\partial F}{\partial A}(r - q_A)A + \frac{\partial F}{\partial D}(r - q_D)D + \frac{\partial F}{\partial t} - rF = 0,\end{aligned} \tag{1}$$

where t is a time variable($0 \le t \le T$). The function of $F(A,D,t)$ represents the value of the deposit insurance premium conditional on the asset value being above the debt value. This partial differential equation is the same as that of an option to exchange one asset for another which was first presented by Margrabe (1978).

In this article, capital forbearance is incorporated in a spirit similar to Ronn and Verma's (1986). Capital forbearance occurs when a bank whose asset value is less than its debt value continues to operate under regulatory supervision. The insuring agency promptly conducts a failure resolution as soon as its asset value A falls to hit ρD, where ρ is a capital forbearance parameter less than 1. This failure resolution can occur at any time in real world. Thus, the bank is assumed to go into bankruptcy at any time during the coverage period in this article, which corresponds to early exercise feature of American option. In the previous studies, the default of the insured bank is assumed to occur only at the end of the period because the bank is assumed to issue a single homogeneous debt. For this reason, most previous studies take advantage of the European option pricing theory to deposit insurance premium pricing. To check out the solvency of the insured bank, monitoring by the insuring agency can occur at times other than regularly scheduled audits. The insuring agency can schedule a special examination at any time. These considerations collectively motivate us to make use of the American option pricing theory in pricing deposit insurance premiums.

The insured banks are audited periodically. On successful completion of a bank audit at the end of the coverage period, the insuring agency assesses and levies a risk-adjusted premium for the next coverage period and the bank continues to operate not liquidates. That is, the successful audit is accompanied by a new issue of the deposit insurance. Of course, if the bank fails to pass the audit, the bank goes into the control of the regulatory authority and resolved through M&A or liquidation, which can be interpreted as an early exercise of American put option. Considering this practice, it is more appropriate for the analysis of the deposit insurance system to adopt a single period pricing framework. In the traditional single period pricing models, the decision for early failure closure or the continuity of operation of the insured bank could not be incorporated because the insured banks are assumed to be necessarily liquidated at the end of the coverage period (Duan & Yu (1999)). But, this assumption does not make sense because it does not reflect the fact that most of the insured banks pass the regularly scheduled audits and continue to operate. The liquidation assumption was generated owing to the fundamental assumption that the bank issues single, homogeneous-term debt. The bank should pay back the matured debt at expiration date by liquidation because there is neither roll-over nor new debt issues. Consequently, the assumption causes the result that the deposit insurance coverage period is equal to the lifespan of the insured bank. From the viewpoint of option pricing theory, the time to expiration of the put option corresponding to the deposit insurance has nothing to do with the lifespan of underlying asset. In general, the underlying asset has no expiration. Thus, the time to expiration of the put option is only related to the lifespan of the put option itself. So to speak, it is only connected with the coverage period of the deposit insurance. To avoid these problems and reflect the real world, this article assumed that a bank issues a lot of diverse debts with different maturities. This includes roll-over or new debt issues. Under this assumption, the insured bank does not necessarily liquidate at the end of the coverage period. Once the insured bank is confirmed to be solvent with more asset value than debt, the bank is assumed to operate continuously. This is in accordance with the basic assumption in finance. Therefore, the deposit insurance payoff at the end of the coverage period can be described by

$$F(A,D,T)=\begin{cases} 0 & \text{if } A(T)>\rho D(T), \\ (1-\rho)D(T) & \text{if } A(T)=\rho D(T), \end{cases}. \tag{2}$$

The insuring agency is assumed to conform to the failure bank resolution regulation rigorously. As soon as the asset value of the insured bank falls to hit ρD at any time during the coverage period, the insuring agency declares the bankruptcy of the insured bank and infuses $(1-\rho)D$ to dissolve the assets of the bank. Neither additional forbearance nor premature closure of the deposit insurance is assumed to be granted. This is described by

$$F(A,D,t)=(1-\rho)D(t) \quad \text{if } A(t)=\rho D(t)\,. \tag{3}$$

This payoff is very similar to that of American digital put option with moving boundary, $\rho D(t)$. If an insured bank has by far more asset value than debt value, it is little possible for the bank to go into bankruptcy. This can be expressed by

$$F(A,D,t)\approx 0, \quad as \quad A(t)\approx\infty\,. \tag{4}$$

To solve the above problem, we need to make a transformation of the Eq. (1). Using the similarity reduction technique by a new change of variable

$$S=\frac{A}{\rho D}, \tag{5}$$

and taking the form

$$F(A,D,t)=\rho DV(S,t), \tag{6}$$

we reformulate the governing equation, Eq. (1), into the following the Black-Scholes-like partial differential equation problem :

$$\frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2+\frac{\partial V}{\partial S}(q_D-q_A)S+\frac{\partial V}{\partial t}-q_D V=0, \tag{7}$$

where

$$\sigma=\sqrt{\sigma_A{}^2+\sigma_D{}^2-2\omega\sigma_A\sigma_D}\ . \tag{8}$$

This equation defines the discounted conditional expectation under the risk-neutral process of a newly introduced variable *S* which is governed by a lognormal diffusion process with the drift of (q_D-q_A) and volatility parameter σ. We can interpret that Eq. (5) represents an option on an underlying asset with continuous dividend yield q_A under the risk-free interest rate q_D.

The change of variables in Eq. (5) and (6) transforms the initial condition of Eq. (2) and boundary condition of Eq. (3) and (4) into the following conditions :

$$V(S,T)=0, \tag{9}$$

$$V(1,t) = \frac{(1-\rho)}{\rho}, \tag{10}$$

$$V(S,t) \approx 0, \quad as \quad S \approx \infty. \tag{11}$$

The problem above is to be solved in the domain Ω={(*S*,*t*) | 1≤*S*<∞, 0≤*t*≤*T*}. The variable change of Eq. (5) has transformed the original problem into a fixed boundary problem. From solving the above problem and making change of variables back with Eq. (5) and (6), we gain the final deposit insurance formula as follows.

$$F(A,D,t) = (1-\rho)D\left[\left(\frac{\rho D}{A}\right)^{\frac{(q_D - q_A)}{\sigma^2}} N(-d_b) + \left(\frac{A}{\rho D}\right) N(-d_a)\right] \tag{12}$$

where

$$d_a = \frac{\log\frac{A}{\rho D} + (q_D - q_A + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_b = \frac{\log\frac{A}{\rho D} - (q_D - q_A + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

$$N(d_a) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d_a} e^{-\frac{1}{2}z^2}\, dz.$$

Here, $N(\cdot)$ stands for the standard Gaussian cumulative distribution function. But, not all the deposits are insured by the deposit insurance. If the insured deposit is denoted by *B*, only the proportion of demand deposits to total debts, *B*/*D*, is insured at the event of bank default. Finally, if we divide them by B, risk-adjusted deposit insurance premium rate, denoted by P, is obtained by

$$P(A,D,t) = (1-\rho)\left[\left(\frac{\rho D}{A}\right)^{\frac{(q_D - q_A)}{\sigma^2}} N(-d_b) + \left(\frac{A}{\rho D}\right) N(-d_a)\right]. \tag{13}$$

Note that Eq. (13) is an exact closed-form solution. It does not need any numerical process like Newton method or Monte Carlo simulation to compute the deposit insurance premium as in Ronn and Verma (1986) and Duan and Yu (1994, 1999).

3. Empirical Analysis

This section describes the empirical results of risk-adjusted deposit insurance premium rate expressed by Eq. (13). Table 1 reports the results of risk-adjusted deposit insurance premium rate for the representative set of parameters. *Panel A* in Table 1 shows that the increasing the insurance coverage period has the effect of enlarging the insurance premium rates. This means that the opportunity for a bank to default increases as the insurance covers longer periods. Figure 1 depicts the deposit insurance premium rate obtained from Eq. (13) as time varies. Specifically, there is no premium rate change on the point of capital forbearance limit. At the point, the payoff is always the same because it is the amount of deposit insurance subsidy in the event of bank failure. *Panel B* in Table 1 shows that the higher the volatility is, so is the deposit insurance premium rate. As the volatility of the insured bank expressed by Eq. (8) grows larger, the probability for the asset value to fall to hit the boundary of the capital forbearance level gets higher. This increased risk of insured bank results in the increased deposit insurance premium rate. This is in accordance with Allen and Saunders (1993). In particular, the premium rate of the bank with A/D=1.05 is valued at zero at low level of σ=0.02. This means that well-managed banks with low risk don't have to pay for deposit insurance. *Panel C* in Table 1 shows the premium rate change as (q_D-q_A) varies. We can observe that the deposit insurance premium increases as the value of (q_D-q_A) decreases. It is mentioned earlier that q_D and q_A are continuous dividend rate of debt and asset respectively. The dividend rate of bank's debts is interpreted as the market cost of debts because a depositor as a bondholder gains interests from the bank like the dividends of shareholder. Meanwhile, the dividend rate of bank's assets is interpreted as the weighted average rate of interests from debts and dividends from equity. That is, q_A conceptually corresponds to the weighted average cost of capital (WACC) required to be greater than the opportunity cost of debts and equities. If (q_D-q_A) is greater than 0, this means that interest rate of the bank is higher than the continuous dividend rate of equity. If q_A becomes greater under the assumption that there is little difference of q_D among banks, the amount of cash outflow increases gradually. From the perspective of the bank, this has a bad influence on the capital structure of the bank. More capital outflow from banks makes capital adequacy worse, and increases the default risk of the bank. Consequently, it may be concluded that the deposit insurance premium rate increases due to the increased risk resulting from reduced (q_D-q_A). *Panel D* in Table 1 shows the premium rate change with respect to the capital forbearance policy. This study assumed that the regulatory authority immediately conducts the failure bank resolution whenever the asset value of the insured bank falls to hit the capital forbearance limit, ρD. According to the capital forbearance policy, the regulatory authority always monitors and supervises the insured banks not only at regularly scheduled audits but also at any time by surprise examinations. If the regulatory authority would employ a strict capital forbearance policy by setting ρ greater than or equal to 1, the insuring agency would not carry out the function of deposit insurance and only play a role of the supervisory of the banks because the asset value of a bank at resolution is always greater than the value of debts. Under this circumstance, there is no capital forbearance and moral hazard problem hardly happens. On the contrary, most previous studies adopting European option approach show that the deposit insurance premium is greater than 0 even under the capital forbearance policy with ρ=1 due to its inherent assumptions. If the regulatory authority would adopt a lenient capital forbearance policy by setting ρ less than 1, the function of deposit insurance eventually burst into activate. *Panel D* reports that more lenient capital forbearance policy requires the insured bank to pay a lower deposit insurance premium. This is due to the fact that lower capital forbearance reduces the probability of the bank default. The probability for the asset value to fall below the baseline for bank default decreases as the boundary of the capital forbearance, ρD, expressed by Eq. (8) gets lower. *Panel E* in Table 1 contains the results for the premium rate change with respect to the asset-to-debt ratio. Note that increasing the asset-to-debt ratio has the effect of decreasing Risk-adjusted deposit insurance premium rate. This has an obvious implication that with a bigger asset-to-debt ratio the bank would be more solvent and safer to reduce its deposit insurance premium. Moreover, it is observed that the premium rate exponentially increases as the asset-to-debt ratio gets closer to the boundary of the capital forbearance. From these observations, we can presume that *delta*, the rate of change of premium rate with respect to the asset-to-debt ratio, gets smaller as asset-to-debt ratio increases. That is,

$$\Delta = \frac{\partial P}{\partial\left(\frac{\rho D}{A}\right)}$$

$$= (1-\rho)\left[\left(\frac{\rho D}{A}\right)^{\frac{(q_D - q_A)}{\sigma^2} - 1}\left(N(-d_b) - \frac{1}{\sigma\sqrt{T-t}}\right) + \left(N(-d_a) - \frac{1}{\sigma\sqrt{T-t}}\right)\right] < 0. \quad (14)$$

Delta is always lower than zero. If the asset-to-debt ratio has infinity, *delta* is approximately equal to 0. Figure 1 also illustrates this relationship between the premium rate and asset-to-debt ratio.

Table 2 compares the results of Allen and Saunders (1993), Duan and Yu (1994), and our model. P_{AS} is the lump sum present value of the premium rate from callable perpetual American put option pricing of Allen and Saunders (1993), P_{DY} is the premium rates of Duan and Yu (1994) computed by Monte Carlo simulation with 5 year coverage horizon, and P is the annual premium rate of Eq. (13). Note that the coverage periods of each model are different from one another. That is, P_{AS}, P_{DY}, P, is for the coverage period of infinity, 5 year and 1 year, respectively. They show the identical increasing pattern with the annual standard deviation. But, the rate of change of premium rate with respect to the volatility differs from one another. P reports the smallest change rate among them. P_{DY} shows approximately linear increase pattern. On the contrary, P_{AS} and P show the pattern of a log-like function. In general, the premium rates of Allen and Saunders (1993) are lower than that of Duan and Yu (1994). This results from the fact that whenever bank asset value falls below premature exercise level, the insuring agency forces exercise of the deposit insurance put and close the bank. Though it seems that the premium rates of Allen and Saunders and those of our model are about the same except the cases of 0.03 and 0.05 in annual standard deviation and 1.0 in asset-to-debt ratio, we need to focus on the fact that the premium rates of them intersect each other as the annual standard deviation increases. This has the obvious implication that the extent of sensitivity with respect to the volatility is much higher in the model of Allen and Saunders (1993). Note that increasing the volatility would enlarge the difference between the premium rates.

Table 3 compares the results of Ronn and Verma (1986), Duan and Yu (1999), and our model. P_{RV} , the premium rate using the Black-Scholes pricing framework of Ronn and Verma (1986), P_{G1}, the premium rates with unit risk premium λ=0.0276 of Duan and Yu (1999), P_{G2}, the premium rates with unit risk premium λ=0.1 of Duan and Yu (1999), and P, the annual premium rate of Eq. (13), show the identical increasing pattern as the asset-to-debt ratio increases as in Table 2. The volatility is calculated with parameters given in GARCH option pricing model of Duan and Yu (1999) to be 0.1176. The insurance coverage horizon is 1 year for all models. P_{G1} is always higher than P_{RV} and P_{G2} is always higher than P_{G1}. This results from the increased risk level of the deposit insurance as the volatility of the underlying asset gets higher. Comparing with P_{RV}, P_{G1} and P_{G2}, P is always the lowest. It appears to be difficult to analyze the reason for the difference between them because the methodologies behind them are different each other. That is, Ronn and Verma (1986) and Duan and Yu (1999) developed the deposit insurance pricing using the European option pricing technique, our model, while, developed using American digital option pricing technique. Therefore, the extent of reflected risk at each asset-to-debt ratio is different among them. But, it is clear that the risk level of more sound and safer bank is estimated lower in our model than others. This result can support more strongly the fact that many sound and safe insured banks have been exempted from deposit insurance premium by FDIC in United States. The results in Table 2 and Table 3 also confirms that the deposit insurance premium increases in the volatility and decreases in the asset-to-debt ratio for all pricing models.

4. Summary and Conclusions

This article has developed an exact closed-form formula for the deposit insurance premium rate reflecting the default risk of the insured bank in the Black-Scholes framework. This formula possesses the attributes of simplicity, fairness and accuracy. This formula enables us to compute the deposit insurance premium easily in terms of a cumulative normal distribution function because it does not need any numerical method. The approach in this article is distinctive from previous studies in four perspectives. First, the deposit insurance premium is priced as an American option with the assumption that the insured bank can go into bankruptcy at any time during the insurance coverage period. Second, we assume that the value of assets and the value of debts follow lognormal processes respectively and bank issues a lot of debts with different maturities. With these assumptions, the deposit insurance pricing model is derived by using the Exchange option pricing technique. Third, it is assumed that the insuring agency promptly conducts a failure resolution as soon as the asset value falls to hit the capital forbearance limit. This assumption leads us to make use of the Digital option pricing technique in deposit insurance premium pricing. Fourth, this article employs a single period pricing framework not assuming that the insured bank is necessarily liquidated at the end of the coverage period. This is because the insuring agency generally levies the deposit insurance premium periodically – annually or quarterly. This article breaks off the correlation between the coverage period of deposit insurance and the lifespan of the insured bank suggested by previous studies.

There are no previous studies for pricing risk-adjusted deposit insurance premium done under the assumption that the insured bank can go into bankruptcy at any time during a limited insurance coverage period. This assumption is important because it is much closer to the real world. Moreover, the assumption of conducting failure resolution immediately on asset value's hitting the capital forbearance limit is came from the fact that the regulatory authority is monitoring the insured banks continuously and adheres to its stated forbearance and resolution policy. It does not wait any longer when the asset value of a financially distressed bank hits the boundary of the capital forbearance. Consequently, this implies that the deep-in-the-money region of put option pricing analysis is not of use any longer. These realistic assumptions motivate us to apply the American digital option pricing technique for the fair deposit insurance pricing.

Our model does not depend on the risk-free interest rate. Instead, the weighted average cost of capital and interest rate of debts are involved in the pricing formula of risk-adjusted deposit insurance premium. As expected, the volatility reflecting the risk of the insured bank plays a crucial role in deposit insurance pricing. The extent of reflecting the default risk in premium pricing differs from literatures. But, our model is less sensitive in the change of the premium rate with respect to the change of volatility than other models. For the sound and safe insured banks, risk-adjusted deposit insurance premiums from our model are smaller than those from any other model. It may be concluded that our model can most strongly support the fact that many sound and safe insured banks in United States have been exempted from deposit insurance premium by FDIC.

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Table 1. Risk-adjusted deposit insurance premium rates

A/D	ρ	q_D-q_A	σ	T	$P(\%)$
Panel A					
1.05	0.90	0.005	0.05	0.25	0.0000
1.05	0.90	0.005	0.05	0.50	0.0001
1.05	0.90	0.005	0.05	1.00	0.0162
Panel B					
1.05	0.90	0.005	0.1	1.00	1.2278
1.05	0.90	0.005	0.06	1.00	0.0884
1.05	0.90	0.005	0.02	1.00	0.0000
Panel C					
1.05	0.90	0.003	0.05	1.00	0.0183
1.05	0.90	0.000	0.05	1.00	0.0221
1.05	0.90	-0.003	0.05	1.00	0.0266
Panel D					
1.05	0.97	0.005	0.05	1.00	0.2993
1.05	0.92	0.005	0.05	1.00	0.0535
1.05	0.89	0.005	0.05	1.00	0.0081
Panel E					
1.03	0.90	0.005	0.05	1.00	0.0565
1.00	0.90	0.005	0.05	1.00	0.2979
0.97	0.90	0.005	0.05	1.00	1.1926

Note : In *Panel D*, ρ=0.97 is widely used in related papers for the value of capital forbearance parameter since Ronn and Verma (1986). ρ=0.92 is selected reflecting the fact that the average loss rate of banks in failure resolution is about 8% in United States. ρ=0.89 is selected reflecting the fact that the average shortage rate of assets of reorganized banks in South Korea at the time of Asia Financial Crisis was about 0.11.

Table 2. Comparison of deposit insurance premium rates of Allen and Saunders (1993), Duan and Yu (1994) and present model.

	Annual Standard Deviation				
	0.03	0.05	0.08	0.1	0.2
asset-to-debt ratio = 1.0000					
P_{AS}	0.0517	0.6953	1.6947	2.0815	2.7380
P_{DY}	0.8088	1.6359	2.6488	3.3206	6.9844
P	0.7902	1.5500	2.0894	2.2796	2.6648
asset-to-debt ratio = 1.1111					
P_{AS}	0.0000	0.0044	0.2350	0.5879	1.9960
P_{DY}	0.4569	0.9572	1.7802	2.4124	6.0441
P	0.0000	0.0160	0.2577	0.5216	1.5642

Note : P_{AS}, the premium rates of Allen and Saunders (1993), and P_{DY}, the premium rates of Duan and Yu (1994), are from Table 2 of Duan and Yu (1994). P is the premium rate of Eq. (13). The premium rates are stated in percent. The capital forbearance parameter, ρ, is set equal to 0.97. For P_{DY}, the risk-taking intensity is 0 and the coverage horizon of P_{DY} is 5 years. The risk-free interest rate equals 6%. The threshold debt-to-asset ratio is 0.92. The annual standard deviation of P_{AS} and P_{DY} is for the underlying asset return. The annual standard deviation of P is for the asset-to-debt ratio. (q_D-q_A) is set equal to 0.05.

Table 3. Comparison of deposit insurance premium rates of Ronn and Verma (1986), Duan and Yu (1999) and present model.

	Asset-to-Debt ratio		
	1.09	1.11	1.13
P_{BS}	1.6469	1.2622	0.9582
P_{G1}	1.9462	1.5899	1.2947
P_{G2}	2.4963	2.1107	1.7870
P	0.9771	0.7670	0.5933

Note : P_{BS}, the premium rates using the Black-Scholes pricing framework, P_{G1}, the premium rates with unit risk premium λ=0.0276 of Duan and Yu (1999), and P_{G2}, the premium rates with unit risk premium λ=0.1 of Duan and Yu (1999) are from Table 2 of Duan and Yu (1999). P is the premium rate of Eq. (13). The premium rates are stated in percent. The capital forbearance parameter, ρ, is set equal to 0.97. σ is set equal to 0.1176 as expressed in Duan and Yu (1999). The insurance coverage horizon is 1 year. For P_{G1} and P_{G2}, the risk-taking intensity is 0.2. The risk-free interest rate equals 6%. For P, (q_D-q_A) is set equal to 0.05.

Figure 1. Comparison of the deposit insurance premium rates with different maturities