Vector-Transformed Circuit Theory and Application to Converter Modeling/Analysis

Soo-Bin Han, Gyu-Hyeong Cho, Bong-Man Jung and Soo-Hyun Choi*

Electric Energy Division
Korea Institute of Energy Research
71-2, Jang-Dong, Yusong-Gu, Taejon 305-343, Korea
(FAX: +82-42-860-3739)

* Department of Electrical Engineering,
Korea Advanced Institute of Science and Technology
373-1, Kusong-Dong , Yusong-Gu, Taejon 305-701, Korea
(FAX : +82-42-869-3410)

Abstract: **New converter modeling method by vector transformation is proposed with the theory and applications. The method is general and derived models have simple but full informed circuit forms. Moreover, the method is equivalent and easily converted to the contemporary per phase analysis and the DQ transformation analysis.**

I. INTRODUCTION

Circuit DQ transformation is obviously a powerful analysis tool and supplies accurate information about important state variables of the multiphase converter [1]. However, resulted circuit models always have a d-axis and a q-axis component. This may be more complex for composite systems in the viewing and model construction process, so the physical insight may deteriorate. It should be noted that the transformed model is a kind of average model, which may restrict real time control capability for the fast response. Another restriction comes from the exclusion of harmonic component so that the circuit DQ transform-ation has difficulty in applications to the unbalanced system and the harmonic analysis.

So new concept is necessary for the general unified modeling that is applicable in both steady-state and transient operation. In the AC system, any variable can be described as a vector composed of arbitrary time varying amplitude and phase. It has been known that three phase variables can be represented as one space vector and its related equations can be much simplified. If such a concept, called space vector theory in the three phase system, can be realized to circuit forms, then the models may be much simpler.

Although various models of power converter [1]-[4] and space vector concepts for the AC motor analysis [5]-[7] have already been presented, a suggested method called the vector transformed circuit theory differs from the previously discussed approaches in the following ways:

1) The proposed method can model entire multiphase circuit variables as one unified vector that describes entire information of systems, and it is reformed as circuits including switch as well as inductor, capacitor and resistor;
2) The method can convert to the per phase equivalent circuit model and the circuit DQ transformed model very easily;
3) The method can apply to both the multiphase converter modeling such as the three phase inverter/ converter and the single phase converter modeling;
4) The method is adequately describing the performance of the converter under both steady and transient operation;
5) The method gives a simple and compact model with clear physical pictures.

II. VECTOR TRANSFORMED CIRCUIT THEORY

2.1 Definition of Vector Transformation

Any electrical values of power converters dominated by the AC component can be represented by a vector $\overline{x(t)}$, which can be an arbitrary time varying function of time t. Then the vector transformation from $\overline{x(t)}$ to $\overline{y(t)}$ is defined as

$$\overline{y(t)} = \overline{x(t)} \cdot e^{j\theta(t)} \tag{1}$$

where $\theta(t)$ is any time varying function but normally $\theta(t) = \pm\omega t + \theta_o$ with the angular velocity ω in AC systems. Any scalar functions, of course, can be represented from the vector function $\overline{y(t)}$ by taking one of the real/ imaginary term or only considering magnitude of $\overline{y(t)}$. It means that the transformation of (1) can be applied to single phase systems as well as balanced three phase AC systems.

Vector circuit transformation is a work taking a simple but full informed circuit model by applying the vector transformation concept of (1) to the circuit theory. If $\theta(t)=0$, transformations are done in a fixed reference, else if $\theta(t)=\omega t$, transformations are done in a rotating reference. This process makes it possible to simplify the model process without the cumbersome matrix manipulations and to give full insight in power converter behaviors.

2.2 *Vector Transformed Circuit Model of 3-Phase System at a Fixed Reference* ($\theta(t)=0$)

Three electrical variables, X_a, X_b, X_c, are given, which can be any time varying current/voltage, i.e., it can be non-sinusoidal functions. If they are balanced and X_a is located at a fixed reference frame, a space vector $\overline{X^s}$ representing a three phase system variable to one position in the vector space is defined as

$$\overline{X^s}=\sqrt{\frac{2}{3}}\left(X_a+X_b\cdot a+X_c\cdot a^2\right) \tag{2}$$

where phase operator $a=e^{j2\pi/3}$. (3)

As a special case, considering balanced sinusoidal functions X_a, X_b, X_c as

$$\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix}=V\cdot\begin{bmatrix} \cos(\omega t+\phi_1) \\ \cos(\omega t-2\pi/3+\phi_1) \\ \cos(\omega t+2\pi/3+\phi_1) \end{bmatrix}, \tag{4}$$

then $\overline{X^s}=\sqrt{3/2}Ve^{j(wt+\phi)}$, and it is a rotating vector with the angular velocity of ω and the magnitude of $\sqrt{3/2}V$ as shown in Fig. 2.

Vector transformed circuit models can be derived by applying (2) to each basic sub-circuit element such as three-phase voltage and current sources, three-phase resistor set, three-phase inductor set, three-phase capacitor set, three-phase current source inverter (CSI) switch and three-phase voltage source inverter (VSI) switch. The transformation sequence is composed of the circuit partition, the sub-circuit transformation and the reconstruction as in circuit DQ transformation. But reconstruction is much simple than the circuit DQ transformation.

A. Transformation of three-phase sources

The three phase voltage and current sources are represented directly as vectors $\overline{v_s^s}$, $\overline{i_s^s}$ by the definition as shown in Fig. 2. So the three-phase source set can be modeled as a single space vectored source.

B. Transformation of three-phase resistor set

Each voltage drops of the three-phase resistor set of Fig. 3 are defined by ohm's low as

$$v_{ar}=R\cdot i_{ar},\ v_{br}=R\cdot i_{br},\ v_{cr}=R\cdot i_{cr}. \tag{5}$$

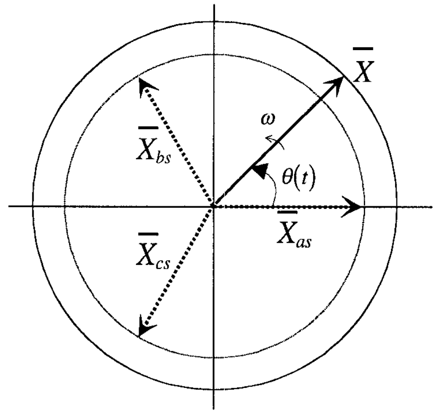

Fig. 1. Space vector obtained from three phase sinusoidal components.

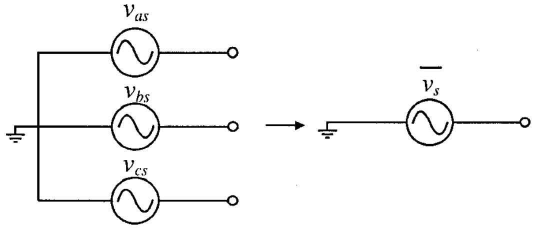

Fig. 2. Transformation of three-phase sources.

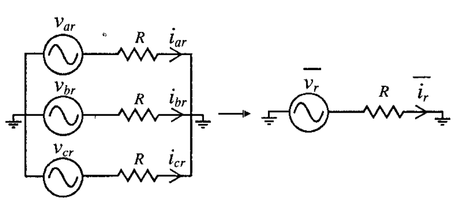

Fig. 3. Transformation of three-phase resistors.

By definition of (2), the following relation is induced.

$$\begin{aligned}\overline{v_r^s}&=\sqrt{\frac{2}{3}}\left(v_{ar}+v_{br}\cdot a+v_{cr}\cdot a^2\right)\\ &=R\cdot\sqrt{\frac{2}{3}}\left(i_{ar}+i_{br}\cdot a+i_{cr}\cdot a^2\right)\\ &=R\cdot\overline{i_r^s}.\end{aligned} \tag{6}$$

So three-phase resistor circuit can be modeled as single vectored resistor circuit.

C. Transformation of three-phase inductor set

Each voltage drop of the three-phase inductor set are defined as

$$v_{aL}=L\frac{di_{aL}}{dt},\ v_{bL}=L\frac{di_{bL}}{dt},\ v_{cL}=L\frac{di_{cL}}{dt} \tag{7}$$

By definition of (2), the following relation is induced.

$$\begin{aligned}\overline{v_L^s} &= \sqrt{\frac{2}{3}}\left(v_{aL} + v_{bL}\cdot a + v_{cL}\cdot a^2\right)\\ &= \sqrt{\frac{2}{3}}\left(L\frac{di_{aL}}{dt} + L\frac{di_{bL}}{dt}\cdot a + L\frac{di_{cL}}{dt}\cdot a^2\right)\\ &= L\cdot\frac{d}{dt}\sqrt{\frac{2}{3}}\left(i_{aL} + i_{bL}\cdot a + i_{cL}\cdot a^2\right)\\ &= L\cdot\frac{d}{dt}\overline{i_L^s}.\end{aligned} \quad (8)$$

So the three-phase inductor set can be modeled as a single vector inductor circuit.

D. Transformation of three-phase capacitor set

The each voltage drop of the three phase capacitor are defined as

$$i_{aC} = C\frac{dv_{aC}}{dt},\ i_{bC} = C\frac{dv_{bC}}{dt},\ i_{cC} = C\frac{dv_{cC}}{dt} \quad (9)$$

By definition of (2), the following relation is induced.

$$\begin{aligned}\overline{i_C^s} &= \sqrt{\frac{3}{2}}\left(i_C + i_C\cdot a + i_C\cdot a^2\right)\\ &= C\cdot\frac{d}{dt}\sqrt{\frac{3}{2}}\left(v_{aC} + v_{bC}\cdot a + v_{cC}\cdot a^2\right)\\ &= C\cdot\frac{d}{dt}\overline{v_C^s}.\end{aligned} \quad (10)$$

So three-phase capacitor circuit can be modeled as single vectored capacitor circuit.

E. Three-phase CSI switch set

Each AC currents of the CSI switch set are defined by the DC side current and the switching functions as

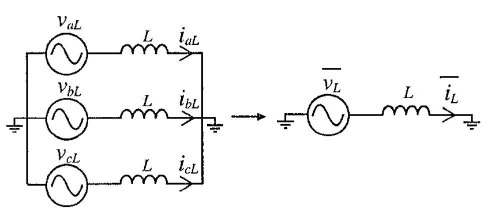

Fig.4. Transformation of three-phase inductor set.

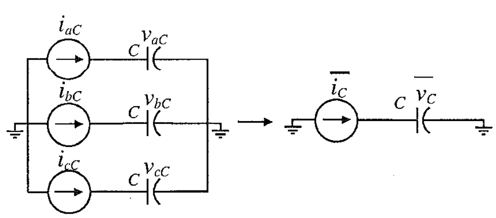

Fig. 5. Transformation of three-phase capacitor set.

$$i_{as} = i_o\cdot S_1,\ i_{bs} = i_o\cdot S_2,\ i_{cs} = i_o\cdot S_3. \quad (11)$$

The output voltage of DC side are defined as

$$v_o = v_{as}\cdot S_1 + v_{bs}\cdot S_2 + v_{cs}\cdot S_3. \quad (12)$$

A relation of AC side voltage and DC output voltage is derived as

$$\begin{aligned}\overline{i_s^s} &= \sqrt{\frac{2}{3}}\left(i_{as} + i_{bs}\cdot a + i_{cs}\cdot a^2\right)\\ &= i_o\cdot\sqrt{\frac{2}{3}}\left(S_1 + S_2\cdot a + S_3\cdot a^2\right)\\ &= i_o\cdot\overline{S_{123}^s}.\end{aligned} \quad (13)$$

A relation between the AC side current and the DC output current is derived from the energy conservation rule, $p_{in} = p_{out}$.

$$\begin{aligned}p_{in} &= i_{as}\cdot v_{as} + i_{bs}\cdot v_{as} + i_{cs}\cdot v_{cs}\\ &= \mathrm{Re}\left(\overline{v_s^s}\cdot\overline{i_s^s}^*\right).\end{aligned} \quad (14)$$

$$p_{out} = v_o\cdot i_o. \quad (15)$$

So $\mathrm{Re}\left(\overline{v_s^s}\cdot\overline{i_s^s}^*\right) = v_o\cdot i_o.$ (16)

From (13) and (16),

$$\mathrm{Re}\left(\overline{v_s^s}\cdot\overline{S_{123}^s}^*\right) = v_o \quad (17)$$

So the three-phase CSI switch set is modeled as a complex transformer with gain relations of (13) and (17).

F. Three-phase VSI switch set

The each AC voltage of VSI switch set are defined by DC side voltage and switching functions as

$$v_{as} = v_o\cdot S_1,\ v_{bs} = v_o\cdot S_2,\ v_{cs} = v_o\cdot S_3 \quad (18)$$

The output current of DC side are defined as

$$i_o = i_{as}\cdot S_1 + i_{bs}\cdot S_2 + i_{cs}\cdot S_3. \quad (19)$$

A relation of AC side voltage and DC output voltage is derived as

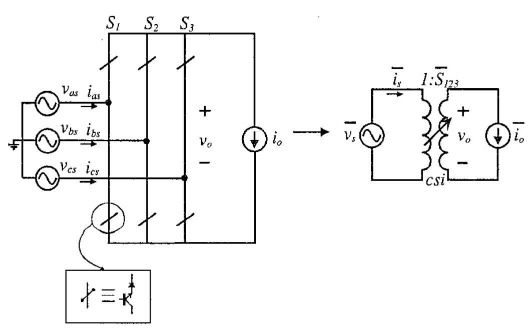

Fig. 6. Three-phase CSI switch set.
($\overline{i_s} = i_o\cdot\overline{S}_{123}$, $\mathrm{Re}\left[\overline{v_s}\cdot\overline{S}_{123}^* = v_o\right]$)

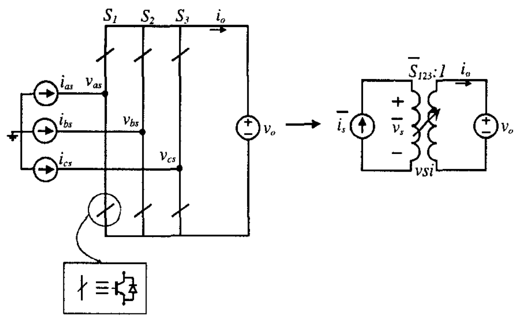

Fig. 7. Three-phase VSI switch set.
($\overline{v_s} = v_o \cdot \overline{S}_{123}$, $\mathrm{Re}\left[\overline{i_s} \cdot \overline{S}_{123}^{*} = i_o\right]$)

$$\overline{v_s^s} = \sqrt{\frac{2}{3}}\left(v_{as} + v_{bs} \cdot a + v_{cs} \cdot a^2\right)$$
$$= v_o \cdot \sqrt{\frac{2}{3}}\left(S_1 + S_2 \cdot a + S_3 \cdot a^2\right) \qquad (20)$$
$$= v_o \cdot \overline{S_{123}^s}.$$

A relation of AC side current and DC output current is derived from energy conservation rule $p_{in} = p_{out}$.

$$p_{in} = i_{as} \cdot v_{as} + i_{bs} \cdot v_{as} + i_{cs} \cdot v_{cs}$$
$$= \mathrm{Re}\left(\overline{v_s^s} \cdot \overline{i_s^s}^{*}\right). \qquad (21)$$

$$p_{out} = v_o \cdot i_o. \qquad (22)$$

$$\text{So } \mathrm{Re}\left(\overline{v_s^s} \cdot \overline{i_s^s}^{*}\right) = v_o \cdot i_o. \qquad (23)$$

From (20) and (23),

$$\mathrm{Re}\left(\overline{i_s^s} \cdot \overline{S_{123}^s}^{*}\right) = i_o \qquad (24)$$

So three-phase VSI switch set is modeled as a complex transformer with gain relations of (20) and (24).

2.3 Vector Transformed Circuit Model at a Rotating Reference ($\theta(t) = \omega t$)

Transformation of the vector of (2) at rotating reference is used for deriving the time invariant form and can be derived by (1). For emphasis, let's introduce again as following.

$$\overline{X}^e = \overline{X^s} \cdot e^{j\theta(t)}. \qquad (25)$$

In (25), $\theta(t)$ normally becomes a linear function of angular velocity as $\theta(t) = \pm\omega t + \theta_o$. In the rotating reference frame, models of three-phase inductor set and three-phase capacitor set must be modified due to the axis transformation, while other models are invariant. From (8) and (25), the following relation in inductor set is induced and modeled as shown in Fig 8.

$$\overline{v_L^e} = \overline{v_L^s} \cdot e^{-j\omega t} = e^{-j\omega t} \cdot L\frac{d\overline{i_L^s}}{dt}$$
$$= L \cdot \frac{d}{dt}\left(\overline{i_L^e} \cdot e^{-j\omega t}\right) + j\omega L\overline{i_L^e} \cdot e^{-j\omega t} \qquad (26)$$
$$= L \cdot \frac{d}{dt}\overline{i_L^e} + j\omega L\overline{i_L^e}.$$

The relation in capacitor set is also induced by the same method and modeled as shown in Fig. 9.

$$\overline{i_L^e} = \overline{i_L^s} \cdot e^{-j\omega t} = e^{-j\omega t} \cdot L\frac{d\overline{v_L^s}}{dt}$$
$$= L \cdot \frac{d}{dt}\left(\overline{v_L^e} \cdot e^{-j\omega t}\right) + j\omega L\overline{v_L^e} \cdot e^{-j\omega t} \qquad (27)$$
$$= L \cdot \frac{d}{dt}\overline{v_L^e} + j\omega L\overline{v_L^e}.$$

It is noted that there exists an imaginary resistor in the rotational circuit as a result of transformation.

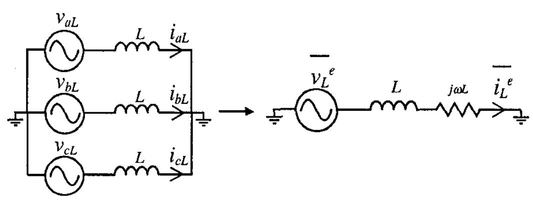

Fig. 8. Transformation of three-phase inductor set at a rotating reference.

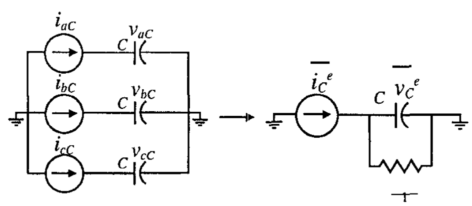

Fig. 9. Transformation of three-phase capacitor set at a rotating reference.

2.4 Relations between Circuit DQ Transformation and Vector Transformation

It is well known that any vectors can be decomposed as a real part and an imaginary part in complex plan, i.e, vectors can be separated as a d-axis and a q-axis component by following relation.

For the fixed reference,

$$\overline{X^s} = X_a + X_b \cdot a + X_c \cdot a^2 = X_d^s + jX_d^s. \quad (28)$$

For the rotating reference,

$$\overline{X^e} = \overline{X^s} \cdot e^{j\theta(t)} = X_d^e + jX_q^e. \qquad (29)$$

Above relations are easily understood through geometry consideration as shown in Fig. 10. So the vector circuit transformation is equivalent to the concept of circuit DQ transformation and can be applied much easily. There are two cases in the selection of rotating reference for power converter systems. The first one is that one phase of the three phase is synchronized to the rotating frame, another is that one of the switching functions is synchronized. In both case, an angular velocity equal to the fundamental frequency of AC system is selected so that the time varying AC variables are transformed to DC variables.

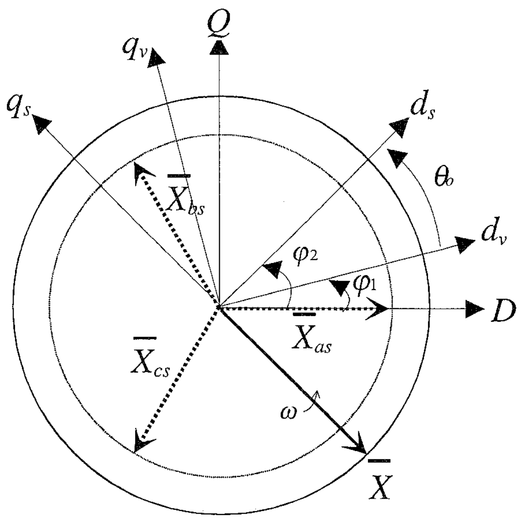

Fig. 10. Relation of vector transformation and DQ transformation at vector space.

III Applications to Power Converter Modeling

3.1 Buck Type Three-Phase PWM Rectifier

The vector circuit transformation theory is applied to the buck type PWM rectifier of Fig. 11. Then, a model of Fig 12 in stationary reference and a model of Fig. 13 in rotating reference are directly induced without any complex mathematical manipulations. The model of Fig. 13 is also expressed as following equations:

AC side; $$\mathrm{L} \cdot \frac{d}{dt}\bar{i}_s = \bar{v}_s - r\bar{i}_s - j\omega L\bar{i}_s + \bar{v}_c \qquad (30)$$

$$\mathrm{C} \cdot \frac{d}{dt}\bar{v}_c = \bar{i}_s - j\omega C\bar{v}_c - \bar{i}_t \qquad (31)$$

$$\bar{i}_t = \overline{S}_{123} \cdot i_o \qquad (32)$$

DC side; $$v_o = \mathrm{Re}\left(\bar{v}_c \cdot \overline{S}_{123}^{*}\right). \qquad (33)$$

Let's consider voltage sources and switching functions as following:

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = V_s \cdot \begin{bmatrix} \sin(\omega t + \phi_1) \\ \sin(\omega t - 2\pi/3 + \phi_1) \\ \sin(\omega t + 2\pi/3 + \phi_1) \end{bmatrix} \qquad (34)$$

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = m \cdot \begin{bmatrix} \sin(\omega t + \phi_2) \\ \sin(\omega t - 2\pi/3 + \phi_2) \\ \sin(\omega t + 2\pi/3 + \phi_2) \end{bmatrix} \qquad (35)$$

If transforming the vectors by (25) with $\theta(t) = -(\omega t - \pi + \phi_2)$, then $\bar{v}_s$, $\overline{S}_{123}$ become as following:

$$\bar{v}_s = \sqrt{\frac{3}{2}}V_s e^{j(\varphi_1 - \varphi_2)} = \sqrt{\frac{3}{2}}V_s e^{j\theta_o} \qquad (36)$$

$$\overline{S}_{123} = \sqrt{\frac{3}{2}}m. \qquad (37)$$

Now, final vector transformed model is shown in Fig. 13 and vector based equations of AC side can be decomposed to real and imaginary parts as followings:

$$\sqrt{\frac{3}{2}}\cos\theta + j\sqrt{\frac{3}{2}}\sin\theta = (R + j\omega L + L\frac{d}{dt})(i_{sd} + ji_{sq}) + v_{cd} + jv_{cq} \qquad (38)$$

$$(j\omega C + C\frac{d}{dt})(v_{sd} + jv_{sq}) = i_{sd} + ji_{sq} - (S_d + jS_q)i_o \qquad (39)$$

$$i_{sd} + ji_{sq} = \sqrt{\frac{3}{2}}m. \qquad (40)$$

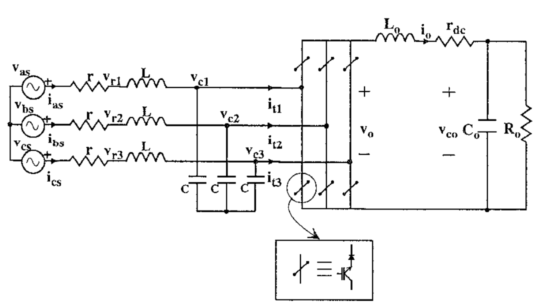

Fig. 11. Three phase buck type PWM rectifier.

Real part is arranged as follows:

$$\sqrt{\frac{3}{2}}\cos\theta_d = R\cdot i_{sd} + L\frac{di_{sd}}{dt} - \omega L i_{sq} + v_{cd} \quad (41)$$

$$C\frac{dv_{cd}}{dt} = i_{sd} - S_d\cdot i_o + \omega C v_{cq} \quad (42)$$

$$i_{sd} = m\cdot i_o \quad (43)$$

Imaginary part is arranged as follows:

$$\sqrt{\frac{3}{2}}\sin\theta = R\cdot i_{sq} + L\frac{di_{sq}}{dt} + \omega L i_{sd} + v_{cq} \quad (44)$$

$$C\frac{dv_{cq}}{dt} = i_{sq} - S_q\cdot i_o - \omega C v_{cd} \quad (45)$$

$$i_{sq} = o \quad (46)$$

DC side; $v_o = v_{cd}\cdot m.$ (47)

It is exactly identical to circuit DQ transformation of Fig 3, i.e., vector transformation is equivalent to the circuit DQ transformation. Moreover, the procedure is much simpler than the matrix manipulation requiring 7*7 matrix manipulations in this case.

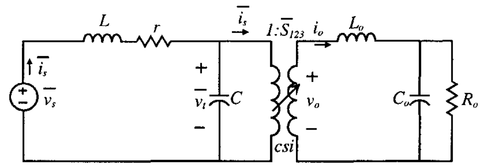

Fig. 12. Vector transformed model of buck PWM rectifier at stationary reference.

$\left(\bar{v}_s = \sqrt{\frac{3}{2}}v_s\cdot e^{j(\omega t-\pi/2+\varphi_1)},\ \bar{S}_{123} = \sqrt{\frac{3}{2}}\cdot m e^{j(\omega t-\pi/2+\varphi_2)}\right)$

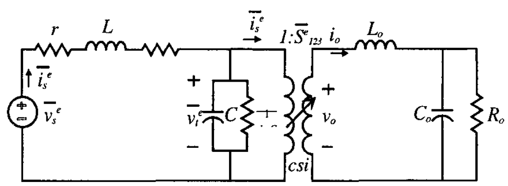

Fig. 13. Vector transformed model of buck PWM rectifier at rotating reference.

$\left(\bar{v}_s = \sqrt{\frac{3}{2}}v_s\cdot e^{j\phi_0},\quad \bar{S}_{123} = \sqrt{\frac{3}{2}}m\right)$

3.2 Single Phase System : Boost Type PWM Rectifier

Single phase system values are not vectors but time varying sinusoidal scalar values and can be derived from the real or imaginary part of vector. let's define single phase system values as the real part of vector by

$$y(t) = \mathrm{Re}\left[\overline{y(t)}\right]. \quad (48)$$

Normally $\overline{y(t)} = f(t)e^{j\theta(t)}$, where $f(t)$ is an arbitrary scalar time varying function. The definition of (48) means that the single phase system can be derived from the vector transformed circuit system, so the vector transformation at rotating reference is also effective as the following:

$$y^e = \mathrm{Re}\left[\bar{y}^e\right] = \mathrm{Re}\left[\overline{y(t)}e^{-j\theta(t)}\right]. \quad (49)$$

It is noted that the form of (49) is now identical to the phasor transformation that was used for the analysis of frequency/phase controlled series resonant converters. Therefore, the vector transformed sub-circuits of previous sections, if their current/voltage relations are operated based on the linear operator, can be used in single phase systems except the switch set due to its nonlinear characteristics. Transformation of the switch set can be performed by (50) as follows at fixed reference:

$$\mathrm{Re}\left[\bar{v}_i\right] = \mathrm{Re}\left[\bar{S}\cdot v_o\right]. \quad (50)$$

By energy conservation rule of $p_{in} = p_{out}$,

$$\begin{aligned} p_{in} &= \mathrm{Re}\left[\bar{v}_i\right]\cdot\mathrm{Re}\left[\bar{i}_i\right] \\ &= \mathrm{Re}\left[\bar{S}\cdot v_o\right]\cdot\mathrm{Re}\left[\bar{i}_i\right] \\ &= v_o\cdot\mathrm{Re}\left[\bar{S}\right]\cdot\mathrm{Re}\left[\bar{i}_i\right] \end{aligned} \quad (51)$$

$$p_{out} = v_o\cdot i_o. \quad (52)$$

so $i_o = \mathrm{Re}\left[\bar{S}\right]\cdot\mathrm{Re}\left[\bar{i}_i\right]$ (53)

In case of the rotating reference frame, where the frame is synchronize to switching function,

$$\mathrm{Re}\left[\bar{v}_i\cdot e^{-j\theta(t)}\right] = \mathrm{Re}\left[\bar{S}\cdot e^{-j\theta(t)}\cdot v_o\right] \quad (54)$$

i.e., $\mathrm{Re}\left[\bar{v}_i^e\right] = \mathrm{Re}\left[\bar{S}^e\cdot v_o\right]$ (55)

$$\begin{aligned} i_o &= \mathrm{Re}\left[\bar{S}\cdot e^{-j\theta(t)}\right]\cdot\mathrm{Re}\left[\bar{i}_i\cdot e^{-j\theta(t)}\right] \\ &= \frac{1}{2}\left(\mathrm{Re}\left[\bar{S}^*\cdot\bar{i}_i\right] + \mathrm{Re}\left[\bar{S}\cdot\bar{i}_i\cdot e^{-2j\theta(t)}\right]\right) \end{aligned} \quad (56)$$

i.e.,

$$\begin{aligned} i_o &= \mathrm{Re}\left[\bar{S}^e\right]\cdot\mathrm{Re}\left[\bar{i}_i^e\right] \\ &= \frac{1}{2}\left(\mathrm{Re}\left[\left(\bar{S}^e\right)^*\cdot\bar{i}_i^e\right] + \mathrm{Re}\left[\bar{S}^e\cdot\bar{i}_i^e\right]\right) \end{aligned} \quad (57)$$

As an example, let's consider the single phase PWM rectifier as shown in Fig. 14, where the voltage source v_s and the fundamental component of switching function is sinusoidal functions of $V\cos(\omega t+\phi_1)$ and

$m\cos(\omega t+\phi_2)$, respectively. Vector transformed model in the stationary frame by (48) is shown in Fig. 15 and this model is time varying system. So transformed model with $\theta(t)=\omega t+\phi_2$ by (49) is necessary for time invariant model as shown in Fig. 16. This model has a similar but more simple form compared to the model of 3-phase buck rectifier of Fig. 13, so all analysis methods applied to buck rectifier are equally applied to the single phase boost converter. Therefore, the complete analysis of steady state and transient can be performed completely. The model equation is as follows;

$$Ve^{j\theta} = r\overline{i_s^e} + L\frac{d}{dt}\overline{i_s^e} + \overline{v_i^e} \quad (58)$$

$$\overline{v_i^e} = m \cdot v_o \quad (59)$$

$$i_o = \mathrm{Re}\left[m \cdot \overline{v_i^e}\right] \quad (60)$$

The solution of (58)-(59) is very simple and final solution of the single phase system is possible by only taking the real part. It is noted that the steady state output voltage ignoring internal resistance r is given by

$$V_o = \frac{R_o \cdot m}{\omega L} V_s \sin\theta_o \quad (61)$$

which shows a complete relation with both circuit parameters and control parameters.

4.4 Conclusion

New modeling method by vector transformed circuit theory is proposed with the theory and applications. Although various approaches to circuit model based on space vector theory have been researched, these approaches cannot show the whole circuit model of PWM converter, which is composed of AC circuit and DC circuit connected through switches. Moreover, the previous approaches cannot solve the problem for application to the single phase systems. The proposed method is general because the variables need not be sinusoidal, and has simple but full informed circuit forms, moreover the method is equivalent and easily converted to contemporary per phase analysis and DQ transformation analysis. As examples of applications, the three-phase buck type PWM rectifier and the single phase boost type PWM converter are completely modeled.

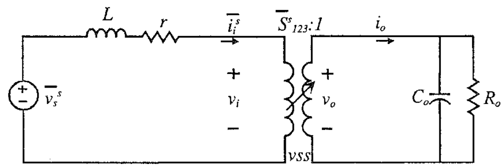

Fig. 15. Transformed model of single phase PWM rectifier at the stationary reference.

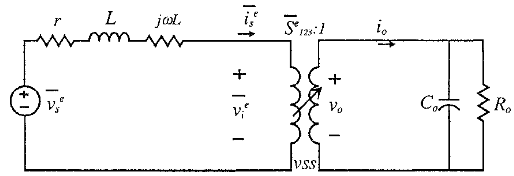

Fig. 16. Transformed model of single phase PWM rectifier at rotating reference.
($\overline{v_s^e} = v_s \cdot e^{j\theta_o}$, $\overline{S_{123}^e} = m$)

REFERENCES

[1] C.T.Rim, D.Y.Hu, and G.H.Cho, "Transformers as Equivalent Circuits for Switches: General Proofs and D-Q Transformation Based Analyses," IEEE Trans. Ind. Applicat., vol. IA-26, no. 4, pp. 777-785, 1990.

[2] S. Hiti, V. Vlatkovic, D.Borojevic, and F.C.Lee, "A New Algorithm for Three Phase Buck Rectifier with Input Displacement Factor Compensation," IEEE Trans. Power Elect. vol. PE-9, no-2, pp. 173-179, 1994.

[3] V.Blasko and V.Kaura "A New Mathematical Model and Control of a Three-Phase AC-DC Voltage Source Converter," IEEE Trans. Power Elect., vol. PE-12, pp.116-123, 1997.

[4] R.Wu, et al., "Analysis of ac-to-dc Voltage Source Converter Using PWM with Phase and Amplitude Control," IEEE Trans. Ind. Applicat, vol. IA-17, pp. 355-363, 1991.

[5] P.C.Krause and J.R.Hake, "Method of Multiple Reference Frames Applied to the Analysis of a Rectifier-Inverter Induction Motor Drive," IEEE Trans. Power Apparatus and System, vol. PAS-88, pp. 1635-1641, 1969.

[6] P.C.Krause, Analysis of Electric machinery, Mc-GrawHill, 1986.

[7] P.Vas, Electrical Machines and Drives: A Space-Vector Theory Approach, Oxford, 1992.