Modeling and Analysis of Buck Type Three Phase PWM Rectifier by Circuit DQ Transformation

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Abstract **— The characteristics of a buck type three phase PWM rectifier are analyzed using circuit DQ transform method. Exact DQ equivalent circuit model is induced and full set of equations are derived in explicit form. With the signal flow graph to describe the internal relations of converter operation, the unique behavior of the converter is analyzed on several practical view points. Various converter characteristics such as gain,real and reactive power,power factor and unity power factor condition including the effects of component values and load variations are analyzed as a function of modulation index and PWM phase angle variation.**

I. Introduction

The forced commutated PWM AC/DC converters have been widely adopted because of the demand of higher power factor and less harmonic pollution. Buck type rectifier is widely used as a PWM based ac/dc converter and has more complex characteristics than boost type rectifier due to the input LC filter. The ac filter is used for the purpose of eliminating switching harmonics which are injected during switching operation. This filter produces phase shift between the line voltage and line current of the power source and degrades the power factor. The phase shift varies not only with the modulation depth and the phase angle of PWM switching strategy but also with the variation of component values and load, which make the control difficult to obtain satisfactory performance in wide operation range.

Many researchers have proposed various control methods but most of the control methods are based on the simple operational characteristics of the converter because the models used for the converter are roughly approximated by only considering the steady state property [1,2],ignoring the input filter characteristics [3],using the per-phase circuit model [4]. It is evident that the applications based on the models are restrictive and not sufficient for high performance. The exact analysis done by full equational model was reported in recent [5] but the dimensions of equations are above 8th order. The analytic method requires the cumbersome matrix manipulation and time consuming in the analysis of various circuit conditions and has difficulty in acquiring the physical meaning of the converter operation.

In this paper, a complete modeling of the buck type PWM rectifier is done using circuit DQ transform method which is very effective in analyzing the multiphase ac circuit [6,7,8]. The exact equivalent linear circuit is induced and various steady state analyses such as DC transfer function, real power, reactive power and power factor as a function of modulation depth, phase angle, component values and load variations are investigated. The clear and exact interpretation about the mechanism of operation in the whole converter is proposed by the analysis based on the model.

II. Circuit DQ transformation of buck type PWM rectifier

A. Process of circuit DQ transformation

Buck type rectifier shown in Fig. 1 is basically time varying system due to switching action although the circuit elements are linear time invariant(LTI). This is the primary barrier to model the whole converter including both ac input side and dc output side with unified linear circuit concept. But the circuit DQ transformation method provides the solution for complete circuit model expressed in LTI form by the following procedures [6].

(1) Partitioning the circuit properly into basic sub-circuits.
(2) Transforming of each sub-circuit based on the DQ equations of (1)-(4) to eliminate the time varying nature of switching system. The voltage, switching function and power invariant DQ transformation matrix are given as follows:

$$V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot V \cdot \begin{bmatrix} sin(\omega t + \phi_1) \\ sin(\omega t - 2\pi/3 + \phi_1) \\ sin(\omega t + 2\pi/3 + \phi_1) \end{bmatrix} \qquad (1)$$

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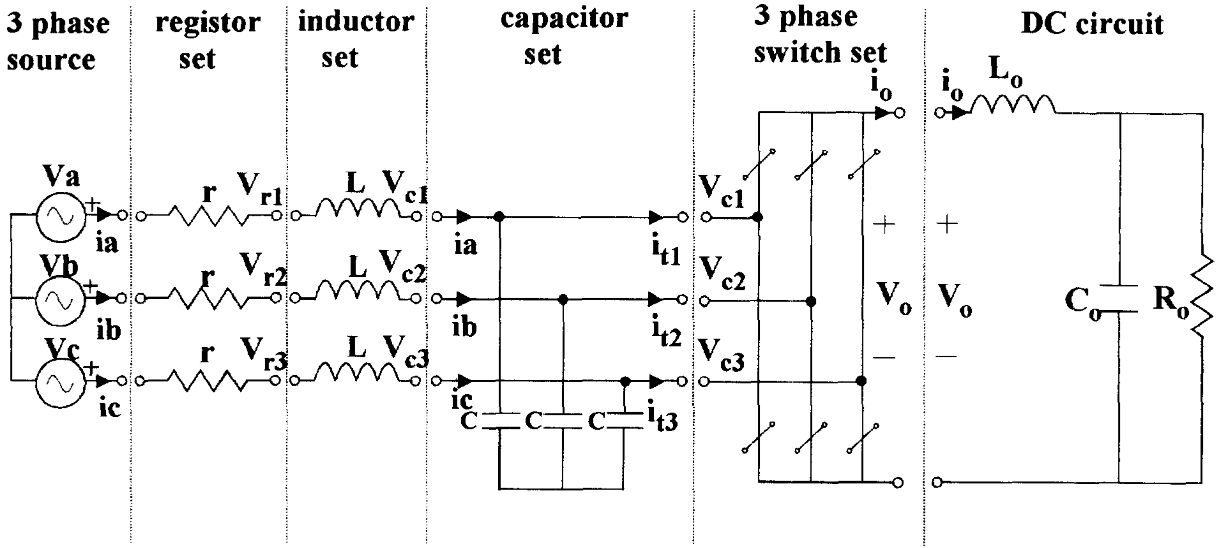

Fig. 1. Partitioned circuit of buck type PWM rectifier.

$$S_{abc} = \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot D \cdot \begin{bmatrix} sin(\omega t + \phi_2) \\ sin(\omega t - 2\pi/3 + \phi_2) \\ sin(\omega t + 2\pi/3 + \phi_2) \end{bmatrix} \quad (2)$$

$$K = K^{-1}$$

$$= \sqrt{\frac{2}{3}} \begin{bmatrix} cos(\omega t + \phi) & \cos\left(\omega t - \frac{2\pi}{3} + \phi\right) & \cos\left(\omega t + \frac{2\pi}{3} + \phi\right) \\ sin(\omega t + \phi) & sin\left(\omega t - \frac{2\pi}{3} + \phi\right) & sin\left(\omega t + \frac{2\pi}{3} + \phi\right) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad (3)$$

$$V_{qdo} = K \cdot V_{abc}. \quad (4)$$

In this case, fundamental component is only considered excluding harmonics.

(3) Reconstructing the transformed subcircuits by connecting nodes of adjacent subcircuits

According to the above rules, buck type rectifier is composed of six basic sub-circuits as shown in Fig. 1. They are voltage source set, resister set, inductor set, capacitor set, current source type switch set and DC circuit. DQ transformed circuits of sub-circuits are summarized in Fig. 2. The three phase inductor set and capacitor set become second order gyrator-coupled system. The equivalent LTI circuit model of the buck type rectifier is obtained as Fig. 3 by cascadely rejoining of sub-circuits.

B. Equivalent circuit model and useful properties.

The DQ transformed equivalent circuit in Fig. 3 shows the obvious physical meaning for whole converter operation as follows.

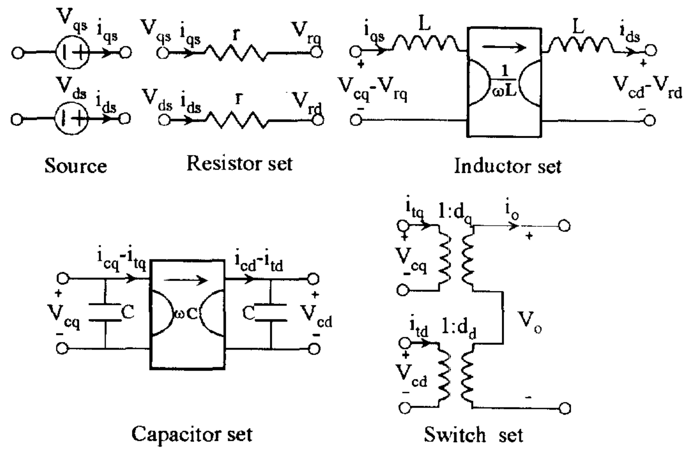

Fig. 2. D-Q transformed partitioned circuit.

o AC real power flows through both the d-axis and q-axis sides and the powers are added into dc side through transformers which represent the switching action.

o There are interactions in AC side between d-axis and q-axis through gyrators which come from 3 phase L and C components in the AC side.

o The interactions between AC side and DC side occur through transformers and the interactions are controlled by two factors : the phase difference and modulation depth of PWM strategy.

These characteristics could also be understood through complicated equational analysis or experience but none of the previous works show the unified and systematic view as this.

All of the powerful analysis tools for linear system can be utilized for further detailed analysis. The signal flow graph method is applied to the equivalent circuit in Fig.4. This graph shows the cause and effect relations between variables.

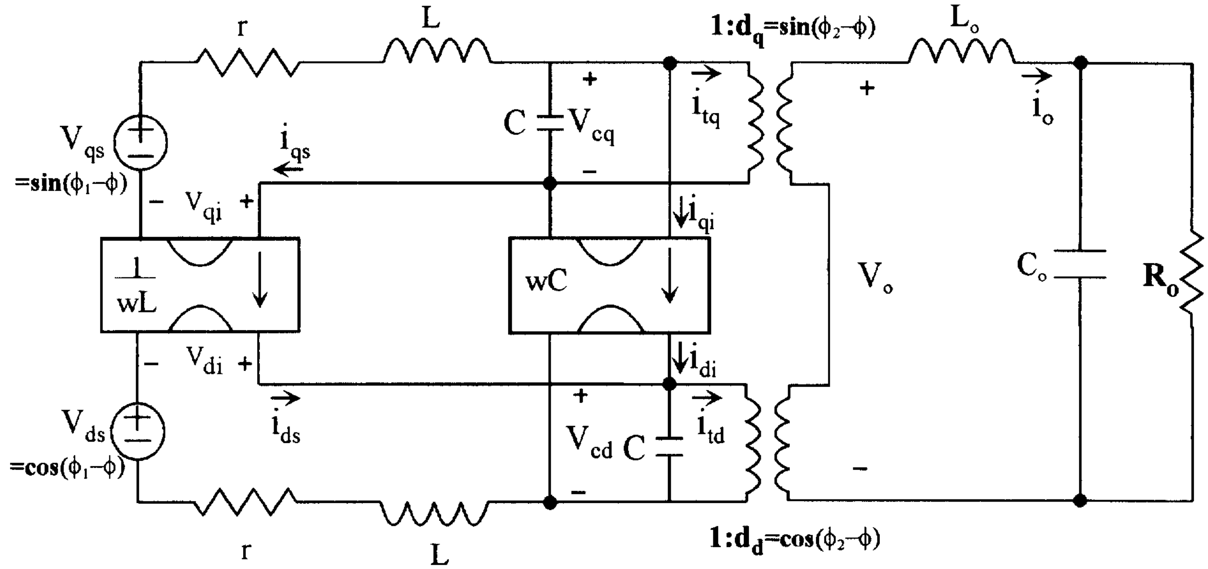

Fig. 3. DQ transformed equivalent circuit of the rectifier.

In this case, the dc gains and ac transfer functions are easily derived and state space equation form can be obtained if needed.

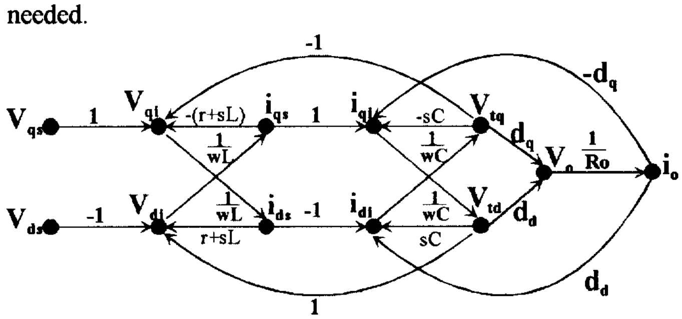

Fig. 4. Signal flow graph of DQ transformed equivalent circuit.

III. Steady state analysis of buck type PWM rectifier

DC equivalent circuit of the rectifier is obtained by shortening the inductors and opening the capacitors from Fig. 3. The general steady state equations of input state variables are easily derived from the DC equivalent circuit as follows:

$$V_{cq} = \frac{\left(1-\omega^2 LC+\frac{rd_d^2-d_d d_q \omega L}{R_o}\right)V_{qs}-\left(\omega rC+\frac{\omega L d_d^2+d_d d_q r}{R_o}\right)V_{ds}}{\Delta} \quad (5)$$

$$V_{cd} = \frac{\left(\omega rC+\frac{\omega L d_q^2-d_d d_q r}{R_o}\right)V_{qs}+\left(1-\omega^2 LC+\frac{rd_q^2+d_d d_q \omega L}{R_o}\right)V_{ds}}{\Delta} \quad (6)$$

$$I_{qs} = \frac{\left(\omega^2 C^2 r+\frac{d_q^2}{R_o}\right)V_{qs}+\left(\omega C(1-\omega^2 LC)+\frac{d_d d_q}{R_o}\right)V_{ds}}{\Delta} \quad (7)$$

$$I_{ds} = \frac{\left(-\omega C(1-\omega^2 LC)+\frac{d_d d_q}{R_o}\right)V_{qs}+\left(\omega^2 C^2 r+\frac{d_d^2}{R_o}\right)V_{ds}}{\Delta} \quad (8)$$

$$\text{where} \quad \Delta = \left(1-\omega^2 LC\right)^2+\left(\omega rC\right)^2+\frac{r\left(d_d^2+d_q^2\right)}{R_o}. \quad (9)$$

The rotation phase ϕ of transformation matrix K in (3) can be selected arbitrary without the loss of generality. It is better that transformation is synchronized with the switching function for simplified analysis because q-axis transformer can be eliminated and d-axis transformer have the constant gain D independent of phase difference. The resulting DC equivalent circuit is shown in Fig. 5. In this case, the magnitude of AC source is depend on the power angle $\theta = \phi_1 - \phi_2$ which means the phase difference between the source voltage and the PWM pattern of switch sets.

A. Ideal condition analysis

Considering the main dominant feature of the converter, we assume that all of the switch and L, C component are ideal. In this condition the parasitic or internal resistance can be ignored and the following results are obtained.

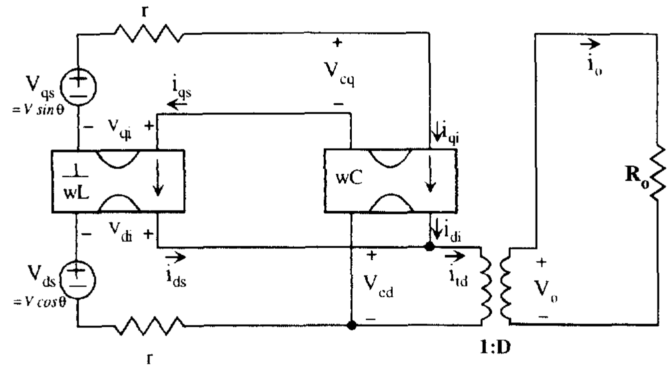

Fig. 5. DC circuit for steady state analysis.

i) DC output characteristics

Output voltage Vo of rectifier in DC side is obtained as

$$V_o = V_{cd} \times d_d = \frac{V \cdot cos\theta}{1-\omega^2 LC} D \ . \qquad (10)$$

This shows that the rectified output voltage is independent of load resistance Ro. This means the output voltage of the rectifier is viewed as an ideal voltage source controlled by phase difference θ and modulation depth D, which is dual characteristic of the boost type rectifier in which the rectified output current is viewed as an ideal controlled current source[7]. In case the system response is very slow, it is possible to approximate the system into a first order system as shown in Fig. 6 since the inductor is very large in practice.

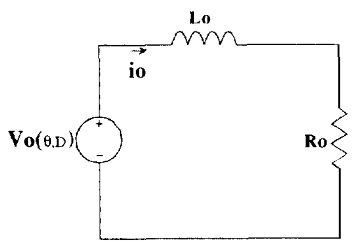

Fig. 6. Quasi steady state output model.

Then DC gain is obtained as

$$G_v = \frac{V_o}{V} = \frac{cos\theta}{1-\omega^2 LC} \cdot D \ . \qquad (11)$$

It is noted that the possible gain can range from zero to infinite and the maximum gain can be much higher than unity if the resonance frequency of ac side LC filter($1/\sqrt{LC}$) is near the main operation frequency. But because the resonant frequency of filter is designed higher than fundamental frequency ω and the internal losses are existed in real system, the gain is limited to some maximum value for given circuit parameters. The voltage gain is mostly dependent on the switching function and the effect is shown in the Fig. 7 which shows the Gv as a function of phase difference θ for different modulation depth D. The parameters used in the analysis are given in the appendix. The voltage gain has a close relation with the real power as shown in (14). Fig. 7 shows that the voltage gain is maximum when the phase difference is π/2 and modulation depth have the highest value in which the real power from the AC source is also maximum.

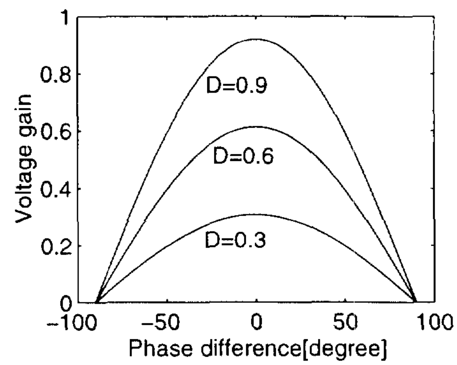

Fig. 7. The DC voltage gain.

ii) Real and reactive power

Real power P_{ac} from AC sources is obtained as

$$P_{ac} = V_{qs}I_{qs} + V_{ds}I_{ds} = \frac{V_s^2 \cdot cos^2\theta}{1-\omega^2 LC} \cdot a \qquad (12)$$

$$\text{where} \quad a = \frac{D^2}{1-\omega^2 LC} \cdot \frac{1}{R_o} \ . \qquad (13)$$

The power dissipated in the DC side is given by

$$P_{dc} = V_o^2 / R_L = \left(\frac{V cos\theta}{1-\omega^2 LC} D \right)^2 \Big/ R_o$$

$$= \frac{V^2 \cdot cos^2\theta}{1-\omega^2 LC} \cdot a = P_{ac} \ . \qquad (14)$$

This means that all real power generated from AC goes to the DC side without any loss.

Reactive power Q is obtained by

$$Q = V_{qs}I_{ds} - V_{ds}I_{qs} = \frac{V^2}{1-\omega^2 LC} \cdot (a\, cos\theta\, sin\theta - \omega C)$$

$$= P \cdot tan\theta - \frac{\omega C V^2}{1-\omega^2 LC} \ . \qquad (15)$$

The first term means the reactive power is generated from the converter by control parameters. The second term is the leading VAR by the input filter C and L. This indicates that the reactive power generated from the converter can be controlled lag or lead by the modulation depth or phase difference. The reactive and real power are shown in Fig. 8 and

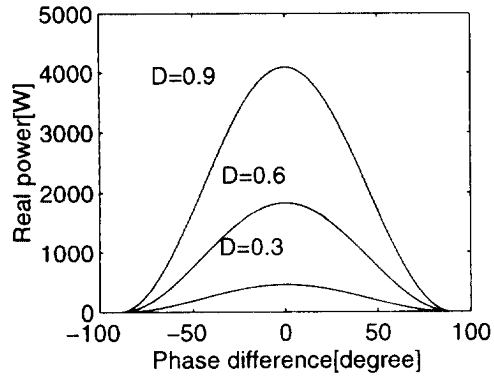

Fig. 8. Real power characteristics.

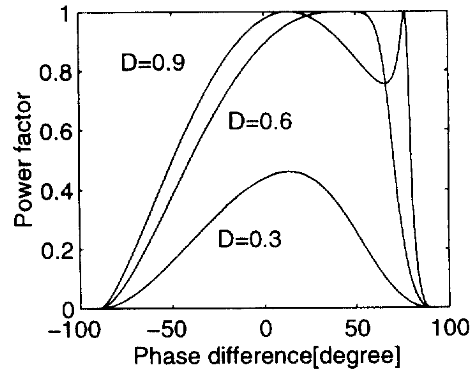

Fig. 10. Power factor characteristics.

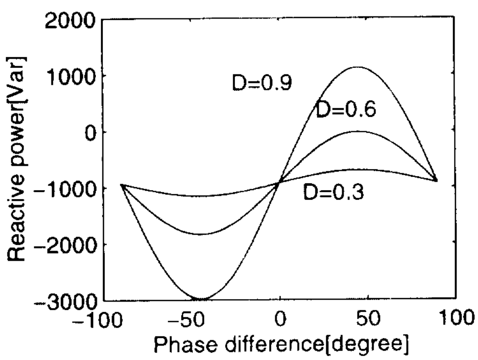

Fig. 9. Reactive power characteristics.

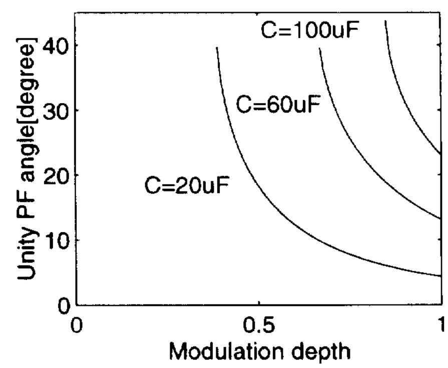

Fig. 11. unity power factor condition.

Fig. 9 as functions of phase and modulation depth of switching function.

iii) Power factor

Power factor is given by

$$PF = \frac{P}{\sqrt{P^2+Q^2}} = \frac{a \cdot cos^2\theta}{\sqrt{(\omega C)^2 - a\omega C\, sin(2\theta) + a^2 cos^2\theta}} \quad . (16)$$

The power factor is controlled by the ratio of real power and reactive power and it can be unity when the reactive power Q is controlled to be 0. From (15), the unity PF condition can be given by

$$a\, cos\theta\, sin\theta - \omega C = 0 \text{ or } \theta = \frac{1}{2} sin^{-1}(2\omega C/a) \text{ for } a \rangle 2. \quad (17)$$

If the coefficient a of (13) is not larger than 2, power factor cannot be unity. This condition can occur when the load resistance Ro is very high, which is dual property of the boost type rectifier in which the power factor cannot be unity when the load resistance is very low [7]. The Fig. 10 shows that there are restricted ranges in which PF cannot be unity and also shows the range of unity PF that can be controlled as functions of control parameter θ and D. The Fig. 11 shows the unity PF condition is changed by circuit parameters.

B Practical characteristics

Practically real system has the parasitic resistance at both ac side and dc side. The trend shown in Fig. 10 and Fig. 11 can be changed somewhat when considering the parasitic resistance. The analyses considering the parasitic resistances and the effects of control parameters are shown in Fig. 12 over whole operating range.

The gain, real power and reactive power are shown as follows considering loss :

$$G_v = \frac{\omega r C\, sin\theta + (1-\omega^2 LC)cos\theta}{\Delta} \quad (18)$$

$$P = \frac{(\omega^2 C^2 r)V_s^2 + \frac{D^2}{R_o^{'}} V_s^2 cos^2\theta}{\Delta} \quad (19)$$

$$Q = \frac{-\omega C(1-\omega^2 LC)V_s^2 + \frac{D^2}{R_o^{'}} V_s^2 cos\theta\, sin\theta}{\Delta} \quad (20)$$

where $R_o^{'}$ is resistance including the load and parasitic at dc side.

The parasitic loss can affect the property of gain,power and power factor if the load is heavy but the unity power factor

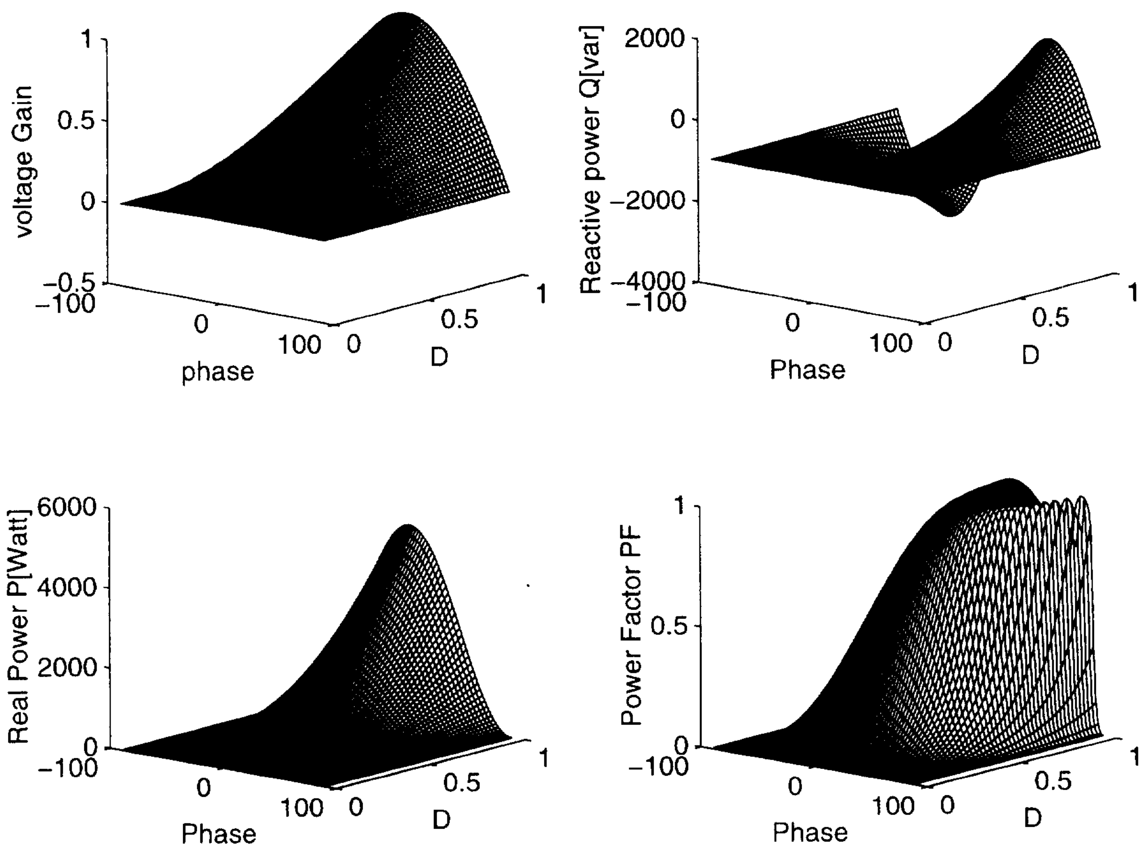

Fig. 12 The operation characteristics considering the loss as a function of phase difference and modulation depth.

condition is not affected by the parasitic loss.

IV. Conclusion

The buck type PWM rectifier is analyzed using circuit DQ transformation. Exact LTI equivalent circuit with unified form is induced and full set of equations are derived in explicit form. The various characteristics such as gain, power, power factor, unity power factor condition and the practical behavior including component and load variations are analyzed. The analysis based on the model shows the unique interpretation about the mechanism of operation of such a buck type.

Appendix

The parameters of buck type rectifier are :

L=3mH,C=50uF,r=1ohm,Co=800uF,Lo=5mH,Ro=10ohm, V=220V.

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