How Does Creditor's Liquidation Decision Affect Debt and Equity Values?

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ABSTRACT

Bankruptcy and liquidation procedures are important to determine the values of equity, debt and firm. During the liquidation process, the creditors play an important role as well as borrowers. This paper models the creditors' liquidation decision and examines its effects on the equity and debt values and credit premiums of risky debt. If the liquidation costs are considerable, the debtholders tend to delay liquidation and it provides the equityholders with an opportunity to behave strategically. Without Chapter 11 of the U.S. Bankruptcy Code, the bankruptcy does not lead to immediate liquidation due to the costly liquidation. Additionally, we address that the tax benefit, in addition to liquidation costs, is one of the factors that cause the strategic debt service. Our model incorporates the equityholders' bankruptcy decision and debtholders' liquidation decision. We suggest how to determine the equity and debt values numerically using finite difference methods. And we show that Leland (1994) model is a special case of ours. Finally, this paper also examines the determinants of values of equity, debt, firm and risky debt's credit premium.

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Keywords: *liquidation costs; strategic behavior; firm value; credit premium.*

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I. Introduction

Over the years the studies about the values of corporate debt and equity have attracted a fair amount of attention. Since Merton (1974) valued the corporate debt and equity using option pricing model, the structural models have emerged as a new approach for the studies of credit risk.

To determine the debt and equity values, it is important to know when the bankruptcy and liquidation occur. Merton assumes that the default and liquidation occur only at the debt's maturity if the firm value is less than the obligation amount. Black and Cox (1976) assume the time-dependent default boundary and regard the firm as going bankruptcy when the boundary is first crossed. Other papers in early days assumed that the bankruptcy event coincides with the liquidation event. (See, for example, Brennan and Schwartz (1978), Leland (1994) and others)

In recent years, however, great attention has been paid to the studies modeling the Chapter 11 of U.S. bankruptcy code. Chapter 11 includes a reorganization process for a distressed firm. If an insolvent firm files for reorganization under Chapter 11, a grace period is given and liquidation by Chapter 7 occurs only if the obligations are not honored during a specified period. In reality, thus, the bankruptcy does not necessarily lead to the liquidation by the creditors. For example, Fan and Sundaresan (2000) show that equityholders have an incentive to stop paying the contractual coupon and start the strategic debt service. They consider the renegotiation as a consequence of Nash bargaining game between borrowers and lenders.

Many studies model the bankruptcy code, Chapter 11 and Chapter 7, by treating bankruptcy and liquidation as separate events explicitly and examine the implication of the bankruptcy codes. Francois and Morellec (2004; henceafter FM) assume that liquidation occurs only if the firm stays in bankruptcy for more than the grace period. Broadie, Chernov and Sundaresan (2005; henceafter BCS) study the conflicts between equityholders and debtholders stemmed from the reorganization in addition to liquidation and show that reorganization does not improve the social welfare. And there are many other articles treating bankruptcy and liquidation as

separate events. (See Galai, Raviv and Wiener (2003), Moraux (2002), Paseka (2003) and others)

In this paper, we address that the liquidation costs make creditors delay the liquidation even without the reorganization forced under Chapter 11 and even after the grace period. Because there is a possibility that the firm recovers the value and that the obligations are fulfilled before the debt's maturity, to wait can be more valuable than to liquidate immediately. In this case, the agency problem that equityholders do not honor the obligation intentionally arises even though the information is symmetric. Anderson and Sundaresan (1996) have already viewed this conflict between the borrowers and lenders as a sequential game. Though they address that the costly liquidation leads the strategic debt services of equityholder, the behavior of debtholders while the firm stay in bankruptcy is not modeled explicitly. In this paper, we focus on modeling the behaviors of both borrowers and lenders during the post-bankruptcy procedure.

We propose the valuation method to implement the strategic debt service of equityholders and the liquidation decision of debtholders in the case of costly liquidation. Previously, Anderson and Tu (1998) solved the PDEs using finite difference methods and calculate the debt and equity values under Anderson and Sundaresan's setting. Broadie and Kaya (2005; henceafter BK) present the method valuing corporate debt using a binomial tree and the way how to incorporate Chapter 11 which implies the path-dependency of debt and equity values.

In this paper, we propose the explicit finite difference methods to determine the debt and equity values under the setting analogous to BCS and BK. Our model can be easily extended to incorporate the automatic stay provision and grace period that are included in Chapter 11 as well. Moreover, we also allow the financial distress costs, tax benefit effects, and liquidation costs as implemented in BCS and BK.

It is worthwhile to note that the bankruptcy is declared by equityholders in their interest and the liquidation is led by debtholders in our model. On the other hand, BCS assume that the bankruptcy leads to liquidation because the firm overstays in the bankruptcy state or equity value reaches zero. In contrast to BCS and BK, our method does not need a numerical maximization to find the optimal bankruptcy boundary. The functional forms of the bankruptcy boundary, which are assumed by BK, are also needless in our method. Another difference between the BCS's framework and ours is that we assume that coupon payments are discrete.

We show that discontinuous coupons deepen the strategic behavior of equityholders.

Without the strategic debt service of debtors, our model is reduced to Leland (1994) or Leland and Toft (1996) that provide the closed-form solutions for infinite and finite maturity debt values. We also show that the values obtained by using our numerical methods converge to the analytic solution given by Leland. The comparison between our model considering the strategic behavior of equityholders and Leland and Toft model shows us the effects of strategic debt services on equity and debt values.

The rest of this paper is organized as follows. Section II sets up the model. Section III presents how to implement the model and calculates equity and debt values using explicit finite difference methods. In section IV, the numerical examples are exhibited and compared with the extended Leland model which includes the finite maturity debt with discrete coupons. Section V examines the determinants of the credit premiums and addresses the implications of our model. Finally, section VI concludes.

II. Model

Unlevered firm value process, $V(t)$, under the risk-neutral measure Q is given by

$$dV(t) = (r - \delta)V(t)dt + \sigma V(t)d\tilde{W}(t), \tag{1}$$

where r is the riskless interest rate, δ is the continuous dividend rate, σ is the asset volatility, and $\tilde{W}(t)$ is a standard Brownian motion under the filtered probability space $(\Omega, \Im, Q)$. We assume that the riskless interest rate, dividend rate and volatility are constant. The firm pays dividends to equityholders continuously with instantaneous rate proportional to the asset value. It is presumed that the dividend rate does not depend on the leverage level. Asset sales are not permitted by the debt covenants and equity dilution is necessary to fulfill

coupon payments.

While Leland and BCS assume the coupon payments are continuous, we assume the coupons are paid discretely n times per year. For instance, if the firm issues the coupon bond that pays coupons every 3 month, the payment frequency *n* is 4. Due to the tax shield effect of leverage, the effective coupon payments are reduced by tax rate τ.

Since the equity has a call option-like property and continuous dividends are paid to equityholders, bankruptcy occurs only when the coupon payments are due. At every coupon payment date, equityholders should decide whether to declare bankruptcy or to serve the coupon payment.

If the equityholders declare bankruptcy, the firm goes into the bankruptcy state and all the dividend payments are stopped. The cash dividends are reserved by the Chapter 11 forcing the automatic stay of assets while in bankruptcy. While the firm is in bankruptcy, a financial distress cost proportional to $V(t)$ reduces the firm value. The distress cost is denoted by w and the unlevered firm value process under bankruptcy is given by

$$dV(t) = (r - w)V(t)dt + \sigma V(t)d\tilde{W}(t). \tag{2}$$

While the firm is in bankruptcy, the equityholdes have an option either to pay the accumulated unpaid coupons plus interest and go into the liquid state or not. On the other hand, the debtholders can choose whether to liquidate the firm or to stay in bankruptcy. If the firm is liquidated, the liquidation cost proportional to the firm value, $\alpha V(t)$, is incurred and debtholders can receive $(1-\alpha)V(t)$.

It is important that the debtholders have a right to liquidate the firm in our model. FM, BCS and BK assume that the firm is liquidated if the firm is still in bankruptcy even after the grace period pre-specified. In reality, however, liquidation costs make the debtholders avoid liquidating and wait for the firm to recover the value before the debt's maturity. If the continuation value of the debt is greater than the liquidation value, the debtholders will delay the liquidation. The behavior of debtholders to avoid liquidation gives the equityholders an opportunity to serve the debt strategically based on their interest. Anderson and Sundaresan

(1996) show that the equityholders' strategic debt service is possible.

We presume that the equityholders and debtholders know the others' behavior each other. Therefore, equityholders can make a bankruptcy decision based on the debtholders' behavior to decide the liquidation during the post-bankruptcy period. If the equity value after paying the contractual coupons is greater than the equity value in the bankruptcy, the coupons are paid to the debtholders. On the other hand, when going into bankruptcy is more profitable to the equityholders, the coupon payments are rejected and the firm will go into bankruptcy.

While the firm is in bankruptcy, equityholders can decide whether to pay the arrears, which are unpaid coupons and interests, and go into the liquid state or to stay in the bankruptcy state. As the firm value increases, going into the liquid state rather than staying in bankruptcy is lucrative to the equityholders, since the likelihood of liquidation is increasing. If the equity value in the liquid state plus the arrears is greater than the continuation value in the bankruptcy state, the equityholder will repay the arrears and the firm will go into the liquid state.

We assume that the coupons are paid n times per year and the coupon payment dates are denoted by $t=\{t_1,t_2,\cdots,t_N\}$ and t_N is the debt's maturity date T. P and c denote the principal amount and coupon rate, respectively. The contractual coupon payment at each due date, t_i, is $C=cP/n$.

$E_L(t,V)$, which denotes the value of equity in the liquid state at time t when the firm value is V, satisfies the following PDE and free-boundary conditions:

$$\frac{1}{2}\sigma^2V^2\frac{\partial^2E_L(t,V)}{\partial V^2}+(r-\delta)V\frac{\partial E_L(t,V)}{\partial V}+\frac{\partial E_L(t,V)}{\partial t}=rE_L(t,V)-\delta V,\ (t_{i-1}<t<t_i) \tag{3}$$

$$E_L(t^-,V)=\begin{cases}E_L(t^+,V)-(1-\tau)C & \textit{if NOT bankruptcy}\\ E_B(t,V) & \textit{if bankruptcy}\end{cases},\quad (t=t_i) \tag{4}$$

where $E_B(t,V)$ is the equity value in the bankruptcy state at time t. $i=1,2,\cdots,N-1$ and $t_0=0$. The boundary and terminal conditions are given by,

$$\lim_{V\to\infty} E_L(t,V) = V(t) - Pe^{-r(T-t)} - (1-\tau)C\sum_{i=1}^{N} 1_{\{t_i \geq t\}} \cdot e^{-r(t_i-t)}, \quad \lim_{V\to 0} E_L(t,V) = 0. \quad (5)$$

$$E_L(T,V) = \begin{cases} V - P - (1-\tau)C & if\ V \geq P + (1-\tau)C \\ 0 & if\ V < P + (1-\tau)C \end{cases}. \quad (6)$$

$D_L(t,V)$, which denotes the value of debt in the liquid state at time t when the firm value is V, satisfies the following PDE and free-boundary conditions:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 D_L(t,V)}{\partial V^2} + (r-\delta)V\frac{\partial D_L(t,V)}{\partial V} + \frac{\partial D_L(t,V)}{\partial t} = rD_L(t,V), \quad (t_{i-1} < t < t_i) \quad (7)$$

$$D_L(t^-,V) = \begin{cases} D_L(t^+,V) + C & if\ NOT\ bankruptcy \\ D_B(t,V) & if\ bankruptcy \end{cases}, \quad (t = t_i) \quad (8)$$

where $D_B(t,V)$ is the debt value in the bankruptcy state at time t. The boundary conditions and terminal conditions are given by,

$$\lim_{V\to\infty} D_L(t,V) = Pe^{-r(T-t)} + C\sum_{i=1}^{N} 1_{\{t_i \geq t\}} \cdot e^{-r(t_i-t)}, \quad \lim_{V\to 0} D_L(t,V) = 0. \quad (9)$$

$$D_L(T,V) = \begin{cases} P + C & if\ V \geq P + (1-\tau)C \\ (1-\alpha)V & if\ V < P + (1-\tau)C \end{cases}. \quad (10)$$

If the borrowers declare bankruptcy, the asset value follows a different stochastic difference equation and the continuous dividends are stopped. Unless the equityholders clear bankruptcy and pay arrears to creditors, the debtholders can liquidate the firm. And equityholders will clear bankruptcy if that is more profitable than staying in a bankruptcy state.

While the firm is in bankruptcy state, $t > t_B$, $E_B(t,V)$ and $D_B(t,V)$ satisfy the following PDEs and the free-boundary conditions:

For $E_B(t,V)$,

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 E_B(t,V)}{\partial V^2} + (r-w)V \frac{\partial E_B(t,V)}{\partial V} + \frac{\partial E_B(t,V)}{\partial t} = rE_B(t,V) \tag{11}$$

$$E_B(t^-,V) = \begin{cases} E_L(t,V) - (1-\tau)A(t) & \textit{if clearing bankruptcy} \\ Max\left[(1-\alpha)V - A(t) - P,\ 0\right] & \textit{if liquidation} \\ E_B(t^+,V) & \textit{otherwise} \end{cases}. \tag{12}$$

For $D_B(t,V)$,

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 D_B(t,V)}{\partial V^2} + (r-w)V \frac{\partial D_B(t,V)}{\partial V} + \frac{\partial D_B(t,V)}{\partial t} = rD_B(t,V), \tag{13}$$

$$D_B(t^-,V) = \begin{cases} D_L(t,V) + A(t) & \textit{if clearing bankruptcy} \\ Min\left[(1-\alpha)V,\ A(t) + P\right] & \textit{if liquidation} \\ D_B(t^+,V) & \textit{otherwise} \end{cases}. \tag{14}$$

where $A(t)$ is the arrears, the unpaid coupons plus interest, and this value does not depend on the path. $A(t)$ is given by,

$$A(t) = C\sum_{i=B}^{N} 1_{\{t_i \le t\}} e^{r(t-t_i)}, \text{ where } t_B \text{ is the moment of bankruptcy.} \tag{15}$$

III. Implementation

The above equations can be solved by the lattice model, for example, the finite difference method or binomial / trinomial tree method. The equity and debt values can be determined by backward induction. In this paper, we address how to determine the debt and equity values using the explicit finite difference method.

For the explicit FDM, suppose that firm value and time are divided into small increments of length ΔV and Δt, respectively. The pair, (k,i), is used when the firm value is $i\Delta V$, $(i=0,\ldots,I)$ and time is $k\Delta t$, $(k=0,\ldots,K)$. The equations for updating the equity and debt values between the coupon payment dates are given by,

$$E(k,i)=a(i)E(k+1,i-1)+\big(1+b(i)\big)E(k+1,i)+c(i)E(k+1,i+1)+d(i), \qquad (16)$$

$$D(k,i)=a(i)D(k+1,i-1)+\big(1+b(i)\big)D(k+1,i)+c(i)D(k+1,i+1). \qquad (17)$$

$a(\cdot)$, $b(\cdot)$, $c(\cdot)$, and $d(\cdot)$ are the time-independent functions that imply the risk-neutral probabilities multiplied by riskless discount factor. The coefficients of liquid state are different from those of bankruptcy state. The concrete equations for $a(\cdot)$, $b(\cdot)$, $c(\cdot)$, and $d(\cdot)$ are given in appendix A.

First, we calculate the equity and debt values at t_{N-1}, the last coupon payment date prior to maturity date. Equityholders will decide whether to declare bankruptcy or not depending on the equity values of each circumstance. If the continuation value of equity after deducting the contractual coupon amount is greater than the value under bankruptcy, equityholders will pay the coupon to debtholders. Otherwise, equityholders will declare bankruptcy. Thus we need to calculate the equity and debt values both in the liquid state and in the bankruptcy state.

Assuming that the firm is not in bankruptcy at t_{N-1}, we can solve $E_L(t_{N-1}^+,i)$ and

$D_L(t_{N-1}^+, i)$ for every i using the updating equations (16) and (17). $E_L(t_{N-1}^+, i)$ and $D_L(t_{N-1}^+, i)$ denote the equity and debt values just after the coupons are paid. The boundary conditions and terminal conditions for equity and debt values are given in equations (5), (6), (9) and (10), respectively.

While in the backward propagation, the values at every grid point should be memorized because the clearing bankruptcy decision depends on the equity values in the liquid state. Equityholders can decide whether to clear bankruptcy or to stay in the bankruptcy state depending on the equity values in the midway between the bankruptcy date and maturity.

After the values in the liquid state are determined, $E_{B(t_{N-1})}(t_{N-1}, i)$ and $D_{B(t_{N-1})}(t_{N-1}, i)$, which are the equity and debt values in the case that the coupons are unpaid and the firm goes into bankruptcy at time t_{N-1}, can be calculated by backward induction as well. While doing this procedure, we should note that equityholders have an option to pay the arrears at any time they want and debtholders have an option to liquidate the firm unless the equityholders fulfill the obligation.

The terminal values in the bankruptcy state are given by,

$$E_B(T,V) = \begin{cases} V - P - (1-\tau) A_{B(t_{N-1})}(T) & \text{if } V \geq P + (1-\tau) A_{B(t_{N-1})}(T) \\ 0 & \text{if } V < P + (1-\tau) A_{B(t_{N-1})}(T) \end{cases} \tag{18}$$

for equity values and,

$$D_B(T,V) = \begin{cases} P + A_{B(t_{N-1})}(T) & \text{if } V \geq P + (1-\tau) A_{B(t_{N-1})}(T) \\ (1-\alpha) V & \text{if } V < P + (1-\tau) A_{B(t_{N-1})}(T) \end{cases} \tag{19}$$

for debt values, where $A_{B(t_{N-1})}(t)$ denotes arrear amounts at time t accumulated from t_{N-1}.

$E_{B(t_{N-1})}(t^+, V)$ and $D_{B(t_{N-1})}(t^+, V)$ are obtained from the following three grid points using

the updating equations (16) and (17). If the equity value in the liquid state, $E_L(t,V)$, is greater than $E_{B(t_{N-1})}(t^+,V)+(1-\tau)A_{B(t_{N-1})}(t)$, equityholders will repay the arrears and the equity values are replaced by $E_L(t,V)-(1-\tau)A_{B(t_{N-1})}(t)$. For this procedure, we need to know the equity values in the liquid state calculated previously.

On the other hand, if the debt value, $D_{B(t_{N-1})}(t^+,V)$, is less than the liquidation value, $Min\left[(1-\alpha)V,\ A_{B(t_{N-1})}(t)+P\right]$, the debtholders will liquidate the firm. Since the creditors' behavior described above is known by equityholders, they will repay the arrears to creditors if the residual value after liquidation is less than $E_L(t,V)-(1-\tau)A_{B(t_{N-1})}(t)$, which is the equity value in the liquid state after deducting the arrears amount with tax savings.

In summary, we update the equity and debt values in the following way.

$$E_B(t^-,V)=\begin{cases} E_L(t,V)-(1-\tau)A(t) & \text{if (a) } E_L(t,V)-(1-\tau)A(t)\geq E_B(t^+,V) \text{ or} \\ & \text{(b) } D_B(t^+,V)<(1-\alpha)V \text{ and} \\ & E_L(t,V)-(1-\tau)A(t)\geq Max\left[(1-\alpha)V-A(t)-P,\ 0\right] \\ Max\left[(1-\alpha)V-A(t)-P,\ 0\right] & \text{if } D_B(t^+,V)<(1-\alpha)V \\ E_B(t^+,V) & \text{otherwise} \end{cases} \tag{20}$$

$$D_B(t^-,V)=\begin{cases} D_L(t,V)+A(t) & \begin{array}{l} if\ (a)\ E_L(t,V)-(1-\tau)A(t)\geq E_B(t^+,V)\ or \\ (b)\ D_B(t^+,V)<(1-\alpha)V\ and \\ E_L(t,V)-(1-\tau)A(t)\geq Max\left[(1-\alpha)V-A(t)-P,\ 0\right] \end{array} \\ Min\left[(1-\alpha)V,\ A(t)+P\right] & if\ D_B(t^+,V)<(1-\alpha)V \\ D_B(t^+,V) & otherwise \end{cases} \tag{21}$$

$E_{B(t_{N-1})}(t_{N-1},V)$ and $D_{B(t_{N-1})}(t_{N-1},V)$, which are the values in bankruptcy at t_{N-1}, the last coupon payment date, are calculated by the backward induction using equations (16), (17), (20) and (21).

Equityholders will decide whether to pay or not depending on the equity values in the liquid state and in the bankruptcy state. If the liquid state value of equity minus the coupon payments after the tax benefit is less than the bankruptcy value, the firm will go into bankruptcy. If else, the firm will stay in the liquid state and the creditors will receive their coupon payment. The equity and debt values at the coupon payment date are updated according to:

$$E_L(t^-,V)=\begin{cases} E_L(t^+,V)-(1-\tau)C & if\ E_L(t^+,V)-(1-\tau)C\geq E_B(t,V) \\ E_B(t,V) & if\ E_L(t^+,V)-(1-\tau)C<E_B(t,V) \end{cases} \tag{22}$$

$$D_L(t^-,V)=\begin{cases} D_L(t^+,V)+C & if\ E_L(t^+,V)-(1-\tau)C\geq E_B(t,V) \\ D_B(t,V) & if\ E_L(t^+,V)-(1-\tau)C<E_B(t,V) \end{cases} \tag{23}$$

Next, the equity and debt values in the liquid state at the second last coupon payment date from the maturity date, $E_L(t_{N-2}^+,i)$ and $D_L(t_{N-2}^+,i)$, are calculated by backward induction

from the values at the last coupon payment date, t_{N-1}. This procedure also uses the same updating equations (16) and (17) in the liquid state. In the same way as the one described previously, $E_{B(t_{N-2})}(t_{N-2}, i)$ and $D_{B(t_{N-2})}(t_{N-2}, i)$, which are the values of equity and debt in the case that the equityholders declare bankruptcy at t_{N-2} can be obtained using the updating equations. Since these values differ from $E_{B(t_{N-1})}(k, i)$ and $D_{B(t_{N-1})}(k, i)$ even in the overlapped period, the propagation must start form the debt's maturity again.

The only difference we have to consider is that the arrears in the above equations should be replaced by $A_{B(t_{N-2})}(t)$, the amounts accumulated from t_{N-2}. As before, equityholders make a decision of clearing bankruptcy depending on the equity values in the liquid state and in bankruptcy. The equity and debt values are updated by equation (22) and (23) at each coupon payment date.

The path-dependency problem stemmed from clearing bankruptcy can be solved by substituting the equity and debt values of the liquid state for those of the bankruptcy state. This method makes the complex problem easy to solve.

The equity and debt values at t_{N-3}, t_{N-4}, …, t_0 can be calculated recursively in the same way. Finally, we can get the equity and debt values at time 0 and calculate the levered firm value by just summing the equity and debt values.

IV. Numerical Examples

This section illustrates simple numerical examples using the methods described in the previous section. First, we show that the Leland (1994) model is the special case of our model and the results of our methods coincide with the analytic value of Leland's. And we extend the Leland model to value the finite maturity debt with discrete coupons, on which this paper mainly focuses. Second, the case that bankruptcy does not lead to immediate liquidation and

debtors behave strategically in order to maximize equity value is examined. The values of strategic behavior of equityholders are measured by the differences between our model and the extended Leland model.

1. Lelnad (1994) Model and Extension

Leland (1994) incorporates the proportional liquidation costs and corporate tax effects into the traditional structural models. Leland assumes that the debt is perpetual and the default occurs if the equity values drop to 0. Since it is assumed that the firm is liquidated as soon as the bankruptcy is declared, the equity value will be 0 immediately. Therefore if the equity value is greater than 0, the firm will keep going. In our model, the Leland model can be generated by replacing the equity values by zero in the bankruptcy state at each coupon payment date. This method is previously suggested by Broadie and Kaya (2005) who use a binomial tree.

The bankruptcy condition is checked at every time step because Leland assumes that the dividends and coupon payments are continuous. Finally, as done in BK, the debt's maturity is assumed to be long enough, such as 200 years, not to affect the current values. The terminal values are given by:

$$E(T,V)=\begin{cases} V-(1-\tau)\dfrac{C}{r} & if\ V>(1-\tau)\dfrac{C}{r} \\ 0 & if\ V<(1-\tau)\dfrac{C}{r} \end{cases} \tag{24}$$

$$D(T,V)=\begin{cases} \dfrac{C}{r} & if\ V>(1-\tau)\dfrac{C}{r} \\ (1-\alpha)V & if\ V<(1-\tau)\dfrac{C}{r} \end{cases}, \tag{25}$$

where C denotes the annual coupon paid continuously.

Table 1 reports the equity and debt values calculated by finite difference methods and

compares them with the values obtained by the analytic formulae in Leland (1994). The analytic formulae are given in appendix B. The results for various parameter values are reported and the absolute and relative differences between the values by finite difference methods and by analytic formulae are presented. We assume that the unlevered firm value is 100 and the maximum firm value for FDM is 500. We set the time interval as 1/6000 and the interval of firm value as 2. This table shows that the errors are small enough to neglect if the intervals are narrow. The magnitude of the error is almost the same with BK's results. This result justifies the usage of the explicit finite difference methods to calculate the risk bond prices and equity values.

[TABLE 1]

Figure 1 shows that the numerical values by finite difference methods converge to the analytic values as the grid gets finer. The relative errors of the equity and debt values are plotted. The axis-X in this figure represents the number of steps between the lowest and highest firm values, from 0 to 500. The equity value converges to the true value faster than debt value. As documented in BK, errors in debt value exhibit an oscillation since the debt value is sensitive to the boundary as in the case of a barrier option.

[FIGUGE 1]

We extend the Leland model so that the debt maturity is finite and coupons are discretely paid. Since the endogenous default boundary is time-dependent, the closed form solution such as the Leland model does not exist in this case. Instead, we calculate the values using the finite difference method which is identical to the one in table 1.

The equity, debt, and firm values in this setting are reported in Table 2. In this table we assume that the model parameters are $V_0 = 100$, $r = 5\%$, $\delta = 3\%$, $\alpha = 0.5$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 5\%$. Since the coupon rate equals to the riskless interest rate, the riskless bond value for each maturity is approximately same with the face value.

This table shows that equity value decreases and debt and firm value increase as the

coupons are paid more frequently. That is because holding equity can be regarded as a long position of option. The time value of equity gets less as the interval between coupon payment dates gets shorter. The present value of payout to the equityholder will get less because the dividends will stop if the firm goes bankrupt. On the other hand, the debt value increases because debt value is concave to the asset value and has negative gamma over all. In other words, the maximum amount debtholders can take is bounded whereas they can lose all invested.

The effects of maturity depend on the debt's face value. In the case of the firm with low leverage, the debt value decrease as the maturity gets longer and vice versa for the distressed firm. However, the debt values do not have monotone trend for the firm with intermediate leverage. Examining the debt values with face value of 80, for different maturities in this table, we can see that the debt value decrease as the maturity gets longer for the short maturities and increase for the long maturities.

[TABLE 2]

These effects are shown in Figure 2 explicitly. Figure 2 plots the equity, debt, and firm values for various leverage levels. In these figures, we assume that the coupons are paid quarterly. As the legend indicates, the circles, rectangles, and triangles present the cases that the face values of outstanding debt are 60, 80, and 100, respectively.

These figures show the equity value grows greater with the debt maturity. On the other hand, the relationship between debt values and maturities depends on the leverage level. In general, the debt values tend to decrease as the maturity gets longer since holing a debt is similar to shorting an option. However, if the liquidation costs exist, it is not the case. A distressed firm might face bankruptcy and immediate liquidation will snatch substantial amounts. Since the default probability tends to decrease as the maturity gets longer, the debt with longer maturity is less affected by the liquidation costs. Thus, the debt values tend to increase in the distressed firm. If the leverage level is intermediate, the two factors – the short position of an option and the liquidation costs, affects the debt and firm values, simultaneously. In figure 2, we can find that the debt and firm values decrease for short maturity and increase for long maturity when the face value is 80.

[FIGURE 2]

2. The Effects of Debtholders' Liquidation Decision

We calculate the equity, debt, and firm values of the model for creditors' liquidation decision and debtors' strategic behavior. These values are compared with those in the case of immediate liquidation and no strategic behavior.

First, we consider the case that the liquidation costs are zero. In this case, the results of our model are the same with those of the Leland model if the tax rate is zero. However, if the tax rate is not zero, the equity and debt values of the two models diverge as the frequency decreases. It is interesting that the differences are caused by the tax advantage of interest payment. If the firm is liquidated immediately, the total value that debtholders will receive is the asset value at the moment of liquidation. The potential tax benefit in the future, which is included in the current firm value, will evaporate. The liquidation will bring on the loss of tax benefit and this result is similar to the debt-equity swap that is addressed in Fan and Sundaresan (2000). The important fact we emphasize is that the tax effect can lead to the strategic debt service of borrowers, though not large.

Second, we examine the effects of high liquidation costs. As assumed in the previous section, the proportional costs are assumed to be 50% of the asset value. As expected, the equity values rise and the debt and firm values fall due to the strategic debt services of debtors.

Table 3 reports the equity, debt, and firm values of our model and compares them with those for the Leland model in which the liquidation occurs immediately. Panel A of this table presents the results of the case with no liquidation costs and the results of high liquidation costs are exhibited in panel B. In the case that the liquidation costs are zero, the values for tax rate 0% and 35% are presented, respectively. These values in panel A shows the effects of tax benefit on

the strategic behavior and pricing implication. We document the values determined by our model and the differences between ours and Leland's, which are our values minus Leland's values, are reported. We assume that there are no financial distress costs, that is $w = 0$. The other parameter values are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 3\%$, $\sigma = 0.2$, and $c = 5\%$.

[TABLE 3]

As noted in the previous paragraph, the results of our model coincide with those of Leland's if the liquidation is costless and there are no corporate taxes. In this case, liquidating the firm immediately is always more profitable to debtholders than staying in bankruptcy. However, if the tax benefit of interest exists, it can be undesirable for debtholders to liquidate immediately. Since equityholders know that the debtholders have an incentive to put off liquidation, the bankruptcy boundary will go up and strategic debt services will take place. As a result, panel A shows that the equity values are greater than those of Leland's and the debt and firm values are less than those of Leland's when tax rate is 35%. As the maturity tends to be longer and the period of coupon payments are lengthened, the differences get greater. Figure 3 shows the differences between our model and Leland's when the liquidation is costless but the tax benefit exists.

[FIGURE 3]

Panel B of table 3 presents the results of high liquidation costs. The results of Leland's with the same parameter values are reported in table 2. The differences of values are presented in this table. We can see the differences are considerable and become greater as the maturity is extended longer. The difference may be regarded as benefits for borrowers and costs for creditors owing to the strategic debt services. In the case that maturity is 20 years, the differences of equity value and debt value are over 10% and over 20% of our model value.

V. The Determinants of Equity, Debt and Credit Premium

We will examine the determinants of credit premium in this section. In our framework, various factors influence the equity, debt and firm. In this section, finally, the effects of these factors are analyzed and the implications are studied.

The various parameters and their base values assumed in this section are presented in the below table:

Notation	Explanation	Value
V_0	Unlevered firm value	100
σ	Volatility of firm value	0.2
r	Riskless interest rate	0.05
δ	Dividend rate	0.01
w	Distress costs	0.05
T	Time to maturity	5, 10, 20
P	Face value	80
c	Coupon rate	0.1
f	Coupon frequency per year	2
α	Liquidation costs	0.5
τ	Effective tax rate	0.3

In the following subsections, we investigate the term premiums for the various leverage levels and the factors to affect the value of equity, debt and firm. Especially, we focus on the effects of (1) coupon frequency, (2) liquidation costs, (3) distress costs, (4) effective tax rate, (5) asset volatility, and (6) coupon rate.

1. Term-Structure of Credit Premiums

Figure 4 plots the term premiums for the different leverage levels and various frequencies

of coupon payments. The face value of debt is assumed to be 40, 60 and 80 in each subplot, respectively. The credit spreads of debt whose contractual coupons are paid once, 2 times, 4 times and 12 times a year are plotted. As shown in the previous section, the frequent coupons play a role to prevent equityholders from behaving strategically. These figures also indicate the precautionary effect of coupon payments. Every figure shows that the spreads go down as the frequency increases. The sensitivity of spread is different depending on maturity and leverage. For high leverage level, the short-term debt's spreads are more sensitive to the coupon frequency than long-term or intermediate-term debts' spreads. For the intermediate leverage firm, the spreads of debt with 3 to 5 years to maturity change more than those of other maturities. These results are related with the expected period remained to bankruptcy.

The overall shapes of term premium are consistent with Leland and Toft's results. For high leverage levels, spreads are high, but decrease as debt maturity increase. For intermediate leverage levels, spreads are humped and the spread increase with debt maturity for low leverage levels.

[FIGURE 4]

2. The Effects of Various Parameters

A. Frequency of Coupon Payments

Figure 5 plots the values of equity, debt, firm and credit spreads as a function of coupon frequency. These figures make sure the effects of coupon frequency. While equity values decrease as the frequency increases, debt and firm values increase with the frequency. This is because the coupons play an important role of precautionary measures not to slack the firm value to fall beyond the bankruptcy boundary further. These effects have been already discussed above.

[FIGURE 5]

B. Liquidation Costs

Figure 6 plots the effects of liquidation costs. As expected, the debt and firm values decrease as the liquidation costs increase. The equity values increase with the liquidation costs whereas liquidation costs have no effects on the equity in the Leland model. The liquidation cost is the most important factor to enable the equityholders to exploit the opportunity of strategic debt services. The equity value thus increases and debt value decreases as shown in these figures. The debt values decrease more sharply than those of the Leland model as a consequence of debtors' strategic behavior. This phenomenon is also shown in table 3.

[FIGURE 6]

C. Distress Costs

Distress costs occurred when the firm stays in bankruptcy have influence on the equityholders to become reluctant to declare bankruptcy. Thus the distress costs have effect on reducing equityholders' strategic debt services. On the other hand, since distress costs reduce the asset value while in the bankruptcy state, the residual value shared by debthoders at the liquidation date decreases as the distress costs increase.

Figure 7 shows the two conflicting influences of distress costs. At the low level of distress costs, equityholders' benefit of refusing the contractual coupon payments is greater than the harm from the distress costs. Consequently, equityholders continue to behave strategically and equity value decreases sharply as distress costs increase. In that case, the debt values decrease with the distress costs because the negative effects exceed the positive one. If the distress costs reach high level, however, the equityhodlers' strategic behavior dwindles as the distress costs

increase. According to this phenomenon, the debt and firm values increase when the distress costs are high. It is interesting that the firm value increases even though the distress costs get greater as shown in panel C.

[FIGURE 7]

D. Tax Rates

The influence of tax benefit is analogous to the positive effect of distress costs. If debtors declare bankruptcy and refuse to pay the contractual interest, the tax benefit will be lost. The level of tax rate indicates the amounts of benefit that will be lost if the bankruptcy is declared. Thus, the possibility of bankruptcy will become lower as the tax rate increase and the debt value as well as equity increases with tax rate. Figure 8 shows these effects of tax benefit.

[FIGURE 8]

E. Asset Volatility

It is straightforward to guess the influence of asset volatility. As expected, the equity value increases and the debt value decreases as the asset volatility increases. The asset substitution effects documented in many articles are also presented in our model. These results are plotted in Figure 9.

[FIGURE 9]

F. Coupon Rates

Figure 10 plots the values as a function of coupon rate for different debt maturities. The equity value decreases with coupon rate since the contractual obligations of equityholders increase. On the other hand, the debt values show distinctly humped shape: debt increases with coupon rate before the coupon rate reaches up to around 10 % but decreases with coupon rates at the high level of coupon rate. These results come from the high default probability of debt with high coupon rate. Similar to the debt, firm values have humped shape and there is a coupon rate which maximize the firm value. These figures give us intuitions of optimal leverage ratio of the firm. While the debt values are humped, the credit spreads increase monotonously as the coupon rate increase.

[FIGURE 10]

VI. Conclusion

This paper develops a model of liquidation decision of creditors and methods to value corporate debt. When the liquidation is costly, bankruptcy does not necessarily lead to the liquidation since immediate liquidation is not optimal to the creditors. In our model, the creditors as well as debtors play an important role to make a liquidation decision while the firm stays in bankruptcy. Unlike the existing literature that models the Chapter 11 of U.S. Bankruptcy Code, this article focuses on the effect of creditors' liquidation decision.

Since we presumed the debt maturity is finite, the bankruptcy and liquidation boundaries are time-dependent and the closed-form solutions do not exist. We suggested the valuation method using finite difference methods to solve the PDE through backward induction. The bankruptcy boundary and recovery rate are determined endogenously while in the backward induction procedure. Accordingly, finding a bankruptcy boundary which maximizes the equity value does not necessary.

The Leland model is a special case of ours. The main feature distinguishing the Leland model from ours is that liquidation is immediate and debt maturity is infinite in the Leland model. We exhibited that the solutions obtained by our numerical method converge to the analytic value of Leland under the assumptions of infinite maturity and immediate liquidation as the grid gets finer. Additionally, we compared our model to the immediate liquidation model applied to the debt with finite maturity and discrete coupons. Even though the liquidation is costless, it might be that not to proceed with liquidation as soon as the bankruptcy is declared is profitable to the debtholders if the tax benefit of interest payment exists. Therefore, we found that the tax benefits would enable borrowers to behave strategically if it were not for liquidation costs.

We plotted the obtained credit spreads as a function of debt maturity. The term structure of credit spreads show similar patterns observed in Leland and Toft. Finally, we examined the effects of various parameters that influence the values of equity, debt, firm and credit spreads. We can confirm that (1) the coupons have a precautionary effect on bankruptcy, (2) liquidation costs deepen the equityholders' profit exploited by strategic debt service, (3) distress costs have two conflicting effects on the debt value: the reluctance to bankrupt of equityhoders and the firm value reduction, (4) the tax benefit contributes the increase of equity, debt and firm values, (5) asset substitution effects are exhibited, and (6) optimal coupon rate that maximizes the firm value is implied.

Our model makes three contributions that are distinct from the previous literatures:

First, the creditors' behavior in the liquidation procedure is modeled. Up to now, the literature focuses more on modeling the bankruptcy boundary and equityholders' decision. This article assumes that debtholders can be actively involved in the liquidation procedure. Thus, the effect of creditors' decision on the debtors' behavior can be analyzed in our model.

Second, the debt and equity values at the moment when bankruptcy is declared are determined endogenously. On the other hand, the renegotiation between debtors and creditors is given exogenously in the previous literature. In Fan and Sundaresan (2000) or Anderson and Sundaresan (1996), for example, the debt and equity values at the bankruptcy state are determined by Nash equilibrium considering the fixed negotiation powers of both creditors and

debtors.

Third, the numerical methods treating the post-bankruptcy procedures are suggested. The path-dependency of equity and debt values can be solved by executing finite difference methods iteratively at every coupon payment date. These methods using finite differences are suitable for the model that incorporates the structural changes in the post-bankruptcy state. However, the shortcoming of this method is that the computational burdens increase almost linearly with coupon frequency.

Appendix A:

$a(\cdot)$, $b(\cdot)$, $c(\cdot)$, and $d(\cdot)$ in equations (16) and (17) are given as follows:

In the liquid state,

$$\begin{cases} a(i) = \frac{1}{2}\left[(\sigma i)^2 - (r-\delta)i\right]\Delta t \\ b(i) = -\left[(\sigma i)^2 + r\right]\Delta t \\ c(i) = \frac{1}{2}\left[(\sigma i)^2 + (r-\delta)i\right]\Delta t \\ d(i) = \delta i \Delta V \Delta t \end{cases} \tag{A1}$$

In the bankruptcy state,

$$\begin{cases} a(i) = \frac{1}{2}\left[(\sigma i)^2 - (r-w)i\right]\Delta t \\ b(i) = -\left[(\sigma i)^2 + r\right]\Delta t \\ c(i) = \frac{1}{2}\left[(\sigma i)^2 + (r-w)i\right]\Delta t \\ d(i) = 0 \end{cases} \tag{A2}$$

Appendix B:

Leland (1994) derives the following analytic formulae for equity and debt:

$$D = \frac{C}{r}\left[1 - \left(\frac{C}{V}\right)^X k\right], \tag{A3}$$

$$E = V - (1-\tau)\frac{C}{r}\left[1 - \left(\frac{C}{V}\right)^X m\right], \tag{A4}$$

where

$$X = \frac{1}{\sigma^2}\left[\left(r-\delta-\frac{\sigma^2}{2}\right)+\sqrt{\left(r-\delta-\frac{\sigma^2}{2}\right)^2+2\sigma^2 r}\right], \tag{A5}$$

$$m = \frac{1}{1+X}\left[\frac{(1-\tau)X}{r(1+X)}\right]^X, \tag{A6}$$

$$k = \left[1+X-(1-\alpha)(1-\tau)X\right]m. \tag{A7}$$

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Table 1. Equity and Debt with Infinite Maturity

This table compares the equity and debt values computed by the finite difference methods with those of analytic solution suggested by Leland (1994). The absolute errors (numerical estimate-analytic value) and relative errors (absolute error/analytic value) are reported. Model parameters are $V_0 = 100$, $r = 5\%$, $\delta = 3\%$, and $\alpha = 0.5$. The results for various volatility, tax rate, and leverage ratio are presented.

σ	τ	C	Value	Leland	Error(Abs)	Error(Rel)
Panel A. Equity Values						
0.1	0.15	3	49.1183	49.1179	0.0004	0.0009%
		4	32.6617	32.6622	-0.0005	-0.0016%
		5	17.5220	17.5261	-0.0042	-0.0238%
	0.35	3	61.0240	61.0236	0.0005	0.0008%
		4	48.1326	48.1324	0.0002	0.0003%
		5	35.5058	35.5052	0.0007	0.0019%
0.2	0.15	3	52.1209	52.1395	-0.0186	-0.0356%
		4	38.5625	38.5969	-0.0344	-0.0891%
		5	26.6764	26.7348	-0.0585	-0.2187%
	0.35	3	62.5649	62.5709	-0.0059	-0.0095%
		4	51.2851	51.3008	-0.0157	-0.0306%
		5	40.8417	40.8717	-0.0299	-0.0732%
Panel B. Debt Values						
0.1	0.15	3	59.4898	59.4627	0.0271	0.0456%
		4	77.1356	76.9811	0.1545	0.2007%
		5	89.0748	88.4838	0.5909	0.6679%
	0.35	3	59.8519	59.8414	0.0105	0.0175%
		4	79.0428	79.1087	-0.0658	-0.0832%
		5	96.6473	96.5999	0.0474	0.0490%
0.2	0.15	3	52.7739	52.9486	-0.1747	-0.3300%
		4	65.1095	65.1830	-0.0735	-0.1127%
		5	73.8179	73.6428	0.1752	0.2379%
	0.35	3	54.9937	55.0040	-0.0103	-0.0188%
		4	69.4893	69.5020	-0.0127	-0.0183%
		5	81.3082	81.3257	-0.0175	-0.0215%

Table 2. Values of Equity and Debt with Finite Maturity and Discrete Coupons

This table reports the equity and debt values for the extended Leland model. In contrast to the Leland model, the debt's maturity is finite and the coupons are paid discretely. Model parameters are $V_0 = 100$, $r = 5\%$, $\delta = 3\%$, $\alpha = 0.5$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 5\%$. The results for various time-to-maturities, frequency of coupon payment, and face value of debt are presented.

Face Value	TTM	Freq	Equity	Debt	Firm
60	5	1	46.1809	55.5553	101.7362
		4	45.8737	56.0189	101.8926
		12	45.8052	56.1153	101.9205
	10	1	51.1121	53.3950	104.5071
		4	50.6400	54.1481	104.7881
		12	50.5346	54.3040	104.8386
	20	1	56.9434	52.4692	109.4125
		4	56.3198	53.4799	109.7997
		12	56.1805	53.7014	109.8819
80	5	1	31.7030	65.0333	96.7363
		4	31.0179	66.6853	97.7032
		12	30.8671	66.9187	97.7858
	10	1	38.1838	63.9538	102.1375
		4	37.3681	65.5988	102.9669
		12	37.1869	65.9002	103.0871
	20	1	45.1740	64.4772	109.6512
		4	44.2356	66.1950	110.4305
		12	44.0261	66.5628	110.5889
100	5	1	20.6775	68.9939	89.6714
		4	19.4680	71.5169	90.9848
		12	19.2013	71.9289	91.1302
	10	1	27.5913	70.5947	98.1860
		4	26.3520	73.0777	99.4297
		12	26.0770	73.5242	99.6012
	20	1	34.9093	73.4067	108.3160
		4	33.6082	75.8284	109.4365
		12	33.3185	76.3012	109.6197

Table 3. Comparison between the Strategic Behavior Model and Immediate Liquidation Model

This table reports the equity, debt, and firm values for our model and the differences from the extended Leland model which assumes immediate liquidation. The results for no liquidation costs are presented in Panel A and those for high liquidation cost, that is $\alpha = 0.5$, are presented in Panel B. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 3\%$, $w = 0$, $\sigma = 0.2$, and $c = 5\%$.

Tax Rate	TTM	Freq	Values			Difference		
			Equity	Debt	Firm	Equity	Debt	Firm
Panel A. Liquidation Costs = 0								
0	5	1	26.4278	73.5723	100.0000	0.0000	0.0000	0.0000
		4	25.3462	74.6538	100.0000	0.0000	0.0000	0.0000
		12	25.1122	74.8879	100.0000	0.0000	0.0000	0.0000
	10	1	29.0063	70.9938	100.0001	0.0000	0.0000	0.0000
		4	27.6791	72.3210	100.0001	0.0000	0.0000	0.0000
		12	27.3869	72.6132	100.0001	0.0000	0.0000	0.0000
	20	1	31.0727	68.9275	100.0002	0.0000	0.0000	0.0000
		4	29.5040	70.4962	100.0002	0.0000	0.0000	0.0000
		12	29.1547	70.8454	100.0002	0.0000	0.0000	0.0000
0.35	5	1	31.7061	73.6667	105.3728	0.0031	-0.0504	-0.0472
		4	31.0195	74.7407	105.7602	0.0016	-0.0315	-0.0299
		12	30.8679	74.9836	105.8516	0.0009	-0.0181	-0.0173
	10	1	38.1937	71.2813	109.4750	0.0099	-0.1224	-0.1125
		4	37.3729	72.6045	109.9774	0.0047	-0.0763	-0.0715
		12	37.1895	72.9045	110.0940	0.0026	-0.0598	-0.0572
	20	1	45.1965	69.6223	114.8188	0.0225	-0.2562	-0.2337
		4	44.2464	71.2033	115.4497	0.0108	-0.1785	-0.1677
		12	44.0333	71.5626	115.5959	0.0072	-0.1588	-0.1516
Panel B. Liquidation Costs = 0.5								
0.35	5	1	32.5483	61.5453	94.0935	0.8453	-3.4881	-2.6428
		4	31.9759	62.7920	94.7680	0.9580	-3.8932	-2.9352
		12	31.8501	63.0679	94.9180	0.9830	-3.8508	-2.8677
	10	1	40.5519	56.8701	97.4220	2.3681	-7.0836	-4.7156
		4	39.8602	58.1841	98.0443	2.4921	-7.4147	-4.9226
		12	39.7060	58.4791	98.1851	2.5191	-7.4211	-4.9020
	20	1	50.3027	52.7391	103.0417	5.1287	-11.7381	-6.6094
		4	49.4587	54.0802	103.5389	5.2231	-12.1148	-6.8916
		12	49.2687	54.4134	103.6821	5.2426	-12.1494	-6.9068

Figure 1. Convergence of Equity and Debt values to the Analytic Solution

This figure plots the relative errors between the numerical estimates and analytic values of equity and debt. The parameter values are $V_0 = 100$, $C = 3$, $r = 5\%$, $\delta = 3\%$, $\alpha = 0.5$, $\sigma = 0.2$, and $\tau = 0.35$. The axis-X represents the number of steps between 0 and 500, minimum and maximum of the asset value. And the axis-Y represents the relative errors of the equity and debt.

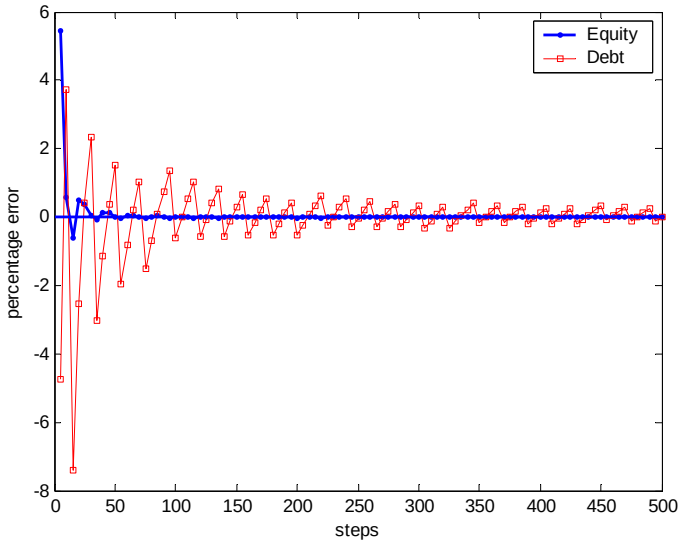

Figure 2. The Values of Equity, Debt and Firm as a Function of Debt's Time-to-Maturity

These figures plot the equity, debt and firm values for various leverage levels. The parameter values are $V_0 = 100$, $r = 5\%$, $\delta = 3\%$, $\alpha = 0.5$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 5\%$. We assume that the coupons are paid quarterly. The axis-X represents the debt's maturity and the axis-Y represents the each values. The marker shapes denote the following different face values of debt: circle (P=60), square (P=80), and triangle (P=100).

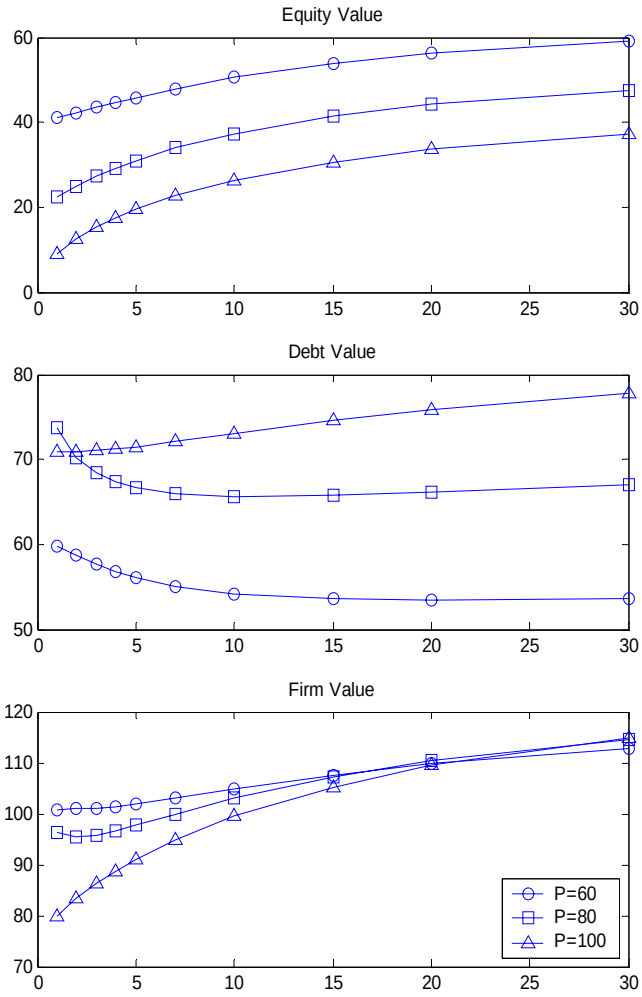

Figure 3. Differences of Immediate Liquidation Model and Debtholders' Decision Model

These figures plot the differences of the equity, debt and firm values between the values of our model and Leland model in the case that liquidation is costless and tax rate is 35%. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 3\%$, $w = 0$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 5\%$. The axis-X represents the coupon frequency and the axis-Y represents the each difference values. The marker shapes denote the following different time to maturity of debt: circle (5 years), square (10 years), and triangle (20 years)

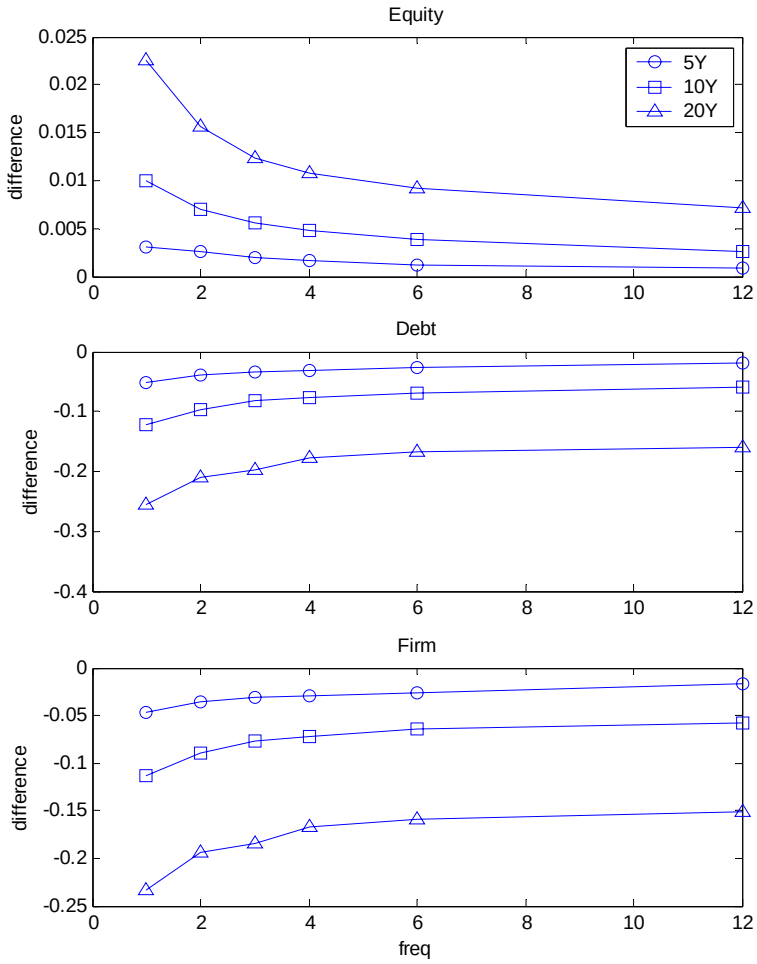

Figure 4. Term Structure of Credit Premium

These figures plot the credit spreads as a function of debt maturity for firms with various leverage levels. Model parameters are $V_0 = 100$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $w = 5\%$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 10\%$. The marker shapes denote the coupon frequency: circle (annually paid), square (semiannually paid), triangle (quarterly paid), and cross (monthly paid).

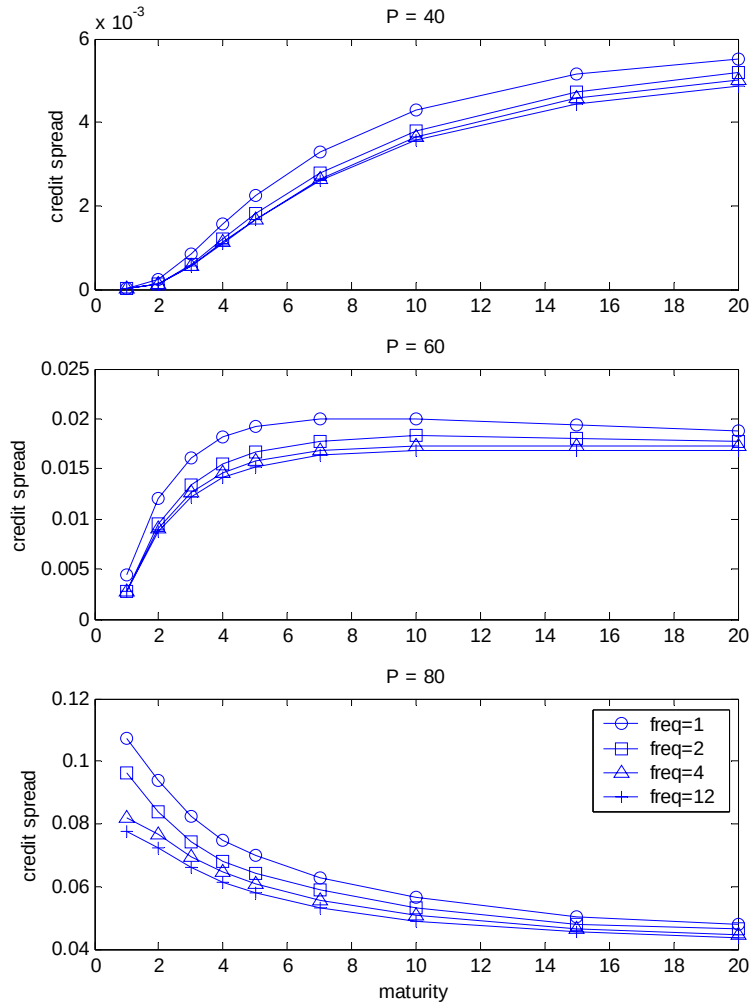

Figure 5. Effects of Coupon Frequency on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of coupon frequency for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $w = 5\%$, $\sigma = 0.2$, $\tau = 0.35$, and $c = 10\%$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).

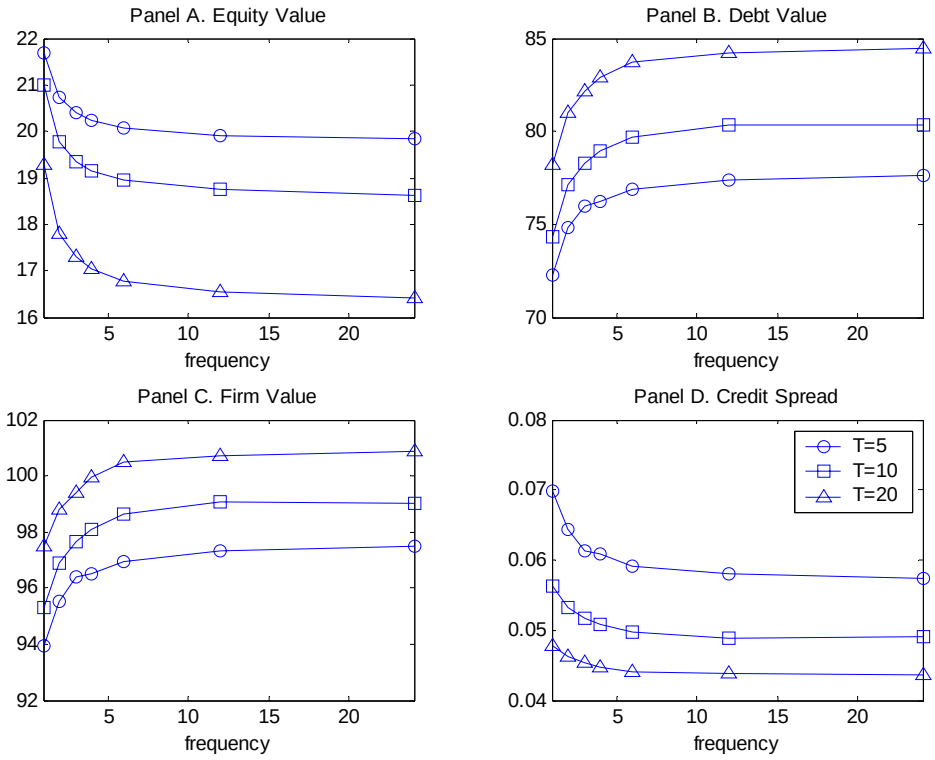

Figure 6. Effects of Liquidation Costs on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of liquidation costs for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $w = 5\%$, $\sigma = 0.2$, $\tau = 0.35$, $f = 2$ and $c = 10\%$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).

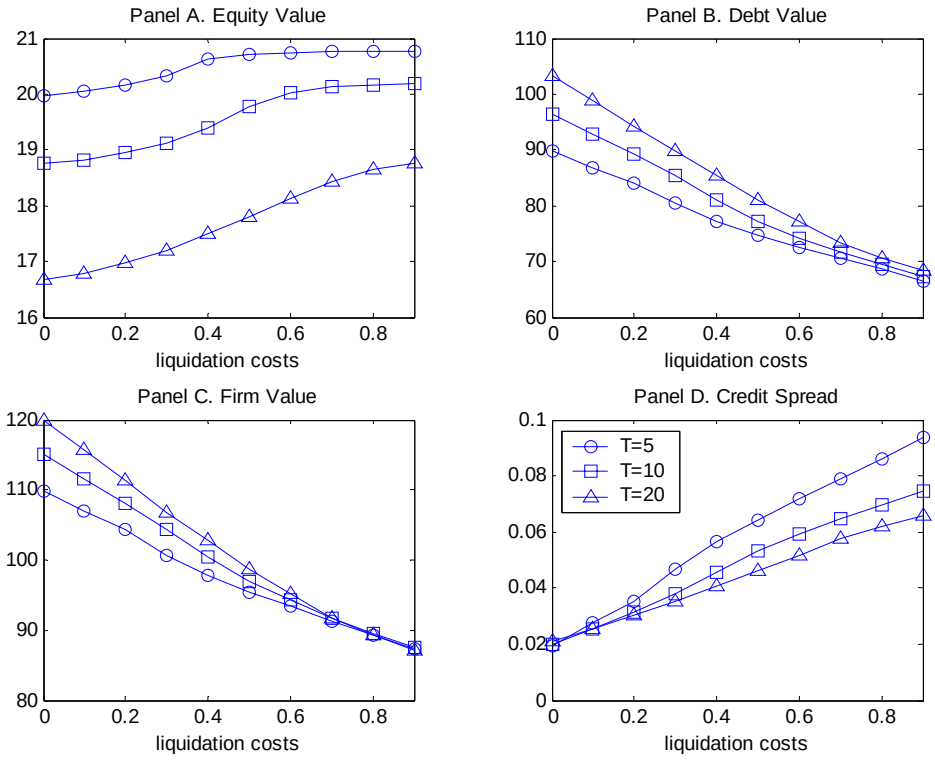

Figure 7. Effects of Distress Costs on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of distress costs for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $\sigma = 0.2$, $\tau = 0.35$, $f = 2$ and $c = 10\%$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).

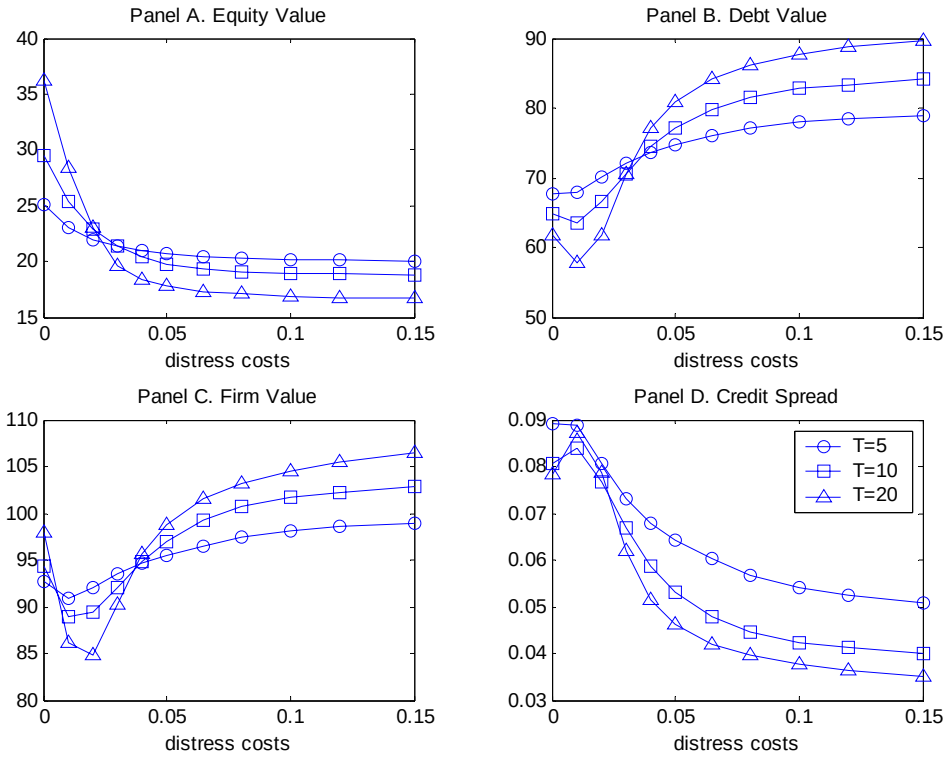

Figure 8. Effects of Tax Rates on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of tax rates for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $w = 5\%$, $\sigma = 0.2$, $f = 2$ and $c = 10\%$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).

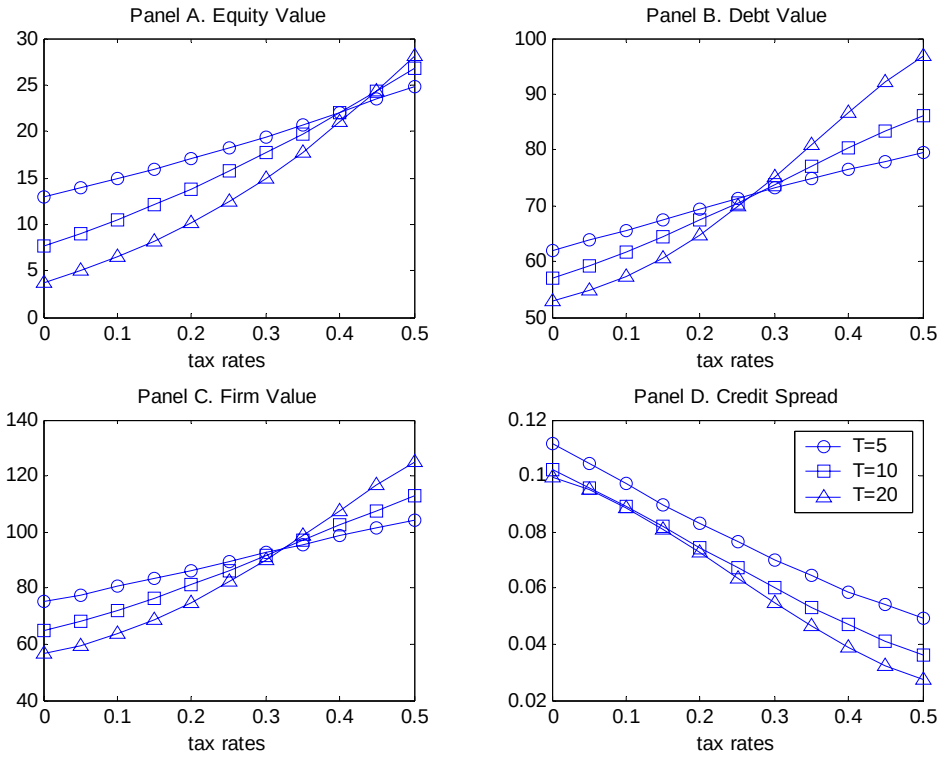

Figure 9. Effects of Asset Volatility on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of asset volatiliy for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $w = 5\%$, $\tau = 0.35$, $f = 2$ and $c = 10\%$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).

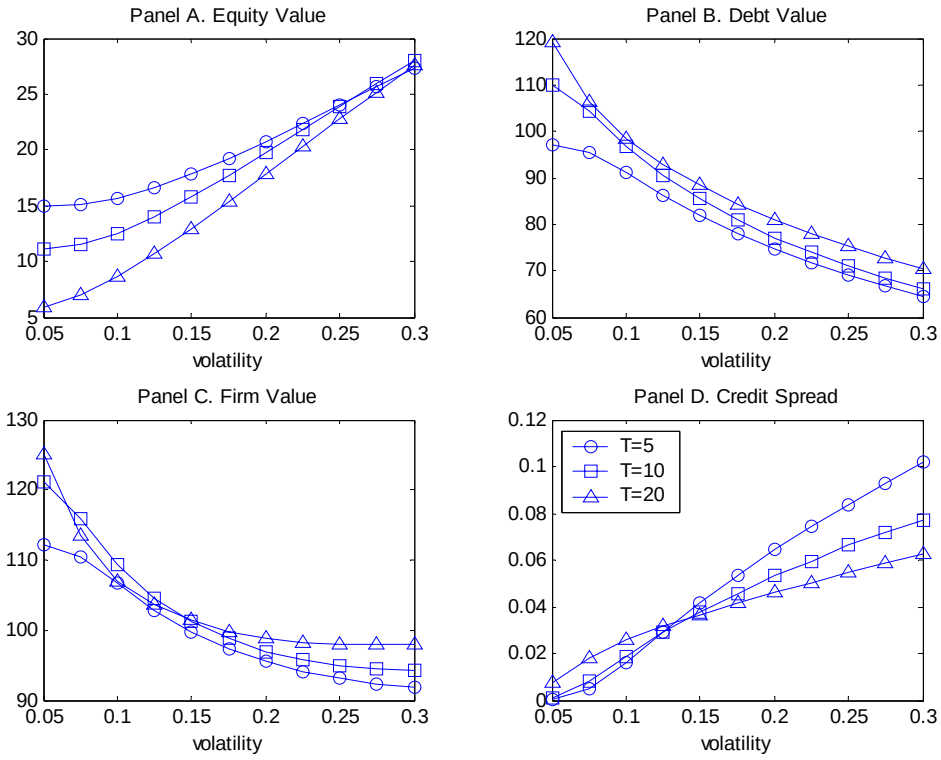

Figure 10. Effects of Coupon Rates on the Equity, Debt, Firm Value and Credit Premium

These figures plot the equity, debt, firm values and credit premium as a function of coupon rates for various debt maturities. Model parameters are $V_0 = 100$, $P = 80$, $r = 5\%$, $\delta = 1\%$, $\alpha = 0.5$, $w = 5\%$, $\sigma = 0.2$, $f = 2$ and $\tau = 0.35$. The marker shapes denote the debt maturity: circle (5 yrs), square (10 yrs), and triangle (20 yrs).