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# SENSING/ACTUATING CHARACTERISTICS OF PIEZO LAMINATED COMPOSITE PLATES USING LAYER-WISE DISPLACEMENT THEORY

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## I. Introduction

Recently the requirements for high performance in a modern aircraft or a spacecraft have led to light weight structures, especially composite structures. However these light weight structures are prone to vibrate excessively, so there have been exuberant study to reduce structural vibration level. Among these efforts the use of piezoelectric materials as distributed sensors and actuators have been drawn attention because they can be easily incorporated with conventional composite structures.

In the process of constructing actively controlled structures, the analysis tools for predicting dynamic and sensing/actuating characteristics of the structures are essential. So far the developed finite element codes have been based on conventional plate models[1] or three dimensional solid models [2]. However these finite element formulations have some limitations due to their improper modeling of in-plane displacements or demanding too many nodal degrees of freedom.

The purpose of this study is to develop a finite element formulation for piezo-laminated composite plates using the layer-wise displacement plate theory (LWPT) proposed by Reddy[3]. LWPT allows layer-wise representation of in-plane displacements, and different strains are predicted in different layers. Piezoelectric actuators are basically induced strain actuators so that the prediction of exact modal strains are essential for accurate estimations of actuating characteristics of piezo actuators. In the same manner exact modeling of modal strains is important for estimating sensing capability of piezo sensors.

## II. Finite Element Formulation

For piezoelectric materials manufactured in a plate

form, the equations governing these material properties can be written as

$$\{\sigma\} = [\bar{Q}]\{\epsilon\} - E_3[\hat{Q}][d]^T \quad (1)$$

$$D_3 = [d][\tilde{Q}]\{\epsilon\} \quad (2)$$

where Eq. (1) is for piezoelectric actuators and Eq. (2) is for piezoelectric sensors. Furthermore no external applied electric field is assumed for piezo sensors. The stress  $\{\sigma\}$  and strain  $\{\epsilon\}$  are expressed in the geometric coordinate and transformed stiffnesses can be given as follows:

$$[\bar{Q}] = [R]^{-1}[Q][R]^{-T} \quad (3)$$

$$[\hat{Q}] = [R]^{-1}[Q] \quad (4)$$

$$[\tilde{Q}] = [Q][R]^{-T} \quad (5)$$

The displacement fields can be expressed as follows:

$$u(x, y, z, t) = u_0(x, y, t) + \sum_{j=1}^{N+1} U_j(x, y, t)\Phi^j(z) \quad (6)$$

$$v(x, y, z, t) = v_0(x, y, t) + \sum_{j=1}^{N+1} V_j(x, y, t)\Phi^j(z) \quad (7)$$

$$w(x, y, z, t) = w_0(x, y, t) \quad (8)$$

where  $u_0, v_0, w_0$  are displacements in the mid plane,  $U_j, V_j$  are in-plane displacement at the  $j$ -th interface, and  $\Phi^j(z)$  is global interpolation function through the thickness.

Using displacement fields equations (6), (7) and (8), and constitutive equation (1) in the Hamilton's principle, the finite element equations of motion can be obtained as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}^{ext} + \sum_{L=1}^{n_{act}} \hat{\mathbf{f}}^L V^L \quad (9)$$

where  $\hat{\mathbf{f}}^L$  is the induced force due to unit applied voltage to  $L$ -th actuator. In the same manner the sensor equations can be obtained as follows:

$$q^M(t) = \tilde{\mathbf{c}}^M \mathbf{u} \quad (10)$$

where  $j$ -th component of  $\tilde{\mathbf{c}}^M$  is the induced charge of  $M$ -th sensor due to  $j$ -th unit nodal displacement of

**II.** Equations (9) and (10) can be transformed to modal space equations as follows:

$$\ddot{\eta}_j + 2\zeta_j \omega_j \dot{\eta}_j + \omega_j^2 \eta_j = \sum_{L=1}^{n_{act}} B_{jL} V_L \quad (11)$$

$$q^M(t) = \sum_{j=1}^m C_{mj} \eta_j \quad (12)$$

### III. Results and Conclusions

A slender cantilevered composite plate with symmetrically fully covered piezo ceramics as shown in Fig. 1 has been analyzed. The material properties of composite lamina are as follows:  
 $E_1 = 130 GPa, E_2 = 9.6 GPa, G_{12} = G_{13} = 4.8 GPa$  (13)  
 $G_{23} = 3.2 GPa, \nu_{12} = 0.31, \rho = 1570 \text{ Kg/m}^3$

Also, the piezo ceramic has the following properties.  
 $E = 59 GPa, \nu = 0.34, \rho = 7400 \text{ Kg/m}^3$  (14)  
 $d_{31} = d_{32} = -260 \text{ pC/N}$

The stacking sequence of laminated plate is  $[0/\pm\theta/90]_s$ , the thickness of each lamina is 0.1 mm, and the thickness of piezo ceramic is 0.2 mm. For various  $\theta$  values the coefficients of governing equations (11) and (12) are obtained. Here, the frequencies  $\omega_j$  and the unit modal control force  $B_{j1}$  for the first three modes are summarized in Fig. 2. In this analysis 9-node (6X1) plate elements were adopted and subspace iteration method was used. As  $\theta$  increases, frequencies become smaller.  $B_{j1}$  has its minimum value near  $\theta = 40^\circ$ . The results based on LWPT have smaller value than those based on FSDT. As  $\theta$  increases, the differences become larger. The closed loop performances can be more exactly predicted using LWPT. For a simple direct velocity feedback with velocity gain of  $G = 10,000 \text{ Volt/Ampere}$ , the real part of the first mode pole is predicted as -3.36 by LWPT modeling, whereas -3.46 by FSDT. In the case of the second mode, -36.0 by LWPT, -37.7 by FSDT is predicted respectively.

In this study a refined finite element analysis based on the layer-wise displacement modeling is proposed. The real parts of the closed loop pole which can be considered as enhanced damping, are analyzed by both FSDT modeling and LWPT modeling. The predicted values based on FSDT are 3 - 4.7 % higher than those based on LWPT for this slender cantilevered plate. LWPT will be very useful for modeling more complex geometry such as thickness

variations and bonding layers. In these cases the proposed finite element formulation can be expected to have strong advantages than conventional formulations for analyzing controlled system performance.

### References

1. Hwang, W.-S., and Park, H.C., "Finite Element Modeling of Piezoelectric Sensors and Actuators," *AIAA J.*, 31(5) 930-937.
2. Ha, S.K., Keilers, C., and Chang, F.K., "Finite Element Analysis of Composite Structures Containing Distributed Piezoelectric Sensors and Actuators," *AIAA J.*, 30(3) 772-780.
3. Reddy, J.N., E. J. Barbero, and Teply, J.L., "A Plate Bending Element Based on a Generalized Laminate Plate Theory," *Int. J. numer. Meth. Engng.*, 28, 2275-2292.

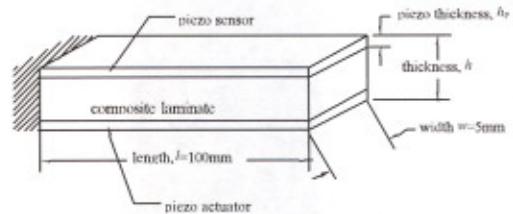


Fig. 1 Configurations.

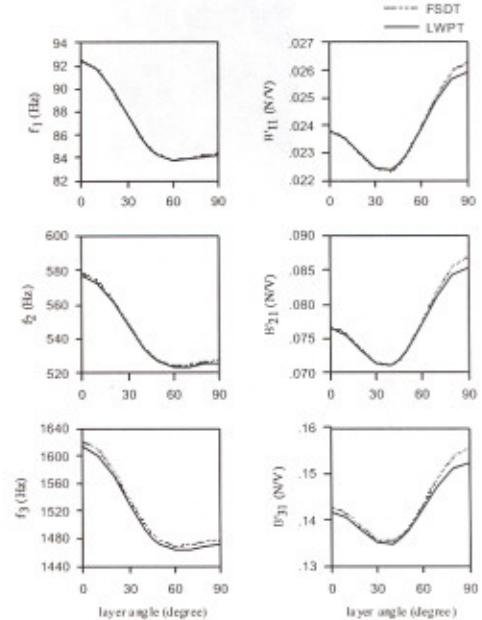


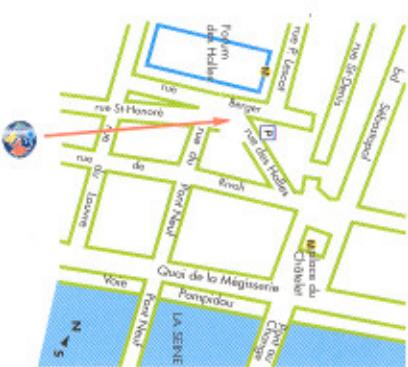
Fig. 2 Frequencies and modal control forces.

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