On the Computation of the Reliability of the Aerodynamic Analysis using the Moment Method

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Abstract: In this study, the reliability analysis was performed for the aerodynamic analysis. Among various reliability analysis methods, the moment method was used, and to parameterize an airfoil shape the PARSEC function was applied. The accuracy of the reliability analysis was compared with other method and it was found that the moment method predicted the probability accurately.

Reliability analysis computes how much the system is reliable using statistical methods. Reliability analysis becomes popular because it reflects more physical phenomena which possess a lot of uncertainties. Reliability based design optimization(RBDO) employs reliability analysis to find optimum values considering uncertainties of design variables.

Popular reliability analysis methods are MCS(Monte Carlo simulation), FORM(first order reliability analysis) and moment methods. MCS computes reliability with a repeated random sampling, and it requires a large number of computations. FORM is a method that approximates the limit state function as a linear equation and finds the most probable point(MPP) that is the nearest point to satisfy the limit state function. Moment methods directly find the probability distribution function and it doesn’t require sensitivity. In this study, Pearson system is used. Reliability analysis is mostly used in the structural analysis, and its use in the aerodynamic analysis is recent. Ahn et al (2005) conducted RBDO of a 3D wing using the FORM. In this study, reliability analysis was performed for the aerodynamic analysis using a moment method.

A parallelized flow solver for the Euler equations is used in the aerodynamic analysis. The flow region is discretized spatially by the finite volume method and the Roe’s FDS and TVD are used in calculating numerical flux. The multigrid method with mesh sequencing is used to accelerate the convergence of the steady calculation.

For the numerical test, the reliability analysis was applied to the two-dimensional aerodynamic analysis. In the aerodynamic optimization, the shape of an airfoil is approximated with a shape function. During the approximation, the shape function uses some parameters and they determine the shape of airfoil. Usually an optimizer finds parameters or design variables which maximize the aerodynamic performance. Among many shape functions, Hicks-Henne and NURBS are generally used. Recently PARSEC was proposed by Sobieczky (1998). Differently from other shape functions, the parameters of the PARSEC function directly control the shape of airfoil. Figure 1 shows those parameters.

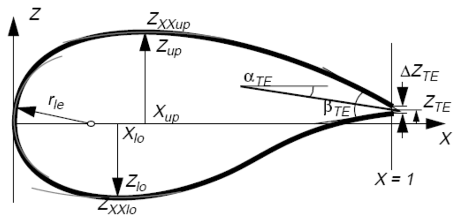

Figure 1. Definition of PARSEC parameters, Sobieczky (1998)

RAE2822 was used in the aerodynamic analysis. Mach number was 0.73 and angle of attack was 2.78 degree. The limit state function was assigned for the lift coefficient to be greater than a reference value. The reliability analysis computes the probability of this function to be violated.

At first, 4 parameters were tested for different DOE(design of experiment) levels. Parameters were assumed to have normal distributions, and their means and standard deviations are shown in Table 1. Test results are shown in Table 2 and as the DOE level increases, the probability of failure converged. The predicted probability is similar to the MCS value and it is within the confidence interval of the MCS.

Secondly, 9 parameters were tested for the same airfoil. Table 3 and 4 show the conditions and results. In this case, the DOE level is 3. As shown in Table 4, the probability of failure is lower than the MCS value and it is out of the confidence interval of the MCS. It is guessed that the increased parameters resulted in the decrease of the accuracy.

Table 1. Mean and standard deviation of 4 parameters

	Xup	Zup	ZXXup	rle
μ	0.42896	0.06301	-0.42818	0.008187
σ	0.01	0.001	0.01	0.001

Table 2. Probability of failure for 4 parameters

Level	Pearson	MCS
3	0.1032	0.107 (0.0878,0.1262)
5	0.0957	
7	0.0971	

Table 3. Mean and standard deviation of 9 parameters

	Xup	Zup	ZXXup	rle
μ	0.42896	0.06301	-0.42818	0.008187
σ	0.01	0.001	0.01	0.001
	αTE	βTE	Xlow	Zlow
μ	-0.12720	-0.15400	0.34201	-0.05921
σ	0.01	0.01	0.01	0.001
	ZXXlow			
μ	0.7			
σ	0.01			

Table 4. Probability of failure for 9 parameters

	Pearson	MCS
Pf	0.0206	0.0331 (0.0296,0.0366)

The reliability of aerodynamic analysis of an airfoil was computed using the Pearson system. The probability of failure of the lift coefficient was computed and the prediction was compared with the MCS. Comparing with the MCS, this method predicted the probability of failure accurately. It is expected that the Pearson system can be used to calculate the reliability of the aerodynamic analysis and the reliability based design optimization.

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