Multi-Point Reliability based Flexible Wing Shape Design Optimization

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ABSTRACT

1. Introduction

The aircraft shape design optimization methods with numerical computations have been used in a lot of fields such as aerodynamics, structures and so on. Conventional deterministic optimization methods are aimed to search the shape or the operating conditions that have the optimum performance when the specific objective function and several constraints are decided. However, sometimes the performance degradation or product failure may happen due to the small changes of the optimum shape or operating conditions. To reduce these unpredictable phenomena, the reliability-based design optimization(RBDO) was proposed. RBDO is one of the design methods that minimize the failure of the product because of uncertain changes of operating conditions or shapes. RBDO considers uncertain effects through transforming the amount of the constraint violations into the probability of failure and it needs many additional efforts to predict the probability of failure compared with the deterministic approaches. Especially, for the aerodynamic shape optimization that requires many computing costs in the flow analysis, it is very difficult to perform RBDO. In this study, multi-point RBDO is performed for a flexible wing with a viscous 3-dimensional Navier-Stokes flow analysis and a NASTRAN based structural analysis. For more efficient design optimization, the design of experiment(DOE) and response surface modeling(RSM) methods are applied under the multi-disciplinary analysis framework with parallel clusters.

2. Reliability based Design Optimization

A general design optimization problem can be expressed as following.

$$\begin{aligned} &\min \quad f(\mathbf{x}) \\ &\text{s.t.} \;\; g(\mathbf{x}) \le \mathbf{0} \end{aligned} \tag{1}$$

This deterministic optimization problem such as Eq.(1) can be transformed into the reliability based design optimization problem as Eq.(2)

$$\begin{aligned} &\min \quad f(\mu) \\ &\text{s.t.}\ P\big(g(\mathbf{x}) \le \mathbf{0}\big) \le \Phi(-\beta) \end{aligned} \tag{2}$$

where **x** is a random design variable whose average is μ and standard deviation is σ. P is the probability that the limit state function g is smaller than 0 about the random design variables x. β is the reliability index of a cumulative probability density function(cdf). RBDO regards that a design optimization fails when the probability not to satisfy the limit state function is smaller than the target probability. To calculate the probability P, FORM(first order reliability method) that approximates the limit state function to a linear equation and find the most probable point(MPP) that is the nearest point to satisfy the limit state function to be zero on the present design point is used[1-3]. In this study, the most probable point is evaluated through the performance measure approach(PMA) as following.

$$\begin{aligned} &\min \quad g(\mathbf{u}) \\ &\textbf{s.t.}\ \|\mathbf{u}\| = \beta_t \end{aligned} \tag{3}$$

where u is a transformed variable defined as $\mathbf{u}=(\mathbf{x}-\mu)/\sigma$. In Eq.(2), constraints are stochastic constraints, not deterministic. To achieve these, RBDO consists of two stages. The first stage is the reliability analysis stage to calculate the probability of failure at the present point. The second stage is the optimization stage through the deterministic optimization method such as FDM(feasible direction method) or SQP(sequential quadratic programming). In the second stage, it is hard to calculate the sensitivity about the probabilistic constraints when the probability of failure itself is used. Therefore, to overcome this, Eq.(2) is changed into the following deterministic formulation.

$$\begin{aligned} &\mathbf{min} \quad f(\mu) \\ &\textbf{s.t.}\ g(\mathbf{x}_{\mathbf{MPP}}) < 0 \end{aligned} \tag{4}$$

x_{MPP} is the most probable point about each constraint in Eq.(4) and it is assumed that a constraint about the probability of failure is satisfied if x_{MPP} is satisfied with this deterministic constraint. If Eq.(2) changes to Eq.(4), the stochastic constraints change to the deterministic constraints, therefore the existent deterministic optimization algorithms are easily applicable.

3. Multi-disciplinary Analysis Framework

3.1 Aerodynamic analysis

For 3-dimensional compressible viscous flow analysis, A parallelized flow solver KFLOW developed at ASDI (aerodynamic simulation and design integration laboratory) of KAIST is used[4,5]. The flow region is discretized spatially by the finite volume method and the Roe's FDS and MUSCL are used in calculating numerical flux. The multigrid method with mesh sequencing is used to accelerate the convergence of steady calculation. An initial wing is ONERA M6 and the Reynolds number is 11.7×10^6.

3.2 Structural analysis

For structural analysis, a well-known FEM based structural analysis program NASTRAN is used. To model the ONERA M6 wing structure skin, spar, rib and sparcap components are used. The structural mesh comes from the CFD mesh, so it doesn't need an interpolation during data transfer between the aerodynamic analysis and the structural analysis.

3.3 Static aeroelasticity analysis framework

The static aeroelasticity is used to model the interaction between the aerodynamic forces and the structural deformations at each cell vertex. For one static aeroelastic analysis, about three or four coupled flow and structural analyses are required. This aeroelastic analysis are performed under the multi-disciplinary analysis framework. The multi-disciplinary analysis framework based on ModelCenter of Pheonix Integration provides an easy GUI interface to integrate two analysis components and can couple two components under different operating systems or different computers. In this study, the KFLOW working on the Linux PC cluster 16 CPU and the NASTRAN operating on the Windows Server PC are integrated by ModelCenter on Windows and static aeroelastic analyses are performed.

4. Design Optimization

4.1 Optimization Procedure

An aerodynamic shape design optimization which requires many computing times for one analysis is hard to apply the direct optimization approach. Therefore, for the efficient optimization the design of experiment(DOE) and the response surface modeling(RSM) are used. First, sample points to represent the design space are constructed using the full-factorial and V-Optimal methods and the aeroelastic analyses are performed at each sample point. Through this DOE procedure, the database about aerodynamic and structural properties can be obtained and it can be expressed as a polynomial equation from RSM. A conventional RSM methods have large approximation errors in case of highly nonlinear data and to reduce these approximation errors RSM with moving least square method(MLSM) is used[6]. Finally, the reliability analysis and main optimization are performed on this response surface without an additional aeroelastic analysis[7-9].

4.2 Design Variables

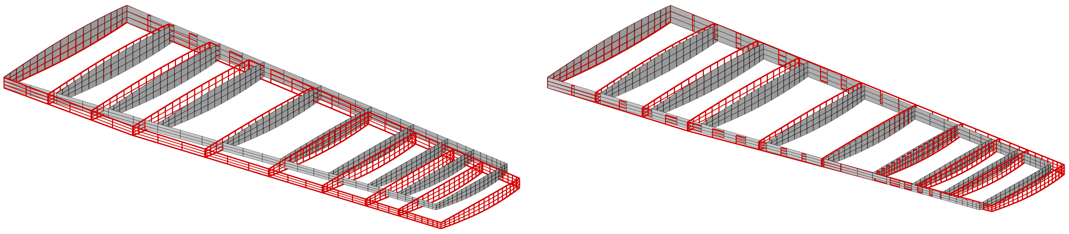
For the multi-point wing shape design optimization with RBDO, the wing planform variables related with process errors and the angle of attack related with the operating condition uncertainty is chosen as design variables. As wing planform variables, 8 design variables are used such as sweep back angle, kink position and kink ratio, taper position and taper ratio, root, kink and taper thickness ratio. Therefore, for 3-point design optimization 8 wing variables and 3 angle of attacks are chosen as design variables.

4.3 Multi-Point Design Optimization

The objective function is the weighted sum of the lift-to-drag ratio at each Mach number and constraints are set to limit the skin and rib maximum stress, maximum deflection and minimum lift coefficient. In this study, 3 point design optimization is performed – target Mach no. are 0.80, 0.82 and 0.84.

$$
\begin{array}{lll}
\text{maximize} & & \sum \omega_i \left(\dfrac{C_L}{C_D} \right)_i \\
\text{subject to} & P(& C_{L,i} < 0.12 \quad) < \Phi(-\beta_t) \\
 & P(& \sigma_{skin,i} > 160\text{MPa} \quad) < \Phi(-\beta_t) \\
 & P(& \sigma_{Rib,i} > 70 \text{ MPa} \quad) < \Phi(-\beta_t) \\
 & P(& \delta_{def,i}/b_{span} > 0.015 \quad) < \Phi(-\beta_t)
\end{array} \tag{5}
$$

Fig. 1 shows the optimized shapes about the deterministic optimiztaion(DO) and RBDO. DO gives better lift-to-drag ratio than that of RBDO but the margin of each constraint violation about DO is so small and this optimized wing is not safe relatively compared with RBDO’s.



(a) DO (b) RBDO

Fig. 1 Optimized wing compared with ONERA M6

Table 1 Optimization Result

	ONERA M6			Deterministic	RBDO
	AoA = 2.0	AoA = 2.5	AoA = 3.0		
Objective	**19.312**	**19.775**	**19.139**	**21.178**	**20.557**

5. Conclusion

In this study, the large-scaled optimization problem such as 3-dimensional aero-elasticticity based RBDO is performed very efficiently using the parallelized multi-disciplinary analysis framework. From the comparison results between the deterministic optimization and the reliability based optimization, RBDO's result gives more robust and stable wing shape.

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