PLANAR ACOUSTIC HOLOGRAPHIC RECONSTRUCTION BY USING MOVING ARRAY

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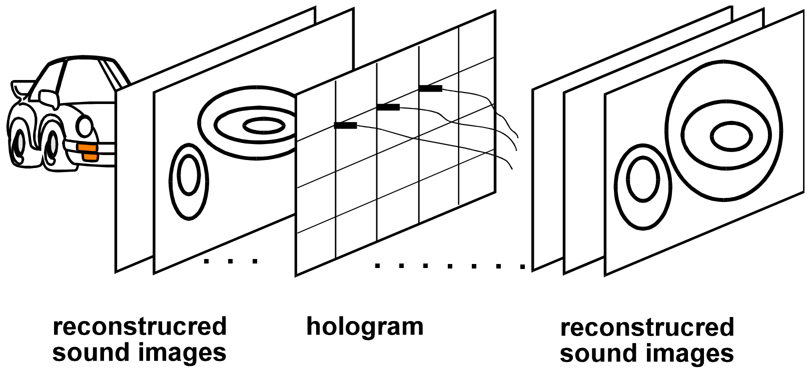
CONTENTS OF PRESENTATION

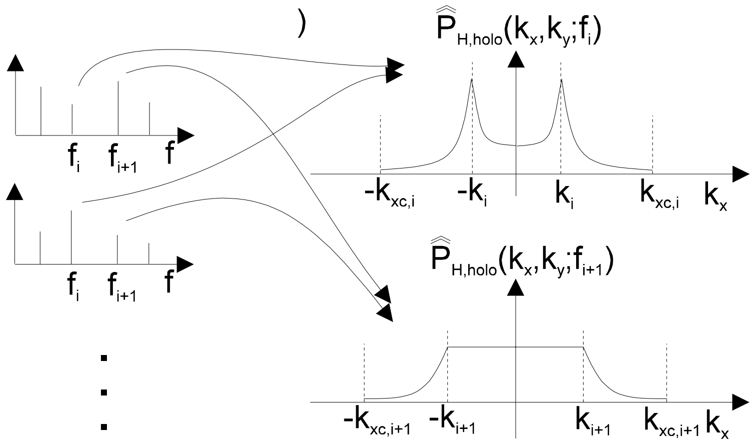
- ♦ THE BASIC DESCRIPTIONS ON HOLOGRAM
- ♦ MEASURED PRESSURE AND HOLOGRAM
- ♦ THEORETICAL BACKGROUNDS
- ♦ SIMULATION
- ♦ EXPERIMENTAL RESULTS
- ♦ CONCLUDING REMARKS

* THEORETICAL BACKGROUND

Plane Wave Decomposition ***Using 2 Dimensional Spatial Fourier Transform***



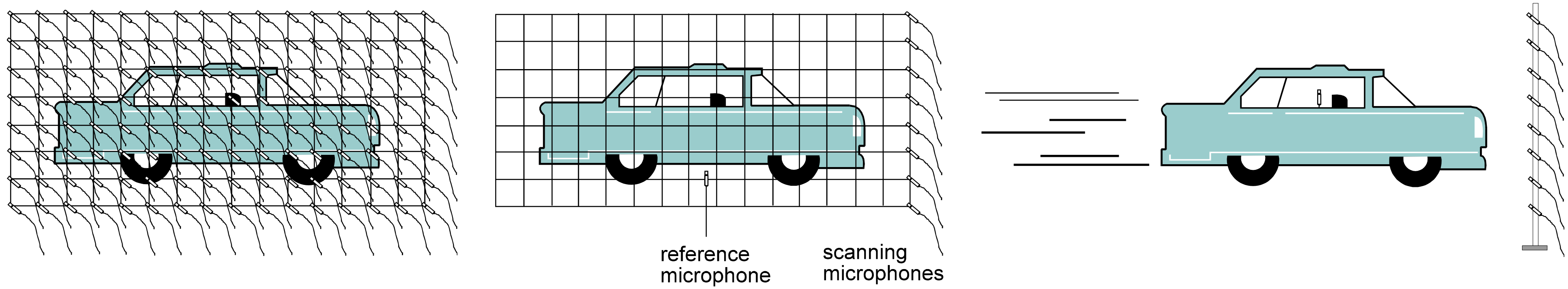
i.e.
J. D. Maynard, E. G. Williams and Y. Lee,
J. Acoust. Soc. Am. 78(4), 1395-1413 (1985)



Hologram;
complex pressur
istribution of each frequency
on hologram plane

MEASUREMENT OF HOLOGRAM

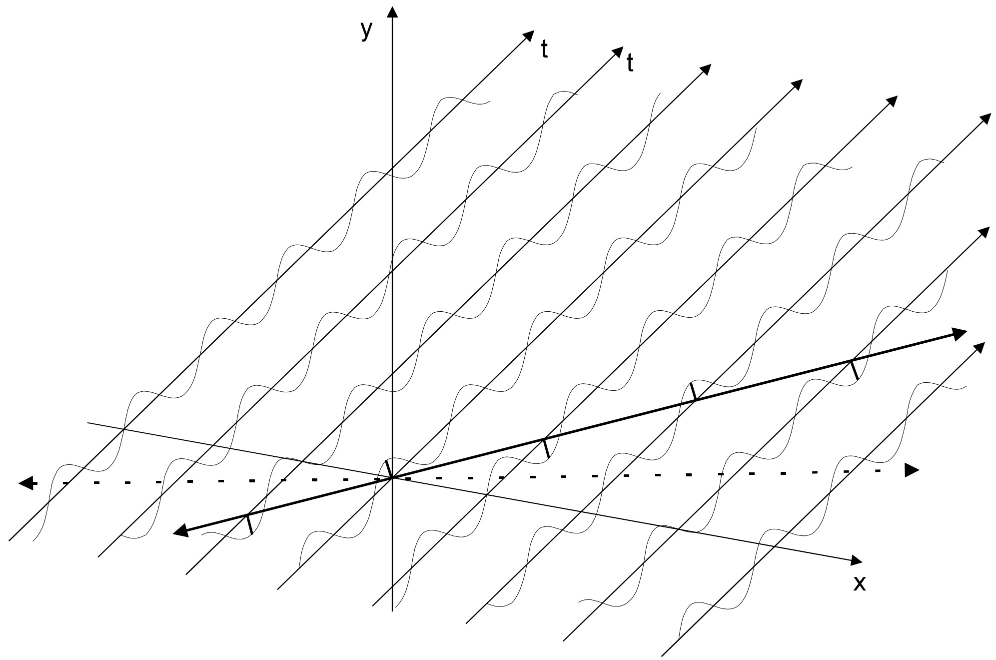
* Requires many microphones



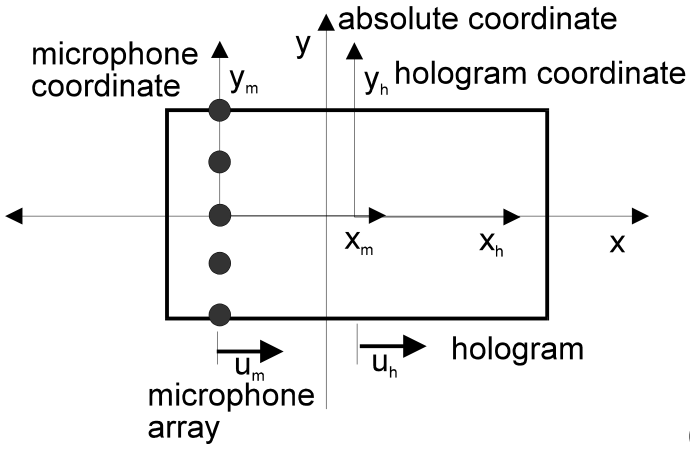
Simultaneous measurement	Conventional scanning technique	Moving frame technique
No. of microphones	requires reference microphone	
= No. of measurement points	time record length	

MOVING FRAME TECHNIQUE (I)

propagating plane wave



COORDINATE



(x, y) : Àý´ëÁÂÇ¥°è, (x_h, y_h) : È¦·Î±×·¥ ÁÂÇ¥°è, (x_m, y_m) : ¸¶ÀÌÅ©·ÎÆù ÁÂÇ¥°è

CASE1 ; MOVING ARRAY

$$u_m = u,\ u_h = 0 \quad x_h = x,\ y_h = y \quad x_m = x - ut,\ y_m = y$$

CASE2 ; MOVING HOLOGRAM

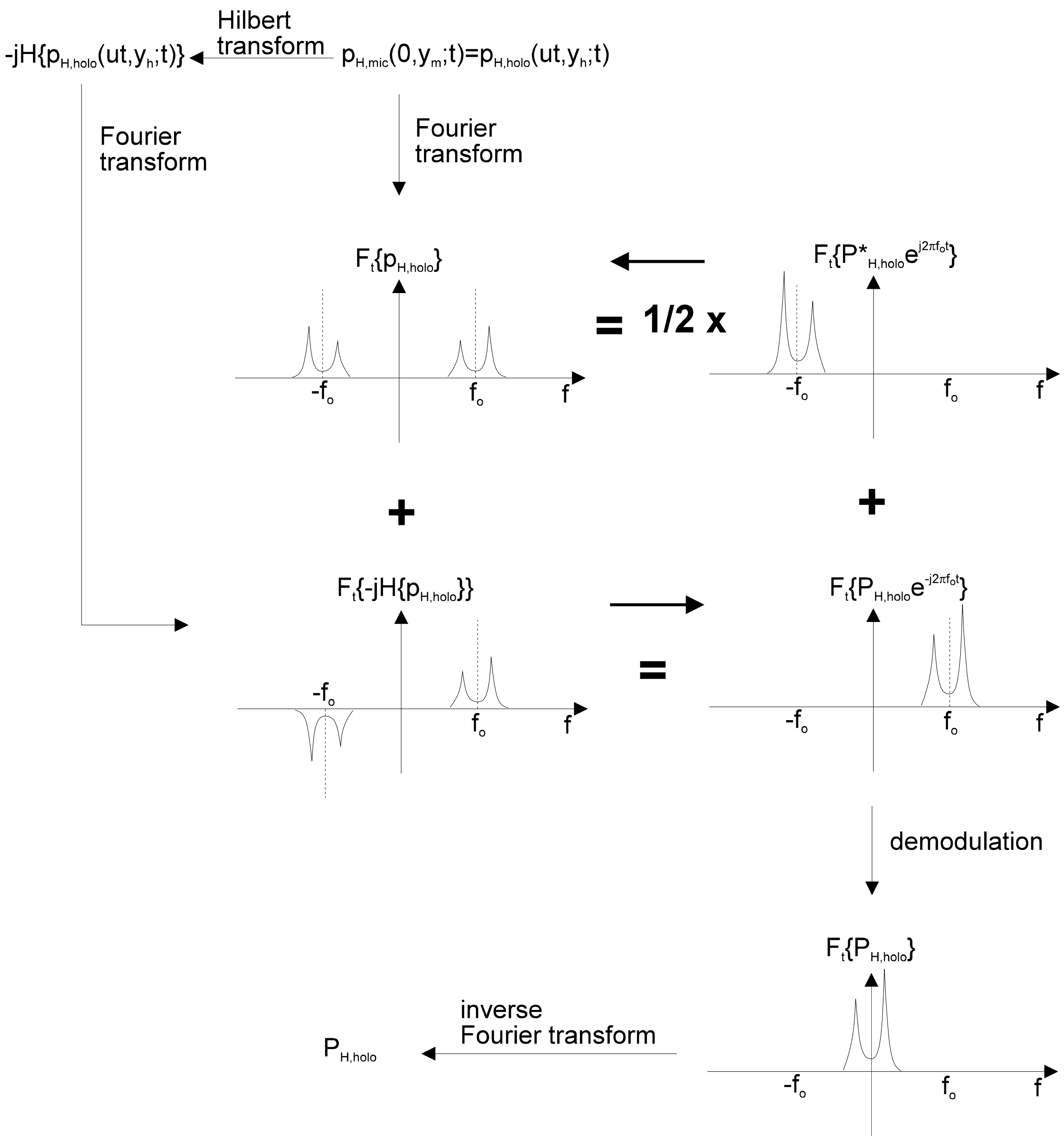
$$u_m = 0,\ u_h = -u \quad x_h = x + ut,\ y_h = y \quad x_m = x,\ y_m = y$$

CASE3 ; MOVING ARRAY & MOVING HOLOGRAM

$$u_m - u_h = u \qquad x_h = x - u_h t,\ y_h = y \quad x_m = x - u_m t,\ y_m = y$$

$$x_m = x_h - ut,\ y_m = y_h \qquad u = u_m - u_h$$

TRANSFORMING


Hilbert transform
-jH{p_H,holo(ut,y_h;t)}
p_H,mic(0,y_m;t)=p_H,holo(ut,y_h;t)
Fourier transform
Fourier transform
F_t{p_H,holo}
= 1/2 x
F_t{P*_H,holo e^j2πf_o t}
+
+
F_t{-jH{p_H,holo}}
=
F_t{P_H,holo e^-j2πf_o t}
-f_o
f_o
f
demodulation
F_t{P_H,holo}
inverse Fourier transform
P_H,holo

ASSUMPTION ;

Expand Unknown True Spectrum by ***Talor Series*** $\tilde{f}(\alpha-\theta)=\tilde{f}(\alpha)-\theta\frac{d\tilde{f}(\alpha)}{d\alpha}+\frac{\theta^2}{2}\frac{d^2\tilde{f}(\alpha)}{d\alpha^2}-\Lambda$

$$\sum_{i=1}^{N}Error_i<\sum_{i=1}^{N+1}Error_i$$

CALCULATION OF WINDOW ;

Expansion of Bias Error (Series Form) $\left(B(\alpha)=\tilde{f}(\alpha)\frac{m_0}{2\pi}-\tilde{f}(\alpha)-\frac{d\tilde{f}(\alpha)}{d\alpha}\frac{m_1}{2\pi}+\frac{d^2\tilde{f}(\alpha)}{d\alpha^2}\frac{m_2}{4\pi}-\frac{d^3\tilde{f}(\alpha)}{d\alpha^3}\frac{m_3}{12\pi}\ \Lambda\right)$

Expression of Moment Terms by Weighting Values $\left(m_k=\int_{-\pi}^{\pi}\theta^k\tilde{W}(\theta)d\theta\right)$

Calculation of Weighting Values by Reducing Moment Terms

* ***MINIMUM ERROR WINDOW*** ; Similar With Tukey Window

EXTRAPOLATION ALGORITHM

ADDITIONAL INFORMATION ;

Identification of Equivalent Sources

NOISE SOURCE IDENTIFICATION ;

Spherical Beamforming Method ; J. W. Choi and Y. -H. Kim (Inter-noise 93, 1339-1342)

Finding The ***Position*** of Equivalent Sources

EXTRAPOLATION ALGORITHM

* Measure The Hologram (On The Hologram Plane)
* Identify The Positions of Point Sources (By Spherical Beamforming)
* Identify The Source Strengths of Point Sources
* Extrapolating The Hologram (By The Identified Point Sources)
* Reconstruction of Sound Images (By Using The Extended Hologram)

No Recursive Process

DETERMINING THE SOURCE STRENGTHS

* M Source Positions (Identified)
* N Measurement Positions

$$\mathbf{P} = \mathbf{TA}$$

P ; N x 1 Hologram Vector,

Measured Pressures

A ; M x 1 Amplitude Vector,

Identified Point Sources

T ; N x M Transfer Matrix

$$\mathbf{T} = \begin{bmatrix} \frac{e^{jkR_{11}}}{R_{11}} & \frac{e^{jkR_{12}}}{R_{12}} & \Lambda \\ \frac{e^{jkR_{21}}}{R_{21}} & \mathrm{O} & \mathrm{M} \\ \mathrm{M} & \Lambda & \frac{e^{jkR_{NM}}}{R_{NM}} \end{bmatrix}$$

MEASURES OF ERROR ASSOCIATED WITH SOUND IMAGES

* PHASE ERROR

$$Phase\ Error = \frac{\left|\mathbf{P}_{\mathbf{True}} \bullet \mathbf{P}_{\mathbf{Est}}\right|}{\left\|\mathbf{P}_{\mathbf{True}}\right\|\left\|\mathbf{P}_{\mathbf{Est}}\right\|}$$

$\left\| \cdot \right\|$; Norm $\mathbf{P}_{\mathbf{True}} \bullet \mathbf{P}_{\mathbf{Est}}$; Inner product

worst : 0 ~ best : 1

* MAGNITUDE ERROR

$$Magnitude\ Error = \frac{\left\|\mathbf{P}_{\mathbf{Est}}\right\|}{\left\|\mathbf{P}_{\mathbf{True}}\right\|}$$

worst : 0 ~ best : 1 ~ worst : ∞

APPLICATIONS TO PLANAR ACOUSTIC HOLOGRAPHY

* SOURCE IMAGE ESTIMATION BY USING

MINIMUM ERROR WINDOW

EXTRAPOLATION (SPHERICAL BEAMFORMING)

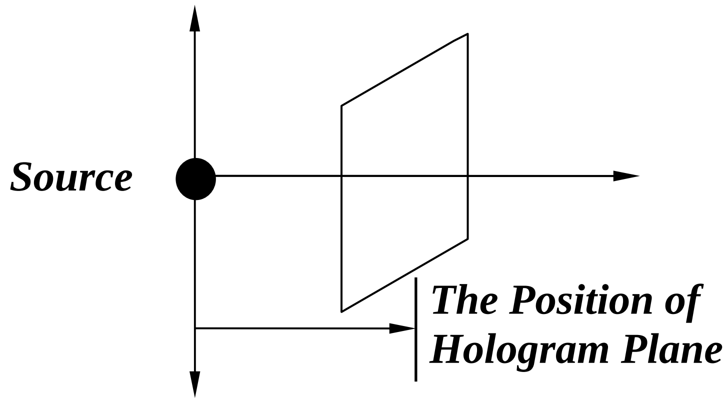
* WINDOWS (with ***Zero Padding***)

No Window (Rectangular Window)

Hann Window

Minimum Error Window

* EXTRAPOLATION



* SOURCES ; Monopole & Dipole

* POSITION OF THE HOLOGRAM PLANE : 0.3 λ

* SAMPLING SPACE : 0.2 λ

* NUMBER OF HOLOGRAM DATA : 14 X 14

* NUMBER OF EXTENDED DATA : 64 X 64 (Zero Padding or Extrapolation)

CONCLUDING REMARKS

* PRACTICAL RESTRICTIONS FOR THE USE OF THE PLANAR ACOUSTIC HOLOGRAPHY ;

The Finite Number of Measurement Points

* FINITE SIZE OF HOLOGRAM APERTURE ; Major Concern

* MINIMUM ERROR WINDOW

To Minimize The Bias Error Due To The Window

* EXTRAPOLATION ALGORITHM

To Extend The Hologram Aperture

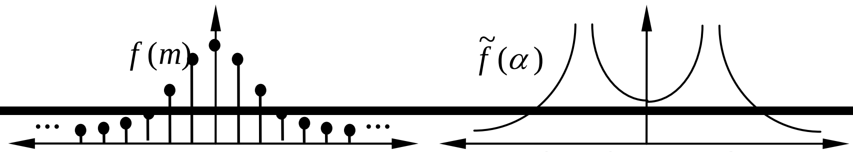
By Using ***Spherical Beamforming Method***

* ENHANCEMENT OF RECONSTRUCTION OF SOUND IMAGES

Comparing Similarity of Reconstructed Sound Images

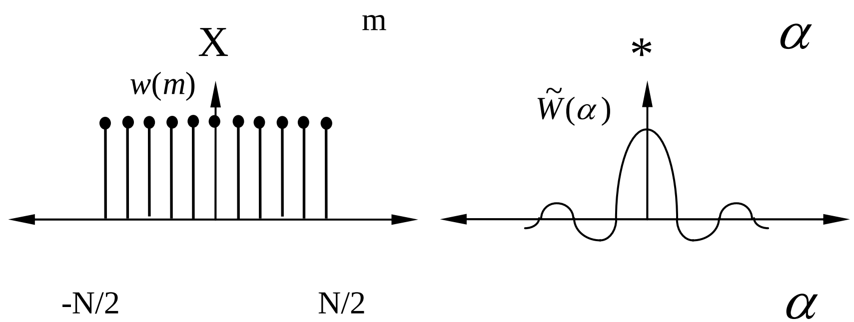
A NEW CLASS OF WINDOW

* DISCRETE FOURIER TRANSFORM PAIR



$$\tilde{f}(\alpha) = \sum_m f(m) e^{-j\alpha m}$$

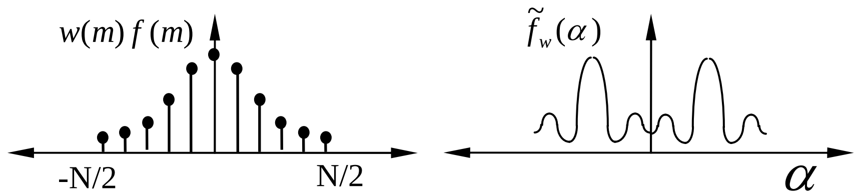
$$f(m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{f}(\alpha) e^{j\alpha m}\, d\alpha$$



* WINDOWED FOURIER TRANSFORM

$$\tilde{f}_w(\alpha) = \sum_m w(m)\, f(m) e^{-j\alpha m}$$

$$\tilde{f}_w(\alpha) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{f}(\alpha-\theta)\tilde{W}(\theta)\, d\theta$$



where $w(m)$, $\tilde{W}(\theta)$ is Discrete Fourier Transform Pair

* BIAS ERROR

$$B(\alpha) = \tilde{f}_w(\alpha) - \tilde{f}(\alpha)$$

ASSUMPTIONS ; *Ref. A. Papoulis, IEEE, **IT-19(1)**, 9-12 (1973)*

* True Spectrum can be Expanded by ***Taylor Series***

* The Magnitude of The Higher Order Derivative is ***Smaller*** Than That of The Lower One

* EXPANSION OF TRUE SPECTRUM

$$\tilde{f}(\alpha-\theta) = \tilde{f}(\alpha) - \theta\frac{d\tilde{f}(\alpha)}{d\alpha} + \frac{\theta^2}{2}\frac{d^2\tilde{f}(\alpha)}{d\alpha^2} - \Lambda$$

* DEFINITION OF MOMENTS

$$m_k = \int_{-\pi}^{\pi}\theta^k \tilde{W}(\theta)d\theta$$

* BIAS ERROR ; SERIES FORM

Papoulis Approach

$$B(\alpha) = \tilde{f}(\alpha)\frac{m_0}{2\pi} - \tilde{f}(\alpha) - \frac{d\tilde{f}(\alpha)}{d\alpha}\frac{m_1}{2\pi} + \frac{d^2\tilde{f}(\alpha)}{d\alpha^2}\frac{m_2}{4\pi} - \frac{d^3\tilde{f}(\alpha)}{d\alpha^3}\frac{m_3}{12\pi} \quad \Lambda$$

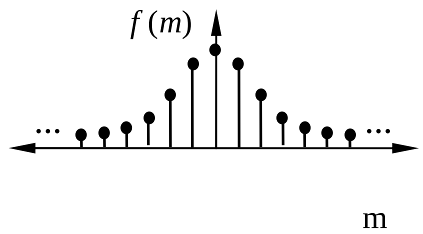
Our Approach

* THE ELIMINATION OF ALL TERMS OF BIAS ERROR

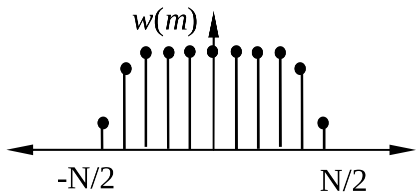
$$\sum_{m=1}^{N} J_r^m w(m) = -\frac{\pi^{2r+1}}{2r+1} = C_r$$

$$M \rightarrow \infty \updownarrow \begin{bmatrix} J_1^1 & \Lambda & J_1^N \\ J_2^1 & \Lambda & J_2^N \\ M & M & M \end{bmatrix} \begin{Bmatrix} w(1) \\ M \\ w(N) \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_2 \\ M \end{Bmatrix} \updownarrow M \rightarrow \infty$$

$\longleftrightarrow$ N



X



$$J_r^m = (-1)^m \frac{4r}{m^2} \pi^{2r-1} - \frac{2r(2r-1)}{m^2} J_{r-1}^m$$

$$J_0^m = 0$$

$$r = 1,2,\Lambda \qquad m = 1,2,\Lambda\ , N$$

NOTE ;

The Number of Terms To Be Eliminated ; ***INFINITE, M***

The Number of Wanted Weighting Values ; ***FINITE, N*** OVERDETERMINED CASE

CALCULATION OF WEIGHTING VALUES

CASE I.

* By The Assumption ;

The Higher Order Derivative is Smaller Than The Lower One

* The Number of Terms To Be Eliminated, M

Equal To The Number of Weighting Values, N

CASE II.

* More Terms Than N are ***Not Negligible***

* M > N

* ***SVD (Singular Value Decomposition)*** is Used