Longitudinal acceleration wave decomposition in time domain with single point axial strain and acceleration measurements

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Abstract

We investigated a longitudinal acceleration wave decomposition method in time domain. The proposed method is useful to separate up- and down-stream acceleration waves with an axial strain and axial acceleration measured at a single point on the 1 dimensional transmission path. The advantages such as low computation load and easy implementation would be accomplished by the extension to time domain under the assumptions; low frequency range, uniform cross sectional area and elastic wave propagation. We confirmed the feasibility and performance of the method through experiment using Split Hopkinson Pressure Bar (SHPB). The method can be effective in several applications, including active vibration power control, where wave separation should be obtained in real time.

Introduction

Longitudinal wave separation methods are classified into two broad groups; two point and single point measurement method.
A single point measurement method was suggested by S.W. Park and M. Zhou [1] for the first time in 1999. This method separated two directional waves based on an analytic solution with fully known boundary condition and a measured strain value at a single point. The two point measurement method introduced by Lundberg and Henchoz [2] in 1977 has been investigated by many researchers recently. This method made use of two measured strain values at different points and spatial information of transducers. Bacon [3] decomposed two directional waves based on the two point measurement method considering three-dimensional effects.
These two types of decomposition methods were performed in frequency domain, because of the dispersion effect that causes the material properties to be a function of frequency. This dispersion phenomena not considered in the classical wave theory was investigated by Pochhammer [4] and Chree [5]. Davies [6] showed the effect of it theoretically and experimentally in 1948.
For this reason, most of wave separation methods were performed in frequency domain and needed additional material properties such as phase velocity as a function of frequency and spatial information of transducers. Therefore, the application of the conventional methods has been restricted to determine and analyze the fundamental one dimensional elastic wave propagation because of large computation load by FFT, spatial constraints and information of phase velocity as a function of frequency.
To cope with these demerits, we investigated a new method of a longitudinal wave decomposition technique that is applicable to one dimensional transmission path with elastic behavior and uniform cross sectional area. Because of the dispersion effect that causes material properties like phase velocity to be frequency dependent and is dominant reason of the frequency domain analysis, we assume that a phase velocity is constant for a low frequency region that does not exceed 10,000Hz. Although such an assumption yields error in high frequency region where the phase velocity is much

dependant on frequency, we use it for the development of real-time time domain method. The method is based on the measurement of axial strain and axial acceleration at a single point on a transmission path.
The feasibility of the method is going to be shown by experiment results using modified compressive Split Hopkinson Pressure Bar.

Theory

In order to develop a new method, we consider a bar depicted in figure 1. It is assumed that the bar is perfectly straight and has a constant cross sectional area that is sufficiently small that the bar can be assumed to be in a state of one dimensional axial stress that is uniform over the cross section. According to the assumptions above, we consider that a longitudinal wave is dominant on the transmission path. There is vibration source at each side of the measurement point, respectively, which generates uniform dynamic stress on the cross sectional area.
An axial strain and axial acceleration are measured by the strain gage and the accelerometer at an arbitrary single point on the bar.

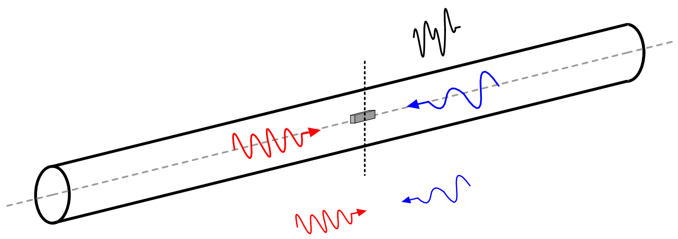

Figure 1. Longitudinal wave propagation

We can express the displacement at $x = l$ as superposition of two displacement waves, namely, rightward and leftward waves.

$$y(x,t) = \sum_m f_m(\gamma_m l - \omega_m t + \phi_m) + \sum_n g_n(\gamma_n l + \omega_n t + \phi_n) \qquad (1)$$

Where m and n are numbers of rightward and leftward waves, $\gamma(\omega) = \omega/c$ is the propagation coefficient, c is the phase velocity as a function of frequency, and ϕ is a constant.
A compressive strain and rightward direction are defined as positive.
The acceleration and strain at the position in figure 1, are expressed as follows, respectively.

$$\begin{aligned} a &= \frac{d^2y}{dt^2} = \sum_{m=1}^{M} \omega_m^2 \cdot \ddot{f}_m(\gamma_m l - \omega_m t + \phi_m) + \sum_{n=1}^{N} \omega_n^2 \cdot \ddot{g}_n(\gamma_n l + \omega_n t + \phi_n) \\ \varepsilon &= \frac{dy}{dx} = \sum_{m=1}^{M} \gamma_m \cdot \dot{f}_m(\gamma_m l - \omega_m t + \phi_m) + \sum_{n=1}^{N} \gamma_n \cdot \dot{g}_n(\gamma_n l + \omega_n t + \phi_n) \end{aligned} \qquad (2)$$

To derive a new method, a strain rate with respect to time, is defined as follows;

$$\dot{\varepsilon} = \frac{d^2 y}{dxdt} = \sum_{m=1}^{M} \left(-\omega_m \gamma_m\right) \cdot \ddot{f}_m \left(\gamma_m l - \omega_m t + \phi_m\right) + \sum_{n=1}^{N} \left(\omega_n \gamma_n\right) \cdot \ddot{g}_n \left(\gamma_n l + \omega_n t + \phi_n\right) \tag{3}$$

The acceleration and strain rate in equation (2) and (3) are rewritten in frequency domain by Fourier transform.

$$\begin{aligned} \tilde{a}\left(x,\omega\right) &= \sum_{m=1}^{M} \omega_m^2 \cdot \Phi_m\left(\omega_m\right) + \sum_{n=1}^{N} \omega_n^2 \cdot \Gamma_n\left(\omega_n\right) \\ \tilde{\dot{\varepsilon}}\left(x,\omega\right) &= \sum_{m=1}^{M} \left(-\omega_m \gamma_m\right) \cdot \Phi_m\left(\omega_m\right) + \sum_{n=1}^{N} \left(\omega_n \gamma_n\right) \cdot \Gamma_n\left(\omega_n\right) \end{aligned} \tag{4}$$

A tilde mark (~) on a letter represents the variable is a Fourier transformed one.

We could separate rightward and leftward acceleration wave from equation (4) by algebraic process at each frequency with the phase velocity as a function of frequency;

$$\begin{aligned} rightward\ accel. &= \frac{1}{2}\left\{\tilde{a}\left(x,\omega_p\right) - \frac{\omega_p}{\gamma_p}\tilde{\dot{\varepsilon}}\left(x,\omega_p\right)\right\} = \omega_p^2 \cdot \Phi_p\left(\omega_p\right), 1 \le p \le M \\ leftward\ accel. &= \frac{1}{2}\left\{\tilde{a}\left(x,\omega_q\right) + \frac{\omega_q}{\gamma_q}\tilde{\dot{\varepsilon}}\left(x,\omega_q\right)\right\} = \omega_q^2 \cdot \Gamma_q\left(\omega_q\right), 1 \le q \le N \end{aligned} \tag{5}$$

This proposed method could be applied without the demerits of conventional ones such as spatial constraints and well-known boundary condition.
The above method in frequency domain, however, could not be easily applied since it still needed large computation load, and frequency-dependent material properties. As shown in the above derivation, the reason for using frequency domain analysis is the dispersion effect that causes the material properties to be a function of frequency. Based on the past research on a dispersion of longitudinal waves ([3], [4] and [7]), the phase velocity could be assumed to be constant in frequency region below 10,000Hz.
Therefore, we could assume a constant phase velocity by considering that the bandwidth of the most mechanical system is less than 10,000Hz. If we assumed a constant phase velocity, elastic behavior and uniform cross section of the transmission path, we could extend the above wave separation theory to time domain assuming a nominal phase velocity with just two steps of multiplication.;

$$\begin{aligned} rightward\ accel. &= \frac{1}{2}\left\{\frac{d^2 y}{dt^2} - c\frac{d^2 y}{dxdt}\right\} = \sum_{m=1}^{M} \omega_m^2 \cdot \ddot{f}_m\left(\gamma l - \omega_m t + \phi_m\right) \\ leftward\ accel. &= \frac{1}{2}\left\{\frac{d^2 y}{dt^2} + c\frac{d^2 y}{dxdt}\right\} = \sum_{n=1}^{N} \omega_n^2 \cdot \ddot{g}_n\left(\gamma l + \omega_n t + \phi_n\right) \end{aligned} \tag{6}$$

As shown in equation (6), there is no need of introducing the phase velocity as a function of frequency which is difficult to find and if we use 3 point backward numerical differentiation to calculate the strain rate with measured strain, computational load was less than 10 times of algebraic multiplication and total computation load for separation was under 20 times. In the case of the conventional methods in frequency domain, the required computation load is $n \log n$ (n is number of data for FFT) for FFT and 5 or 6 times of multiplication for separation at each frequency. There are several advantages as follows;

(1) utilization of practical transducers; strain gage and accelerometer
(2) no spatial constraints and information for transducers
(3) time domain method without frequency-dependent material properties
(4) significant low computational load

We can expect that the proposed method in time domain can be implemented easily and applied to several applications.

Experiment and its results

In order to confirm feasibility and performance of method proposed in chapter 2, we performed the experiment using Hopkinson pressure bar test based on the proposed method with measured strain and acceleration. The SHPB(Split Hopkinson Pressure Bar) depicted in figure 2, introduced by Kolsky [8], is widely used in measuring material properties with high strain and monitoring longitudinal pulse shape and propagation on one dimensional transmission path. Other application of it is one of the techniques for inducing ideal longitudinal impact loading and generating evident up- and down-stream waves on the path.

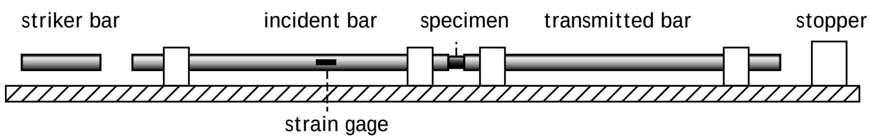

Figure 2. general compressive SHPB setup

The general compressive SHPB setup consists of three bars; striker bar, incident bar, and transmitted bar. The specimen is installed between the incident bar and transmitted bar for finding out its material properties and protecting both bars from impacting. In this paper, we employed modified compressive SHPB setup proper to the experiment.

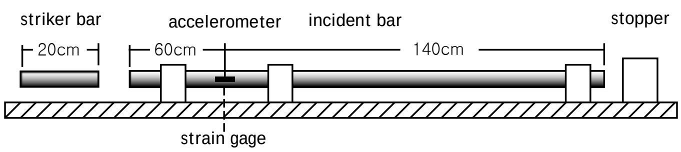

Figure 3. modified SHPB setup

Owing to needlessness of transmitted wave, there are only striker bar and incident bar without transmitted bar as shown in the figure 3. The striker bar was crashed head-on to the incident bar with velocity, about 10m/s to 20 m/s by compressed gas to generate a single incident wave on the incident bar. The shapes of two bars made of steel alloy whose nominal velocity is 5100m/s, are as follows;

(1) striker bar : circular cross section area with 20mm diameter and 25cm length

(2) incident bar : circular cross section area with 20mm diameter and 200cm length

The strain gage and accelerometer for wave separation was installed at incident bar as shown in figure 3 and the sampling frequency is 1,000 kHz. The leftward reflected wave generated at the free boundary condition is distinguished from the rightward incident wave by time delay between two waves.

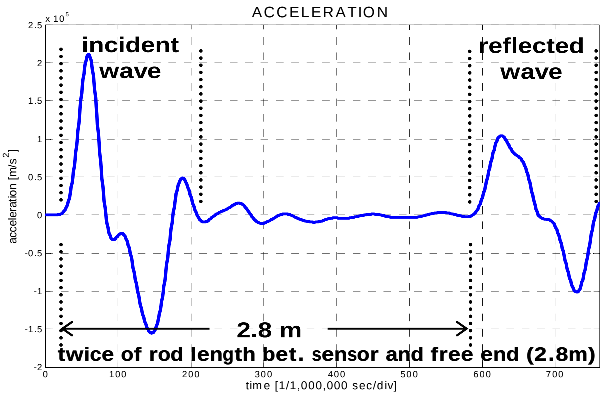

Figure 4. measured acceleration signal

The result of measured acceleration is shown in figure 4. The travel distance of incident wave is 2.8m, twice of length between transducer and free end. We confirmed the coincidence between analytic and experiment results of the travel distance.

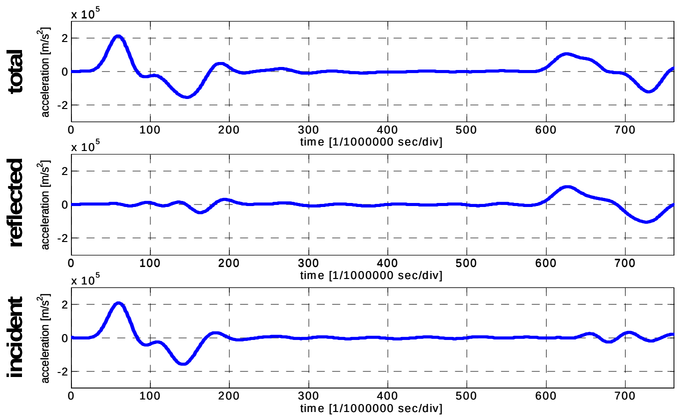

Figure 5. results of separated incident and reflected waves

We could obtain the results of separated up- and downstream waves by means of proposed method with the strain rate calculated by backward numerical differentiation and measured acceleration, as depicted in figure 5.
From the results shown in figure 5, we confirmed the performance of proposed method with some error which is below than 4% of RMS values. The error in the region where only one directional wave exists is due to measurement error and the assumptions used in deriving the method and acceptable to apply in various real time areas.

Conclusions

The longitudinal vibration wave decomposition technique is method for discriminating between bidirectional ones on one dimensional waves. The conventional methods, however, are improper to apply to real time fields with several constraints.
In this paper, new longitudinal wave decomposition method in time domain with single point measurements of an axial strain and axial acceleration is developed under the assumptions of a constant phase velocity, elastic behavior and uniform cross section of the transmission path with specific frequency range less than 10,000Hz.
The proposed method can be simply applied to various branches of vibration with several advantages, such as significantly low computational load and no spatial restriction and information of transducers.
For the feasibility test of the proposed method, we tried successfully to separate up- and down-stream waves using modified compressive SHPB. The proposed method requires no detailed information of surrounding and little computational power compared to the conventional approaches.

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