Robust Preview Control of a Vehicle Suspension

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Abstract

This study deals with the problem of robust preview control of a vehicle suspension system. The system is described by a state-space model with linear nominal parts and norm-bounded linear uncertainties, which are functions of states and control input of the state and controlled output equations. Using the augmentation of preview information to the state, the robust preview controller is designed using the well-developed robust feedback control algorithms. The proposed controller guarantees the robust stability of the vehicle suspension system and simulated results are presented.

Introduction

Most control systems utilize the feedback structure, whose control input depends only on the error signals in the current and past states without considering future information, such as tracking commands and measurable disturbances into the systems. Thus, if we know the information about future disturbances and utilize them to be fully reflected on a control law, better performance maybe obtained.
The idea of preview was first proposed by Bender(1968). Bender used the spectral technique and Wiener filter theory to derive a preview controller of 1DOF system. Tomizuka developed a preview control algorithm for discrete-time systems using the dynamic programming method for 1DOF system(1976). Hac also solved the problem using the calculus of variation(1992).
Dynamic systems have always uncertainties in the plant parameters. Aforementioned researches are based on a LQ framework show the limitation on considering robustness specifications for designing the controller in systematic manner. Some preview control methods based on the H_∞ control theory have been reported. Kojima and Ishijima derived an H_∞ control law with preview compensation in the continuous time domain(1997). Mianzo and Peng suggested a unified framework based on Hamiltonian formulation(1999). This approach provided a general framework to solve both the LQ and H_∞ preview control problems. Choi and Tsao(2001) presented a design method of the preview controller based on discrete-time H_∞ control with augmentation of preview information to the state.
The problem of robust H_∞ preview control for linear systems has not been studied. In this paper, a robust H_∞ preview controller is proposed and applied to the vehicle suspension system. The class of systems is described by a state-space model with linear nominal parts and time-varying norm-bounded uncertainties of the state and output equations except previewable disturbance input matrices. To design the robust H_∞ preview controller which guarantees the robust stability, the previewable disturbance input which is assumed to be exactly known is augmented to the state. Then well-developed robust feedback controller design techniques are applied. The Hamiltonian approach is used to obtain a solution to the problem of robust H_∞ preview control of the system.

Robust Preview Control

Consider the following stabilizable uncertain system:

$$(\Sigma): \begin{array}{ll} x(k+1) = Ax(k) + B_1u(k) + B_2r(k) + \Delta_1(x(k),u(k)) \\ z(k) \quad = Cx(k) + D_1u(k) + D_2r(k) + \Delta_2(x(k),u(k)) \end{array} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $r(k) \in \mathbb{R}^p$ is the previewable disturbance input, $z(k) \in \mathbb{R}^s$ is the controlled output, A, B_1, B_2, C, D_1 and D_2 are known real constant matrices of appropriate dimensions that describe the nominal system. $\Delta_i(x(k),u(k)), i = 1,2$ are real time-varying vectors representing norm-bounded uncertainties. $k \in \mathbb{Z}$ is the integer which represents the discrete-time step.
Using the following augmented state,

$$x_r(k+1) = A_r x_r(k) + B_r r(k + N_p + 1) \tag{2}$$

where

$$x_r(k) = \begin{bmatrix} r(k) \\ \vdots \\ r(k+N_p) \end{bmatrix} \in \mathbb{R}^{p(N_p+1)}, A_r = \begin{bmatrix} \mathbf{0} & I & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}, B_r = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ I \end{bmatrix}.$$

The state and controlled output equations are represented as

$$(\Sigma_a): \begin{array}{ll} x_a(k+1) = A_a x_a(k) + B_{a1}u(k) + B_{a2}r_a(k) + \Delta_{a1}(x_a(k),u(k)) \\ z_a(k) \quad = C_a x_a(k) + D_{a1}u(k) \qquad\qquad + \Delta_{a2}(x_a(k),u(k)) \end{array} \tag{3}$$

where

$$x_a(k) = \begin{bmatrix} x(k) \\ x_r(k) \end{bmatrix} \in \mathbb{R}^{n+p(N_p+1)},\ r_a(k) = r(k+N_p+1),\ A_a = \begin{bmatrix} A & \bar{B}_2 \\ \mathbf{0} & A_r \end{bmatrix},\ B_{a1} = \begin{bmatrix} B_1 \\ \mathbf{0} \end{bmatrix},$$

$$B_{a2} = \begin{bmatrix} \mathbf{0} \\ B_r \end{bmatrix},\ C_a = \begin{bmatrix} C & \bar{D}_2 \end{bmatrix},\ D_{a1} = D_1,\ \bar{B}_2 = \begin{bmatrix} B_2 & \mathbf{0} \end{bmatrix},\ \bar{D}_2 = \begin{bmatrix} D_2 & \mathbf{0} \end{bmatrix},$$

$$\Delta_{a1}(x_a(k),u(k)) = \begin{bmatrix} \Delta_1(x(k),u(k)) \\ \mathbf{0} \end{bmatrix},\ \Delta_{a2}(x_a(k),u(k)) = \Delta_2(x(k),u(k)).$$

Let us assume the admissible uncertainties are of the form

$$\|\Delta_{ai}(x_a(k),u(k))\| \le a_i\|x_a(k)\| + b_i\|u(k)\|,\ i = 1,2$$

where $a_i \ge 0, b_i \ge 0,\ i = 1,2$ are known constant numbers, $\|\bullet\|$ represents the Euclidean vector norm.
Denote the corresponding uncertainty sets by

$$\Omega_i(x_a(k),u(k)) = \left\{ \begin{array}{r} \Delta_{ai}(x_a(k),u(k)) : \|\Delta_{ai}(x_a(k),u(k))\| \\ \le a_i\|x_a(k)\| + b_i\|u(k)\| \end{array} \right\}, i = 1,2.$$

Using the Lemma 1(Wang and Zhan, 1996), the uncertainty sets are represented as

$$\Omega_1(x_a(k),u(k)) \triangleq \left\{ \begin{aligned} & aM_{1k}x_a(k)+bM_{2k}u(k): M_{1k} \in \mathbb{R}^{(n+p(N_p+1))\times(n+p(N_p+1))} \\ & , M_{2k} \in \mathbb{R}^{(n+p(N_p+1))\times m}, \bar{\sigma}(M_{ik}) \le 1,\ i=1,2 \end{aligned} \right\},$$

$$\Omega_2(x_a(k),u(k)) \triangleq \left\{ \begin{aligned} & a\bar{M}_{1k}x_a(k)+b\bar{M}_{2k}u(k): \bar{M}_{1k} \in \mathbb{R}^{s\times(n+p(N_p+1))} \\ & , \bar{M}_{2k} \in \mathbb{R}^{s\times m},\ \bar{\sigma}(\bar{M}_{ik}) \le 1,\ i=1,2 \end{aligned} \right\}.$$

Assumption 1: Let us introduce the following structure assumption for the uncertain parameters $M_{ik}, \bar{M}_{ik}, i=1,2$.

$$\begin{bmatrix} aM_{1k} & bM_{2k} \\ a\bar{M}_{1k} & b\bar{M}_{2k} \end{bmatrix} = \begin{bmatrix} H_{a1} \\ H_{a2} \end{bmatrix} M_k \begin{bmatrix} aE_1 & bE_2 \end{bmatrix}$$

where $H_{a1} \in \mathbb{R}^{(n+p(N_p+1))\times n}$, $H_{a2} \in \mathbb{R}^{s\times n}$, $E_1 \in \mathbb{R}^{i\times(n+p(N_p+1))}$ and $E_2 \in \mathbb{R}^{i\times m}$ are known constant matrices which characterize how the uncertain parameter in M_k enters the nominal matrices A, B_1, B_2, C, D_1 and D_2. $M_k \in \mathbb{R}^{n\times i}$ is a unknown time-varying matrices satisfying $\bar{\sigma}(M_k) \le 1$; $\bar{\sigma}(\bullet)$ stands for its largest singular value.

We provide a solution to the problem of robust H_∞ preview control for the system (Σ_a) in order to guarantee the stability of the closed-loop system in the presence of uncertainty. From Assumption 1, the system (Σ_a) can be represented as

$$(\Sigma_{al}): \begin{aligned} x_a(k+1) &= (A_a + H_{a1}M_kE_{a1})x_a(k) + (B_{a1}+H_{a1}M_kE_{a2})u(k) + B_{a2}r_a(k) \\ z_a(k) &= (C_a + H_{a2}M_kE_{a1})x_a(k) + (D_{a1}+H_{a2}M_kE_{a2})u(k) \end{aligned} \tag{4}$$

where $H_{a1} = \begin{bmatrix} I & \mathbf{0} \end{bmatrix}^T$, $H_{a2} = H$, $E_{a1} = \begin{bmatrix} aE_1 & \mathbf{0} \end{bmatrix}$, $E_{a2} = bE_2$.

Then we examine the robust H_∞ control problem for system(Σ_{al}). To do this, let us introduce the following auxiliary system:

$$(\tilde{\Sigma}_{al}^a): \begin{aligned} x_a(k+1) &= A_ax_a(k) + \varepsilon H_{a1}\tilde{w}(k) + B_{a1}u(k) \\ \tilde{z}_a(k) &= \begin{bmatrix} C_a \\ \varepsilon^{-1}E_{a1} \end{bmatrix} x_a(k) + \begin{bmatrix} \varepsilon H_{a2} \\ 0 \end{bmatrix} \tilde{w}(k) + \begin{bmatrix} D_{a1} \\ \varepsilon^{-1}E_{a2} \end{bmatrix} u(k) \end{aligned} \tag{5}$$

where $x_a(k) \in \mathbb{R}^{n+p(N_p+1)}$ is the augmented state, $\tilde{w}(k) \in \mathbb{R}^n$ is the disturbance input, $\tilde{z}_a(k) \in \mathbb{R}^{s+i}$ is the controlled output, and $\varepsilon > 0$ is a scaling parameter to be chosen.

For an proper $\varepsilon > 0$, if we design the H_∞ feedback controller for the auxiliary system$(\tilde{\Sigma}_{al}^a)$, the system (Σ_a) is quadratically stabilizable with unitary H_∞ disturbance attenuation(Shi, and Shue, 1999). $r_a(k)$ is ignored because it is the distant future signal which has no effect on the system at the current state.

The required controller can be obtained via the Hamiltonian approach. The cost function to be minimized by the control vector is assumed to be $J = \frac{1}{2}\sum_0^{N_f}(\tilde{z}_a^T(k)\tilde{z}_a(k) - \tilde{w}(k)^T\tilde{w}(k))$, and we assume that disturbance input, control input and previewable disturbance input are uncorrelated.

< *Theorem 1* >: The optimal solution of the system $(\tilde{\Sigma}_{al}^{a})$ is achieved when

$$u(k) = -R_a^{-1}\left\{N_a^T + B_{n2}^T\Phi^{-1}(k)P(k+1)\Gamma_1\right\}x_a(k) \tag{6}$$

where
$$P(k) = \Gamma_2 + \Gamma_2\Phi^{-1}(k)P(k+1)\Gamma_1 \tag{7}$$

$$\Phi(k) = I + P(k+1)\left\{B_{n2}R_a^{-1}B_{n2}^{-1} - B_{n1}\left(I - Q_{a2}\right)^{-1}B_{n1}^{-1}\right\},$$

$$\Gamma_1 = A_n - B_{n2}R_a^{-1}N_a^T + B_{n1}\left(I - Q_{a2}\right)^{-1}Q_{a12}{}^T,\ \Gamma_2 = Q_{a1} - N_aR_a^{-1}N_a^T + Q_{a12}\left(I - Q_{a2}\right)^{-1}Q_{a12}{}^T,$$

$$A_n = A_a, B_{n1} = \varepsilon H_{a1}, B_{n2} = B_{a1},\ C_n = \begin{bmatrix} C_a \\ \varepsilon^{-1}E_{a1}\end{bmatrix}, D_{n1} = \begin{bmatrix}\varepsilon H_{a2} \\ 0\end{bmatrix}, D_{n2} = \begin{bmatrix} D_{a1} \\ \varepsilon^{-1}E_{a2}\end{bmatrix}.$$

Proof : Details of proof are given in (Ryu *et al.*, 2006).

Target System

The plant to be controlled is the 2-DOF vehicle suspension model as shown in Hac(1992). Using the following state vector x, and the controlled output vector z

$$x = \left[x_1, \dot{x}_1, x_2, \dot{x}_2\right]^T,\quad z = \left[\ddot{x}_1, \rho_1(x_1 - x_2), \rho_2(x_2 - r), \rho_3 u\right]^T$$

where the superscript T denotes transposition, the state-space and the controlled output equation with uncertainty can be written as:

$$\begin{aligned}\dot{x} &= \bar{A}_\Delta x + \bar{B}_2 u + \bar{B}_3 r \\ z &= C_\Delta x + D_2 u + D_3 r\end{aligned} \tag{8}$$

where

$$\bar{A}_\Delta = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/m_1 & -b_\Delta/m_1 & k_1/m_1 & b_\Delta/m_1 \\ 0 & 0 & 0 & 1 \\ k_1/m_2 & b_\Delta/m_2 & -(k_1+k_2)/m_2 & -b_\Delta/m_2 \end{bmatrix},\ \bar{B}_2 = \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ -1/m_2\end{bmatrix},\ \bar{B}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m_2\end{bmatrix}$$

$$C_\Delta = \begin{bmatrix} -k_1/m_1 & -b_\Delta/m_1 & k_1/m_1 & b_\Delta/m_1 \\ \rho_1 & 0 & -\rho_1 & 0 \\ 0 & 0 & \rho_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ D_2 = \begin{bmatrix} 1/m_1 \\ 0 \\ 0 \\ \rho_3\end{bmatrix},\ D_3 = \begin{bmatrix} 0 \\ 0 \\ -\rho_2 \\ 0\end{bmatrix}$$

Here, the parameters $m_1 = 1,000[kg]$, $m_2 = 100[kg]$, $k_1 = 36,000[N/m\bullet kg]$, $k_2 = 360,000[N/m\bullet kg]$, $b = 3,000[N\bullet s/m\bullet kg]$. And an uncertain parameter $b_\Delta(= b + \Delta)$ is subject to the bound of $0.4 * b \le b_\Delta \le 1.6 * b$.

The vehicle is assumed to travel over a single bump described by:

$$r(t) = \begin{cases} 0.025(1 - \cos 20\pi(t - 0.3)) & \text{for } 0.3 \le \mathrm{t} \le 0.4 \\ 0 & \text{otherwise}\end{cases}$$

Simulation Results

Using the above auxiliary system (5), the robust H_∞ preview controller is designed by Hamiltonian formulation. The weighting parameter is selected as $\rho_1 = 10^3$, $\rho_2 = 10^4$, $\rho_3 = 0$ with a preference for ride comfort and the vehicle velocity $v = 20m/\sec$. a and ε is selected as $1.02*10^{-3}$ and $1*10^{-4}$, respectively.
Figures 1 and 2 show the simulated behavior of the conventional LQ preview and the robust H_∞ preview control with preview time 0.2sec. For the case when b has uncertainty of -60%, Figure 1 shows the vertical acceleration of vehicle body which is related with ride comfort. Figure 2 shows the position of the wheel which is related with vehicle handling. Figure 3 and 4 show the feedback and feedforward input to the system. The feedforward input of robust preview controller is increased with uncertainty, because no uncertainty exists at previewable disturbance input and therefore the controller gives more weight to it. If, however, the uncertainty exists at matrix E, then the feedforward input is decreased with increasing uncertainty. From this simulation results, we conclude that the proposed robust H_∞ preview controller can stabilize the uncertain system, while the conventional LQ preview controller fails.

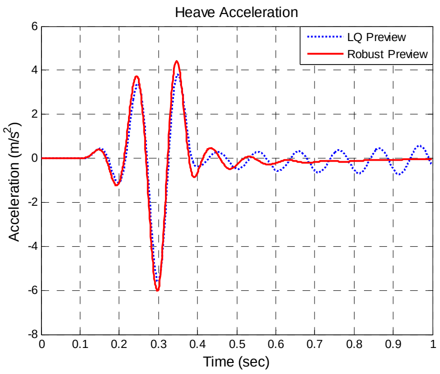

Figure 1. Vertical Acceleration of the Vehicle Body

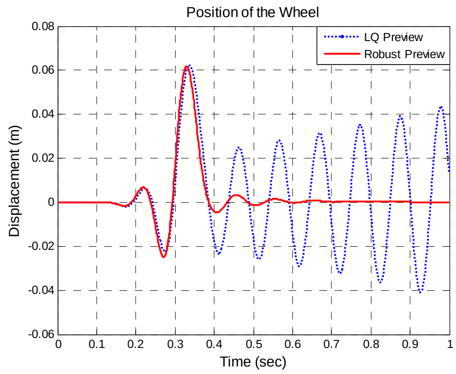

Figure 2. Position of the Wheel

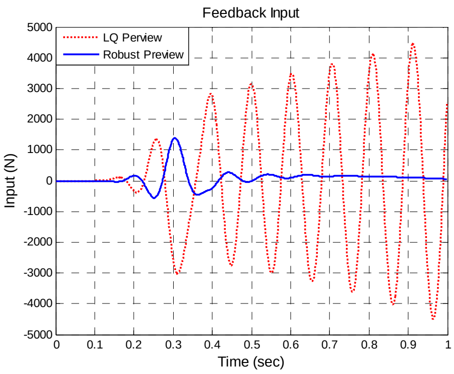

Figure 3. Feedback Input

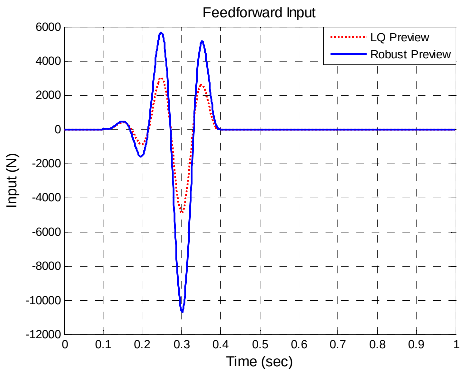

Figure 4. Feedforward Input

Conclusions

This paper proposed the robust H_∞ preview control method, which guarantees the robust stability, in the feedback form for vehicle suspension systems. The results show that the robust H_∞ preview controller can be easily designed using the well-developed robust feedback control method with the augmentation of preview information to the state. The proposed controller has a unitary H_∞ disturbance attenuation for the auxiliary system to satisfy the small gain theorem. Moreover, the simulated results show that the proposed robust H_∞ preview controller assures robust stability even with relatively large uncertainties.

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References

Bender, E. K., 1968, “Optimal linear preview control with application to vehicle suspension,” ASME Journal of Basic Engineering, Ser. D, Vol. 90, No. 2, pp. 213-221.

Tomizuka, M., 1976, “Optimum linear preview control with application to vehicle suspension,” ASME J. of Dynamic Systems, Measurement, and Control, Vol. 98, No. 3, pp. 309-315.

Hac, A., 1992, “Optimal linear preview control of active vehicle suspension,” Vehicle system dynamics, Vol. 21, pp. 167-195.

Kojima, A. and Ishijima, S., 1997, “H_∞ control with preview compensation,” Proceedings of the American Control Conference, Albuquerque, New Mexico, June 4-6, pp.1692-1697.

Mianzo, L. and Peng, H., 1999, “A unified Hamiltonian approach for LQ and H_∞ preview control algorithms,” ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 121, pp. 365-369.

Choi, C. and Tsao, T., 2001, “H_∞ preview control for discrete-time systems,” ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 123, pp. 117-124.

Wang, L. Y. and Zhan W., 1996, “Robust disturbance attenuation with stability for linear systems with norm-bounded nonlinear uncertainties,” IEEE Transactions on Automatic Control, Vol. 41, pp. 886-888.

Shi, P. and Shue, S., 1999, “Robust H_∞ control for linear discrete-time systems with norm-bounded nonlinear uncertainties,” IEEE Transactions on Automatic Control, Vol. 44, pp. 108-111.

Ryu, S. Park, Y. and Park, Y., 2006, “Robust H_∞ preview control for linear discrete-time systems with time-varying norm-bounded uncertainty,” IEICE Transactions on Fundamentals, 2006. (submitted)