

# A Robust Saturation Controller Design Method Considering Structured Real Parameter Uncertainties

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## ABSTRACT

In this paper, we focus on developing robust saturation controller for the linear time-invariant (LTI) system involving both actuator's saturation and structured real parameter uncertainties. The traditional suboptimal bang-bang control method only guarantees stability for nominal LTI system. The controller suggested in this paper can analytically prescribe the upper and lower bounds of parameter uncertainties, and guarantee the closed-loop robust stability of the system in the presence of actuator's saturation. The suboptimal bang-bang control method is extended to LTI system with parameter uncertainties. Based on affine quadratic stability and multi-convexity concept, a robust saturation controller is newly proposed and the linear matrix inequality (LMI)-based sufficient existence conditions for this controller are presented. The effectiveness and the availability of the proposed controller are investigated by an example of three-story building model. Also, some characteristics of the proposed controller are examined. Through numerical simulations, it is confirmed that the proposed robust saturation controller is robustly stable with respect to parameter uncertainties over the prescribed range defined by the upper and lower bounds.

## INTRODUCTION

In active control system, actuator's saturation must be considered for most of actuation devices. Because the physical inputs such as force, torque, thrust, stroke, voltage, current, and flow rate of all conceivable applications of current technology are ultimately limited. Unexpected large amplitude disturbances can push systems' actuators into saturation, thus forcing the system to operate in a nonlinear mode for which it was not designed and in which it may be unstable (Bernstein *et al.* 1995). In recent years, research on the active control of civil engineering structures such as bridges and

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buildings has received increasing attention (Kobori 2002, Spencer Jr. *et al.* 1997, Housner *et al.* 1994). Many control strategies have been developed with the goal of protecting buildings subjected to weak and moderate earthquakes. But one of the main difficulties in realizing the active control systems to protect the building against strong earthquakes is the demand of unrealistic large control force. With the linear active controller such as linear quadratic regulator (LQR) which has been used extensively in many structural control applications, the applied control force is a linear combination of the structural responses. Therefore, the maximum control force will correspond to the peak response which actually occurs only for a few times during strong earthquakes. Thus the design of an actuator based on the maximum demand of control force is inefficient and uneconomical. Saturation control considering the limit of control force is known to be able to embrace this problems and, furthermore, to be more effective in the reduction of structural response than the linear control algorithms under strong earthquakes (Mongkol *et al.* 1996, Indrawan *et al.* 1994, Wu *et al.* 1996, Lim *et al.* 2003, Yang *et al.* 1995a). Most of the existing saturation control algorithms are developed in nominal LTI system. Because inherent modeling errors between mathematical models and real-world systems are unavoidable, active controller for systems with actuator's saturation should be designed to be robust with respect to system uncertainties.

The bang-bang control, which minimizes a performance index subjected to the control force constraint, has been continuously investigated by several authors in optimal control theory (Wonham *et al.* 1964, Friendland 1971, Meirovitch 1990). The main shortcoming of the bang-bang control becomes clear when one wants to apply this control method to the structural control. First, because control force is not a function of state but of co-state, the on-line computation process of it will significantly increase the time delay and may lead to instability due to the accumulated error in on-line numerical evaluation. Second, the undesirable control chattering near the origin of state-space due to high frequency switching of control force often occurs and great care should also be taken against spillover instability at higher modes. Some studies have been investigated to overcome these shortcomings. Mongol *et al.* (1996) proposed the linear saturation control method which consists of a low-gain linear control when the system is close to the zero state and the bang-bang control otherwise. They showed a scheme to synthesize the switching surface that is needed to implement the bang-bang control as well. Wu *et al.* (1996) introduced the suboptimal bang-bang control described by a function of state. In the suboptimal bang-bang control, the control force is determined by minimizing the time derivative of a quadratic Lyapunov function under the control force constraint. Wu *et al.* (1996) also proposed the modified bang-bang control method which overcomes control chattering problem of the suboptimal bang-bang control. This method is found to be effective under a certain range of control force but it can be unstable outside of this range. To overcome this instability Lim *et al.* (2003) proposed an adaptive bang-bang control algorithm. All of the aforementioned bang-bang type control algorithms guarantee only stability for nominal LTI system and don't explain analytically the robustness with respect to parameter uncertainties of the system.

To explain robust stability of the saturation control, Yang *et al.* (1995a, 1995b) presented the saturated sliding mode control method based on the theory of the sliding mode control (SMC) and proved it to be effective method in vibration control for civil building structures. Even though this method is robust with respect to parameter uncertainties of the system, it cannot prescribe the bounds of parameter uncertainties of the system within which closed-loop robust stability is guaranteed for certain. Also the robustness is not guaranteed over the complete response of a SMC system because robustness of the SMC with respect to parameter uncertainties is guaranteed only in the sliding mode.

In this paper, we focus on developing robust saturation controller guaranteeing robust stability of uncertain LTI system over the prescribed upper and lower bounds of structured real parameter uncertainties. In high-rise building, masses, stiffnesses, and damping coefficients are physical system parametric uncertainties. Because these uncertainties are regarded as time-invariant, we can model high-rise building as LTI vibrating system with uncertain masses, stiffnesses, and damping coefficients (or uncertain natural frequencies and damping ratios). Approach of expressing system uncertainties as structured real parameter uncertainties is known to be an effective way of describing the modeling

errors in state space. Through this approach, the controller can be designed to guarantee robust stability and/or performance for given bounds of each parameter uncertainty. To reduce conservatism of the classical quadratic stability test, both uncertain LTI system and its Lyapunov function are assumed to be affine in time-invariant uncertain real parameters. Suboptimal bang-bang control method, which is designed based on Lyapunov stability condition for nominal LTI system, is extended to this uncertain LTI system. Based on affine quadratic stability (AQS) definition and multi-convexity concept (Gahinet *et al.* 1996) to reduce the problem to an LMI problem, a robust saturation controller is proposed and the LMI-based sufficient existence conditions are presented to design this proposed controller. A practical numerical example is illustrated to verify the availability and the effectiveness of the proposed controller.

## AFFINE QUADRATIC STABILITY

We first review the AQS test which is used as an analytical tool for our robust saturation control. Even though it is much more difficult to deal with its analysis mathematically, the AQS test can analyze robust stability of linear systems with uncertain real parameters which are time-invariant or time-varying. In robust controller design, the classical quadratic stability test guarantees robust stability against arbitrarily fast parameter variations (Khargonekar *et al.* 1990). As a result, this quadratic stability test can be very conservative for constant parameters or slow-varying parameters. However, the AQS test is applicable to both constant and time-varying uncertain parameters and much less conservative than the quadratic stability test in the case of constant parameters or slow-varying parameters. This AQS test is an extension of the notion of quadratic stability where the fixed quadratic Lyapunov function is replaced by a Lyapunov function with affine dependence on uncertain parameters.

This paper is concerned with the LTI system with constant uncertain real parameters that can be described by state space equation of the form

$$\dot{x}(t) = A(\theta)x(t), \quad x(0) = x_0 \quad (1)$$

where state vector is  $x \in \mathfrak{R}^n$ ,  $\theta = (\theta_1, \theta_2, \dots, \theta_K) \in \mathfrak{R}^K$  is a vector of uncertain real parameters, and the system matrix  $A(\theta)$  is assumed to be stable and depends affinely on the parameters of  $\theta_i$ . That is

$$A(\theta) = A_0 + \theta_1 A_1 + \theta_2 A_2 + \dots + \theta_K A_K \quad (2)$$

where  $A_0, A_1, A_2, \dots, A_K$  are known fixed matrices.

We assume that lower and upper bounds are available for the parameter values. Specifically, each parameter  $\theta_i$  ranges between known external values  $\underline{\theta}_i$  and  $\bar{\theta}_i$ .

$$\theta_i \in [\underline{\theta}_i, \bar{\theta}_i] \quad \text{for } i = 1, 2, \dots, K \quad (3)$$

This means that the parameter vector  $\theta$  is valued in a hyper-rectangle called the parameter box. In the sequel

$$\Theta := \{(\omega_1, \omega_2, \dots, \omega_K) : \omega_i \in \{\underline{\theta}_i, \bar{\theta}_i\}\} \quad (4)$$

denotes the set of the  $2^K$  vertices or corners of this parameters.

The following notion of parameter dependent Lyapunov function is introduced to reduce conservatism of the classical quadratic stability test when system (1) is affine in  $\theta$  with time-invariant parameters.

$$V(x, \theta) = x^T P(\theta) x \quad (5)$$

where  $P(\theta)$  is an affine function of  $\theta$ .

$$P(\theta) = P_0 + \theta_1 P_1 + \theta_2 P_2 + \cdots + \theta_K P_K \quad (6)$$

Using this parameter dependent Lyapunov function we can define AQS for the LTI systems with constant uncertain real parameters (1) as followings.

**Definition 1.** Affine Quadratic Stability (AQS) (Gahinet *et al.* 1996): The linear time-invariant system with constant uncertain real parameters (1) is said to be affinely quadratically stable if there exist  $K + 1$  symmetric matrices  $P_0, P_1, P_2, \dots, P_K$  such that

$$P_0 + \theta_1 P_1 + \theta_2 P_2 + \cdots + \theta_K P_K > 0 \quad (7)$$

$$A(\theta)^T P(\theta) + P(\theta) A(\theta) < 0 \quad (8)$$

hold for all admissible trajectories of the parameter vector  $\theta = (\theta_1, \theta_2, \dots, \theta_K)$ . ■

The Definition 1 expresses that  $V(x, \theta) > 0$  and  $dV(x, \theta)/dt < 0$  for all admissible parameter trajectories. Recall that the AQS test is much less conservative than the classical quadratic stability test seeking a fixed parameter independent Lyapunov function along all admissible parameter trajectories. By imposing additional “multi-convexity” constraints on the parameter dependent Lyapunov function, finding an affine Lyapunov matrix  $P(\theta)$  can be turned into an LMI problem with variables  $P_0, P_1, P_2, \dots, P_K$  (Gahinet *et al.* 1996). Efficient polynomial-time optimization algorithms are available to solve this (Boyd *et al.* 1994) because LMI problems are convex.

## ROBUST SATURATION CONTROLLER

We focus on designing robust saturation controller, which guarantees stability of uncertain LTI system with actuator saturation, over the prescribed upper and lower bounds of structured real parameter uncertainties analytically. Suboptimal bang-bang control method (Wu *et al.* 1996), which is designed based on Lyapunov stability condition for nominal LTI system, is extended to uncertain LTI system. To reduce conservatism of classical quadratic stability, both uncertain LTI system (1) and its Lyapunov function (5) are affine in time-invariant uncertain real parameters.

In this section, we propose sufficient conditions for the existence of robust saturation controller. To focus our attention on designing robust saturation controller based on AQS, we add control force term to (1). We consider a following uncertain LTI system

$$\dot{x}(t) = A(\theta)x(t) + Bu(t), \quad x(0) = x_0 \quad (9)$$

with control force constraint.

$$|u(t)| \leq u_{\max} \quad (10)$$

where  $B$  is control input vector, and  $u$  is scalar control force.

For uncertain LTI system (9), a parameter dependent Lyapunov function (5) is defined. The time

derivative of this Lyapunov function is of the following form.

$$\dot{V}(x(t), \theta) = x^T(t)[A(\theta)^T P(\theta) + P(\theta)A(\theta)]x(t) + 2x^T(t)P(\theta)Bu(t) \quad (11)$$

To guarantee robust stability, the time derivative of parameter dependent Lyapunov function (11) under the control force constraint (10) is always less than 0. In this paper, a robust saturation controller (12) is considered.

$$u(t) = -\text{sat}[\delta B^T P_0 x(t)] \quad (12)$$

This controller (12) has similar form to the suboptimal bang-bang controller ( $u(t) = -u_{\max} \cdot \text{sgn}[B^T P_0 x(t)]$ ) for nominal LTI system and is expressed by using saturation function instead of sign function. Note that using saturation function is a typical choice to overcome control chattering problem occurring in sign function type controllers. To guarantee robust stability of controller (12), we propose the following Theorem 1 which gives the LMI-based sufficient existence conditions for this controller.

**Theorem 1.** Consider an uncertain linear time-invariant system (9) where  $A(\theta)$  depends affinely on the parameter vector  $\theta = (\theta_1, \theta_2, \dots, \theta_K)$ ,  $\theta_i$  satisfies (3), and control force has constraint of (10). Let  $\Theta$  denotes the sets of vertices of the parameter box (4). Robust stability of the saturation controller (12) is guaranteed if there exist  $K+1$  symmetric matrices  $P_0, P_1, P_2, \dots, P_K$ , and positive-definite symmetric matrix  $M_a$  satisfying (13), (14), and (15), and if there exists  $\delta > 0$  which satisfies (16) for these matrices  $P_0, P_1, P_2, \dots, P_K$  and  $M_a$ .

$$P(\omega) > 0 \quad \text{for all } \omega \in \Theta \quad (13)$$

$$A(\omega)^T P(\omega) + P(\omega)A(\omega) + M_a < 0 \quad \text{for all } \omega \in \Theta \quad (14)$$

$$A_i^T P_i + P_i A_i \geq 0 \quad \text{for } i = 1, 2, \dots, K \quad (15)$$

$$M_a + \delta \cdot \{2P_0 B B^T P_0 + \sum_{i=1}^K \theta_i (P_0 B B^T P_i + P_i B B^T P_0)\} > 0 \quad \text{for all } \omega \in \Theta \quad (16)$$

**Proof:** We can express controller (12) as the following form introducing  $\beta(x(t))$ .

$$\begin{aligned} u(t) &= -\beta(x(t)) \cdot \delta B^T P_0 x(t) \\ \beta(x(t)) &= \frac{\text{sat}(\delta B^T P_0 x(t))}{\delta B^T P_0 x(t)} \\ \beta(x(t)) &= 1 \quad \text{if } B^T P_0 x(t) = 0 \end{aligned} \quad (17)$$

where  $0 < \beta(x(t)) \leq 1$ .

Along the trajectories of system (9) with the controller given in (17), the time derivative of  $V(x, \theta)$  in (5) is obtained

$$\begin{aligned}\dot{V}(x, \theta) &= x^T [A(\theta)^T P(\theta) + P(\theta)A(\theta)]x \\ &+ x^T [-\beta \cdot \delta \cdot \{2P_0 BB^T P_0 + \sum_{i=1}^K \theta_i (P_0 BB^T P_i + P_i BB^T P_0)\}]x\end{aligned}\quad (18)$$

To show  $\dot{V}(x, \theta) < 0$ , we first seek  $K + 1$  symmetric matrices  $P_0, P_1, P_2, \dots, P_K$  which satisfy that the first term in the right-hand term of (18) is less than 0, and then substitute the found matrices into the second term in the right-hand term of (18). Robust stability of the saturation controller (12) is guaranteed if the second term in the right-hand term of (18) is less than 0 with  $\delta > 0$ . Unfortunately, it is not guaranteed that the second term in the right-hand term of (18) is less than 0 with  $\delta > 0$ . However, we can obtain  $\delta > 0$  by introducing positive-definite symmetric matrix  $M_a$  and modifying two terms in (18) with  $M_a$ . (19) is obtained from adding and subtracting  $M_a$  each term in the right-hand term of (18).

$$\begin{aligned}\dot{V}(x, \theta) &= x^T [A(\theta)^T P(\theta) + P(\theta)A(\theta) + M_a]x \\ &+ x^T [-M_a - \beta \cdot \delta \cdot \{2P_0 BB^T P_0 + \sum_{i=1}^K \theta_i (P_0 BB^T P_i + P_i BB^T P_0)\}]x\end{aligned}\quad (19)$$

We assume that there exist symmetric matrices  $P_0, P_1, P_2, \dots, P_K$ , and  $M_a$  satisfying that the first term in the right-hand term of (19) is less than 0. Robust stability of the saturation controller of (12) is guaranteed if the second term in the right-hand term of (19) is less than 0 with  $\delta > 0$  when we substitute these matrices into the second term in the right-hand term of (19). The first term in the right-hand term of (19) is always less than 0 if there exist symmetric matrices  $P_0, P_1, P_2, \dots, P_K$ , and positive-definite symmetric matrix  $M_a$  satisfying (13), (14), and (15). Let  $\rho_i = \delta \theta_i (i = 1, 2, \dots, K)$ , then  $\underline{\delta \theta}_i \leq \rho_i \leq \overline{\delta \theta}_i$  and the second term in the right-hand term of (19) is rewritten as (20).

$$-x^T [M_a + \beta \{\delta (2P_0 BB^T P_0) + \sum_{i=1}^K \rho_i (P_0 BB^T P_i + P_i BB^T P_0)\}]x\quad (20)$$

Here we first consider the case of  $\beta = 1$ . For given  $\delta > 0$  the following LMI of (20) is a convex constraint on the variables  $\rho_i$  because  $M_a$ ,  $P_0 BB^T P_0$ , and  $P_0 BB^T P_i + P_i BB^T P_0$  are the symmetric matrices respectively.

$$M_a + \delta (2P_0 BB^T P_0) + \sum_{i=1}^K \rho_i (P_0 BB^T P_i + P_i BB^T P_0) > 0\quad (21)$$

When we define  $\Phi$  as the set of the  $2^K$  vertices of  $\rho_i$ , (21) is satisfied for all  $\rho_i$  if and only if (21) is satisfied in  $\Phi$  by convexity of (22).

$$\Phi := \{(\psi_1, \psi_2, \dots, \psi_K) : \psi_i \in \{\underline{\delta \theta}_i, \overline{\delta \theta}_i\}\}\quad (22)$$

(22) is equivalent to (16). Next we consider the case of  $0 < \beta < 1$ . For given  $\delta > 0$  we can easily show that (20) is less than 0 if (21) is satisfied. Therefore (20) is always less than 0 if (16) is satisfied for given  $\delta > 0$ . ■

**Remark 1.** In (14), we must solve LMIs with  $K + 2$  variables including matrix  $M_a$ . To reduce numerical computational burden we can reduce them to LMIs with  $K + 1$  variables by setting  $M_a$

in an arbitrary matrix. And we can seek  $\delta_{\max}$  by setting  $\delta$  in a fixed value and sweeping through  $\delta$  in (16), because the maximum value of  $\delta$  satisfying (16) is finite ( $\delta \leq \delta_{\max}$ ).  $\delta$  is used to maximize the utilization of the available control force. The system performance can be improved by using large  $\delta$ .

**Remark 2.** There is a difficulty in the numerical implementation of Theorem 1. It comes from the multi-convexity constraint of (15). As suggested in Gahinet *et al.* (1996), we can relax the multi-convexity of the function  $dV(x, \theta)/dt < 0$  by only requiring that it be bounded by a multi-convex function. In this case, for nonnegative definite symmetric matrix  $N_i (i = 1, 2, \dots, K)$  LMI conditions of (14) and (15) are replaced by (23), (24), and (25).

$$A(\omega)^T P(\omega) + P(\omega)A(\omega) + M_a + \sum_{i=1}^K \omega_i^2 N_i < 0 \quad \text{for all } \omega \in \Theta \quad (23)$$

$$A_i^T P_i + P_i A_i + N_i \geq 0 \quad \text{for } i = 1, 2, \dots, K \quad (24)$$

$$N_i \geq 0 \quad \text{for } i = 1, 2, \dots, K \quad (25)$$

A simple remedy is to choose  $N_i = \lambda_i I$  with  $\lambda_i > 0$ .

## NUMERICAL EXAMPLE

In this section, a practical numerical example for a linear building is illustrated to verify the feasibility of the proposed robust saturation controller (12) and simulation results are presented. LMIs in Theorem 1 are solved using Matlab<sup>®</sup> and LMI control toolbox (Gahinet *et al.* 1995). Controller design parameter  $M_a$  is chosen at an arbitrary value by trial and error as suggested in Remark 1.

A three-story scaled building model studied by Kobori *et al.* 1992 and Yang *et al.* 1995a, in which every story unit is identically constructed and an active brace system (ABS) is installed in the first-story unit, as shown in Fig. 1, is considered. The mass, stiffness and damping coefficient of each story unit for nominal system are  $m_i = 1000kg$ ,  $k_i = 980kN/m$ , and  $c_i = 1.407kNs/m$ , respectively, for  $i = 1, 2, 3$ . The El Centro earthquake (north-south component, 1940) scaled to a maximum acceleration of  $0.112g$  is used as the input excitation. The maximum control force  $u_{\max}$  is  $700N$  and uncertainties of the system are stiffnesses and damping coefficients of each floor. Let uncertainties of stiffnesses be  $\theta_i (i = 1, 2, 3)$  and the uncertainties of damping coefficients  $\theta_i (i = 4, 5, 6)$ , then the admissible trajectories are given by  $k_i(1 + \theta_i)$  for  $i = 1, 2, 3$  and  $c_{i-3}(1 + \theta_i)$  for  $i = 4, 5, 6$  specified in multiplicative form. This uncertain system can be described by state space equation as follows.

$$\dot{x}(t) = A(\theta)x(t) + Bu(t) + E\ddot{x}_g \quad (26)$$

where  $x_i (i = 1, 2, 3)$  are the relative displacement of each floor to ground, state vector  $x = [x_1 \ x_2 \ x_3 \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_3]^T$ , control input vector  $B = [0 \ 0 \ 0 \ 1/m_1 \ 0 \ 0]^T$ , the disturbance input vector  $E = [0 \ 0 \ 0 \ -1/m_1 \ -1/m_2 \ -1/m_3]^T$ , and uncertain system matrix  $A(\theta)$  is

$$A(\theta) = A_0 + \theta_1 A_1 + \theta_2 A_2 + \theta_3 A_3 + \theta_4 A_4 + \theta_5 A_5 + \theta_6 A_6 \quad (27)$$

$$\text{where } A_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} & 0 \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_2} & \frac{c_2}{m_2} & -\frac{c_2+c_3}{m_2} & \frac{c_3}{m_2} \\ 0 & \frac{k_3}{m_3} & -\frac{k_3}{m_3} & 0 & \frac{c_3}{m_3} & -\frac{c_3}{m_3} \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{k_3}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_3}{m_3} & -\frac{k_3}{m_3} & 0 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_2}{m_1} & \frac{c_2}{m_1} & 0 \\ 0 & 0 & 0 & \frac{c_2}{m_2} & -\frac{c_2}{m_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{c_3}{m_2} & \frac{c_3}{m_2} \\ 0 & 0 & 0 & 0 & \frac{c_3}{m_3} & -\frac{c_3}{m_3} \end{bmatrix}.$$

Before we confirm the robust stability of the proposed controller, we ascertain general trend of the proposed controller according to the value of the controller design parameter  $M_a$ . For convenience we replace  $M_a$  by  $\mu_a I$  with  $\mu_a > 0$ . We investigated the maximum value of  $\delta$  according to  $\mu_a$ . The control performance can be improved by using large  $\delta$  because  $\delta$  is used to maximize the utilization of the available control force. Let  $|\theta_i| \leq \theta_e$  ( $i=1,2,\dots,6$ ) for all parameter uncertainties. Here we set  $\theta_e$  arbitrarily. Table 1 shows values of  $\delta_{\max}$  according to  $\mu_a$  in the case of  $\theta_e = 0.4$ . The larger  $\mu_a$  is, the larger  $\delta_{\max}$  is. So the larger  $\mu_a$  makes the control performance better because  $\delta_{\max}$  is closely related to the control performance of the proposed controller.

Next, we confirm the robust stability and the effectiveness of the proposed controller. For given bounds of uncertain parameters  $\theta_e = 0.4$ , the controller design parameter  $M_a = \text{diag}(5e5, 5e5, 5e5, 5e2, 5e2, 5e2)$  is chosen for good performance of the controller. The computed value of  $\delta_{\max}$  is about 519. We designed the robust saturation controller (12) with  $\delta = 519$ . Control performance of the proposed controller is compared with other controllers from the viewpoint of maximum responses (maximum interstory drifts  $d_i$  and maximum absolute accelerations  $\ddot{x}_{ai}$ ).

For nominal system, Table 2 shows control performance of the proposed controller on maximum response values of the system in comparison with those of the classical LQR controller, the modified bang-bang controller (MBBC), and the saturated sliding mode controller (SSMC). The LQR controller is adjusted so that maximum control force is about 700N. It is observed from Table 2 that control performances of saturation controllers such as the MBBC, the SSMC, and the proposed controller are quite remarkable in comparison with that of the LQR controller. Time histories for absolute acceleration of the third-story unit, drift of the first-story unit, and control force using the LQR controller and the proposed controller are presented in Fig. 2 in comparison with the responses without control. Saturation controllers considering the limit of control force are known to be more effective than the LQR controller in maximum response reduction under the same maximum control force (see Wu *et al.* 1996 for the MBBC and Yang *et al.* 1995a for the SSMC). The proposed controller also produces better performance than the LQR controller in terms of maximum response reduction under the same maximum control force as shown in Table 2 and Fig. 2. In the reduction of interstory drifts, the MBBC is the most effective and the proposed controller and the SSMC have almost the same effectiveness.

For uncertain system, we ascertained that our proposed controller guarantees robust stability within all the range of parameter uncertainties considered in controller design. Robust stability of the proposed controller, which is guaranteed in Theorem 1 analytically, was verified through numerical simulations for the cases with various parameter uncertainties. Table 3 shows control performance of the proposed controller on maximum response values of the system in comparison with that of the



SSMC in the case of parameter uncertainties with  $\theta_i = 0.4(i = 1, 2, \dots, 6)$  and  $\theta_i = -0.4(i = 1, 2, \dots, 6)$ . The control performances of these two controllers are almost similar. Through extensive numerical simulations, it is checked within considered bounds of parameter uncertainties that the proposed controller has almost the same effectiveness in maximum responses reduction in comparison with the SSMC. But from the viewpoint of robust stability, the proposed controller can only address bounds of parameter uncertainties analytically within which robust stability is guaranteed over the complete response of system.

Our proposed controller uses saturation function instead of sign function. And for given bounds of parameter uncertainties, maximum value of  $\delta$  guaranteeing robust stability of this controller is finite. The larger bounds of parameter uncertainties are, the smaller maximum value of  $\delta$  is. So the slope of saturation function is gentler as bounds of parameter uncertainties are larger as shown in Fig. 3. This trend can be checked through the following Fig. 4 in our example. Fig. 4 shows control forces of three cases ( $\theta_e = 0$ ,  $\theta_e = 0.2$ , and  $\theta_e = 0.4$ ) for nominal system. The case of  $\theta_e = 0$  shows bang-bang type control force of the MBBC because it corresponds to nominal system. The slope of  $\theta_e = 0.4$  is gentler than that of  $\theta_e = 0.2$  because of smaller maximum values of  $\delta$ . Therefore, our proposed controller moves away further from bang-bang type controller and its control performance may be worse in comparison with the MBBC for nominal system as bounds of parameter uncertainties are larger. However, we can guarantee robust stability at the cost of performance degradation.

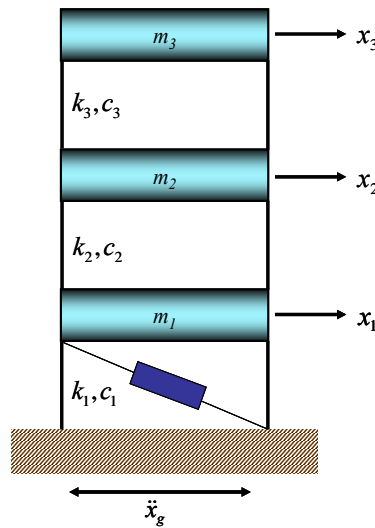


Fig. 1. Three-story building model with ABS

Table 1 Values of  $\delta_{\max}$  according to  $\mu_a$  ( $|\theta_i| \leq 0.4$ )

$\mu_a$	$\delta_{\max}$
5e1	0.9
5e2	1.1
1e3	2.1
5e3	4.2
1e4	21.6
5e4	88.3
6e4	104.5

Table 2 Maximum response values for nominal system

Story	No Control		LQR		MBBC		SSMC		Proposed	
	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$
1	1.34	3.13	0.88	2.25	0.51	1.64	0.52	1.62	0.53	1.60
2	1.02	4.75	0.66	3.24	0.47	2.27	0.49	2.27	0.48	2.20
3	0.60	5.84	0.35	3.41	0.32	3.20	0.33	3.27	0.32	3.17

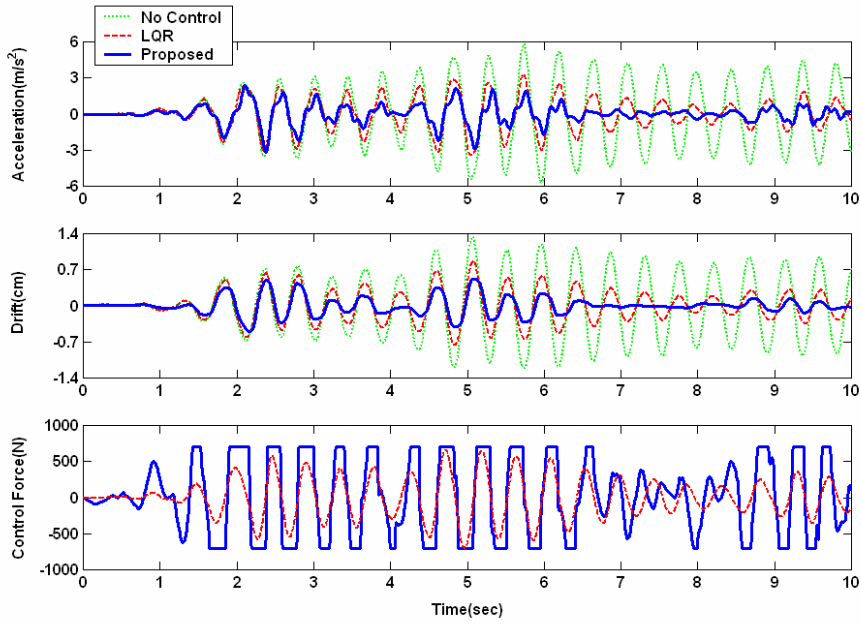


Fig. 2. Comparison of responses and control forces for nominal system applying the LQR controller and the proposed controller

Table 3 Maximum response values for uncertain system

Story	$\theta_i = 0.4(i = 1, 2, \dots, 6)$						$\theta_i = -0.4(i = 1, 2, \dots, 6)$					
	No Control		SSMC		Proposed		No Control		SSMC		Proposed	
	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$	$d_i(cm)$	$\ddot{x}_{ai}(m/s^2)$
1	0.70	2.11	0.31	1.86	0.31	1.82	1.64	2.82	1.05	1.91	1.06	2.01
2	0.57	3.39	0.28	1.91	0.28	1.94	1.34	3.45	0.97	2.37	0.97	2.47
3	0.34	4.64	0.20	2.69	0.19	2.64	0.79	4.63	0.61	3.60	0.61	3.59

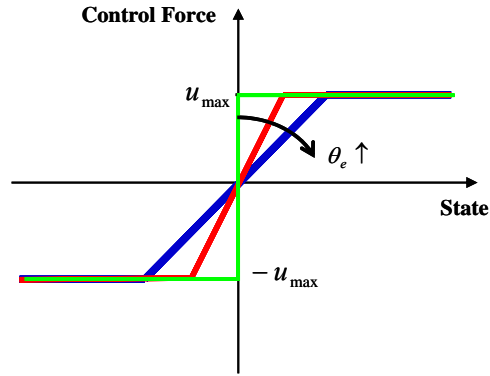


Fig. 3. The slopes of saturation function of the proposed controller according to given bounds of parameter uncertainties

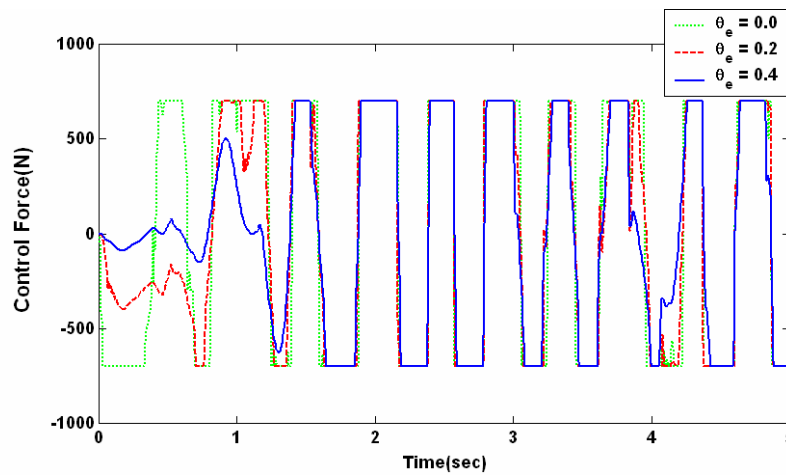


Fig. 4. Comparison of control forces of the proposed controller for nominal system according to given bounds of parameter uncertainties

## CONCLUSIONS

The objective of this paper is to develop robust saturation controller guaranteeing robust stability of uncertain LTI system over the prescribed upper and lower bounds of structured real parameter uncertainties analytically. Based on affine quadratic stability and multi-convexity concept, a robust saturation controller was newly proposed. Theorem 1 suggested in this paper gives LMI-based sufficient conditions for the existence of this controller by introducing controller design parameter  $M_a$ .

A general trend of the proposed controller was examined through numerical simulations. The larger bounds of parameter uncertainties are, the smaller maximum value of  $\delta$  in controller (12) is. Therefore, while the proposed controller guarantees robust stability within bounds of parameter uncertainties, its control performance may be worse in comparison with the any other saturation controllers for nominal system. Note that to guarantee the robust stability for the system uncertainties, which is inevitable in real world, sacrificing the performance a little can be sensible and practical.

The availability and the effectiveness of the proposed controller were also verified through numerical simulations. Simulation results show that the proposed controller is robustly stable with respect to parameter uncertainties over the prescribed upper and lower bounds and the proposed controller can be easily applicable for civil engineering structures.

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