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Development of an indicial function approach for the two-dimensional incompressible/compressible aerodynamic load modelling

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Abstract: By using a combined analytical–computational methodology, a unified modelling of aerodynamic indicial functions covering the incompressible, subsonic compressible, transonic, and supersonic flight speed regimes is presented. The procedure is carried out in conjunction with a computational fluid dynamic analysis. For a plunging–pitching airfoil, selected unsteady aerodynamic load expressions have been supplied, and appropriate procedures enabling one to obtain these loads via the indicial function approach have been presented. While a single indicial function is needed to describe the aerodynamic loads in the incompressible flight speed regime, for cases where the compressibility effects play a dominant role, four indicial functions are needed. Having in view the usefulness of indicial functions towards determination of unsteady aerodynamics loads in both time and frequency domains, and implicitly for aeroelastic response and flutter predictions, the advantages of their implementation and use appear evident. Comparisons and validations of the aerodynamic model against numerical, analytical, and experimental results are presented, and pertinent conclusions are drawn.

Keywords: linear/non-linear indicial aerodynamic functions, incompressible and compressible flight speed regimes, two-degrees-of-freedom airfoil aerodynamics

1 INTRODUCTION

In this paper, the problem of determination of unsteady aerodynamic loads on a two-degrees-of-freedom (two-DOF) airfoil using a methodology based on the linear and non-linear aerodynamic indicial functions, in conjunction with the computational fluid dynamics (CFD), is addressed. Incompressible, subsonic compressible, transonic, supersonic, and hypersonic flight speed regimes are considered. The unsteady aerodynamic loads via indicial function concept can be expressed in time and frequency domains. While the time-domain representation

is necessary for the open-closed-loop aeroelastic response analyses, the frequency-domain representation is required for the determination of the flutter instability boundary [1]. However, as it was shown in reference [2], also the time-domain representation of aerodynamic loads can be applied for the same purpose. Unsteady compressible aerodynamic loads are obtained by using the compressible counterparts of Wagner and Theodorsen's incompressible indicial functions, respectively references [1], [3], and [4], while in the transonic flight speed regime, the unsteady lift and aerodynamic moment are derived in terms of the non-linear indicial functions. The indicial expressions can involve the arbitrary time variation of the angle-of-attack and/or inflow velocity, and are carried out using Duhamel's superposition principle. In reference [5], the aeroelastic response of a single-DOF control surface obtained by the linearized unsteady transonic flow code LTRANZ was presented,

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and to this end, a simultaneous time integration of both the structural and aerodynamic equations was carried out. Assuming linear aerodynamics, an indicial method was also used and, in that context, an aerodynamic impulse function was first calculated by the aerodynamic code and then used in flutter studies.

The flutter of the same airfoil, but with two-DOF was analysed in reference [6]. The aerodynamic forces were obtained by three methods: time-integration, indicial function concept, and via harmonic analysis. In general, all three methods agree well for the range of the considered parameters. Isogai [7] used the time-integration method for evaluating the transonic aerodynamic forces that were converted to linearized harmonic time-dependent representation for flutter calculation. Linearity can be assumed if the amplitude of the airfoil oscillation is sufficiently small [7, 8], even though in the transonic flowfield, the governing aeroelastic equations are inherently non-linear. Non-linear effects of the transonic aerodynamic forces on catastrophic or benign character of flutter boundary (i.e. stable and unstable limit cycle oscillations (LCOs)) of a typical section airfoil were studied by Ueda and Dowel [9], using a variation of the describing function method that takes into account the first fundamental harmonic of the non-linear oscillatory motion.

Within the methodology developed here, a formulation based on the linear and non-linear indicial functions for the unsteady aerodynamics covering the entire speed range from the incompressible to the hypersonic one is presented. Herein, a two-dimensional wing section featuring the plunging and pitching degrees of freedoms is considered (Fig. 1). Validation of the aerodynamic model and comparisons with the results obtained via CFD-based unsteady Euler codes with moving grid techniques, experimental data, and linear/non-linear theories are also supplied.

Among the advantages provided by the use of the aerodynamic indicial function concept, the following are taken into consideration.

1. Accurate approach towards describing the aerodynamic characteristics in incompressible and compressible flight speed regimes.

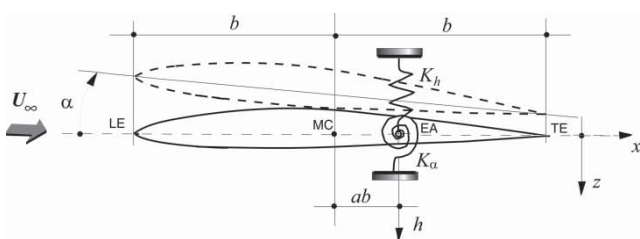


Fig. 1 Typical wing section

2. Possibility to obtain the unsteady airloads on lifting surfaces undergoing arbitrary time-dependent motion.
3. Unified unsteady aerodynamic formulation in incompressible and compressible flight speed regimes.
4. Possibility of approximating the indicial functions by analytical, computational fluid dynamics, or through experimental means.
5. The linear (non-linear) aerodynamic indicial functions in the frequency and time domains can be used in flutter (post-flutter) analyses, as well as addressing the aeroelastic response and the feedback control of two-dimensional lifting surfaces, respectively.
6. As indicated in reference [10], the indicial approach can also be used for the study of dynamics of aircraft, carrying out large amplitude manoeuvres.

2 INDICIAL FUNCTION MODELLING: LINEAR VERSUS NON-LINEAR FORMULATION

A brief description of the linear and non-linear indicial function theory is presented next. In this context, for illustration purposes, the incompressible flight speed regime is considered.

2.1 Linear indicial function formulation

The linear theory is based on the premise that the flow characteristics, such as the lift and the aerodynamic moment coefficients, $C_L(\tau)$ and $C_M(\tau)$, respectively, can be linearized with respect to the downwash velocity $w(\tau)$ if the variations of $C_L(\tau)$ and $C_M(\tau)$ are smooth functions of $w(\tau)$ [4, 10]. This enables one to represent the lift and moment coefficients in terms of a Taylor series about $w = w_0 = w(0)$. Assuming that $C_L(0)$ and $C_M(0)$ are zero, then the approximate solutions for the lift and moment coefficients are

$$C_L(\tau) = \Delta w \left. \frac{\partial C_L}{\partial w} \right|_{w=w_0} \quad (1a)$$

$$C_M(\tau) = \Delta w \left. \frac{\partial C_M}{\partial w} \right|_{w=w_0} \quad (1b)$$

This approximate expression becomes more accurate as $\Delta w \rightarrow 0$ and exact if $C_L(\tau)$ and $C_M(\tau)$ are linear functions of $w(\tau)$. If the responses $\partial C_L/\partial w$ and $\partial C_M/\partial w$ depend only on the elapsed time $(\tau - \sigma)$, then it may be shown that the solution for $C_L(\tau)$ and $C_M(\tau)$ is [3, 4, 10]

$$C_L(\tau) = \phi_L(\tau)w(0) + \int_0^\tau \frac{dw}{d\sigma} \phi_L(\tau - \sigma) d\sigma \quad (2a)$$

$$C_M(\tau) = \phi_M(\tau)w(0) + \int_0^\tau \frac{dw}{d\sigma} \phi_M(\tau - \sigma) d\sigma \quad (2b)$$

In equations (2a) and (b),

$$\phi_L(\tau) \equiv \left. \frac{\partial C_L}{\partial w} \right|_{w=w_0} \quad (3a)$$

$$\phi_M(\tau) \equiv \left. \frac{\partial C_M}{\partial w} \right|_{w=w_0} \quad (3b)$$

define the indicial functions. For the incompressible flowfield, both functions coincide to the well-known Wagner's function $\phi(\tau)$ that expresses the lift due to a unit change of the airfoil angle of attack [3, 4]. Equations (2a) and (b) are semi-analytic relations between $C_L(\tau)$, or $C_M(\tau)$, and $w(\tau)$. Hence, if the downwash velocity $w(\tau)$ and $\phi(\tau)$ are known, equations (2a) and (b) provide $C_L(\tau)$ and $C_M(\tau)$, respectively, for any schedule of $w(\tau)$.

Since the indicial functions can be further approximated, for example by exponential functions, it may be shown that the entire unsteady aerodynamic prediction amounts to solving a low-dimensional system of non-homogeneous coupled first-order ordinary differential equations. This yields a further reduction of the computational time.

2.2 Non-linear aerodynamic indicial function formulation

Following the developments by Tobak [11] and Reisentel [10], a non-linear formulation based on the indicial function as to obtain the non-linear unsteady aerodynamic lift and moment in the transonic flight speed regime is presented.

By extending the discussion of the previous section, the approximate solutions of $C_L(\tau)$ and $C_M(\tau)$ are

$$C_L(\tau) = C_L(0) + \Delta w \left. \frac{\partial C_L}{\partial w} \right|_{w=w_0} + \dots \quad (4a)$$

$$C_M(\tau) = C_M(0) + \Delta w \left. \frac{\partial C_M}{\partial w} \right|_{w=w_0} + \dots \quad (4b)$$

The linear formalism yielding equations (2a) and (b) can be retained in the form of a generalized superposition integral, provided that the non-linear indicial functions ϕ_L and ϕ_M are now taken to be functionals involving $w(\zeta)$, i.e. $\phi_L[w(\zeta); \tau, \sigma]$ and $\phi_M[w(\zeta); \tau, \sigma]$. Assuming that $C_L(\tau)$ and $C_M(\tau)$ are Fréchet differentiable [4, 10], one can write

$$C_L(\tau) = C_L[\tau, w(0)] + \int_0^\tau \frac{dw}{d\sigma} \phi_L[w(\zeta); \tau, \sigma] d\sigma \quad (5a)$$

$$C_M(\tau) = C_M[\tau, w(0)] + \int_0^\tau \frac{dw}{d\sigma} \phi_M[w(\zeta); \tau, \sigma] d\sigma \quad (5b)$$

Equations (5a) and (b) are generalizations of the linear Duhamel's convolution model, in the sense of their representation in terms of the prior motion indicated by any arbitrary schedule of $w(\zeta)$. The non-linear indicial functions $\phi_L[w(\zeta), \tau, \sigma]$ and $\phi_M[w(\zeta), \tau, \sigma]$ are defined in terms of Fréchet derivatives as [4, 10]

$$\phi_L[w(\zeta); \tau, \sigma] \quad (6a)$$

$$\begin{aligned} &= \lim_{\Delta w \rightarrow 0} \frac{\Delta C_L(\tau)}{\Delta w} \\ &= \lim_{\Delta w \rightarrow 0} \left\{ \frac{C_L[w(\zeta) + H(\zeta - \sigma)\Delta w] - C_L[w(\zeta)]}{\Delta w} \right\} \end{aligned} \quad (6b)$$

$$\phi_M[w(\zeta); \tau, \sigma]$$

$$\begin{aligned} &= \lim_{\Delta w \rightarrow 0} \frac{\Delta C_M(\tau)}{\Delta w} \\ &= \lim_{\Delta w \rightarrow 0} \left\{ \frac{C_M[w(\zeta) + H(\zeta - \sigma)\Delta w] - C_M[w(\zeta)]}{\Delta w} \right\} \end{aligned} \quad (6c)$$

In equations (6a) and (b), the step in Δw is applied at time $\tau = \sigma$ and H denotes the Heaviside step function. As it was remarked in references [10] and [11], the non-linear indicial function approach differs from the linear one in several respects: (a) $\phi[w(\zeta); \tau, \sigma]$ has a separate dependence on τ and σ , rather than on the elapsed time $(\tau - \sigma)$ alone, implying that $\phi[w(\zeta); \tau, \sigma]$ depends on the past history of $w(\zeta)$; (b) the functional dependency on $w(\zeta)$ itself distinguishes the non-linear indicial response from its linear counterpart. When the system is linear time invariant, implying that $\phi[w(\zeta); \tau, \sigma] \equiv \phi[\tau - \sigma]$, the Duhamel's convolution integral of the linear indicial theory, equations (2a) and (b), is obtained. In the incompressible case, the only indicial function necessary to describe the aerodynamic loads is the Wagner function $\phi (= \phi_L = \phi_M)$.

3 FORMULATION OF THE AEROELASTIC PROBLEM

As shown by Edwards *et al.* [12] for linear systems, the aeroelastic governing equations of lifting surfaces can be converted from the time domain to the frequency domain via a Laplace transform. In the case of zero initial conditions, using the correspondence $s \rightarrow ik$, the time-domain representation of the aerodynamic loads passes into the frequency-domain representation. Herein, s and $k(= \omega b/U_\infty)$ are the Laplace variable and the reduced frequency, respectively, while $i (= \sqrt{-1})$ is the imaginary unit. As it is well known, Wagner's function $\phi(\tau)$ is connected with Theodorsen's function $C(k)$ via a Laplace

transform [1, 2, 3]. Using it, from the indicial function pertinent to the incompressible speed range, one can obtain the expressions of unsteady aerodynamic coefficients in terms of Theodorsen’s function $C(k)$ and of its components $F(k)$ and $G(k)$ [1]. Whereas within the linear indicial theory the linear kernel or linear impulse response can be convoluted with the input to predict the output of a linear system, the non-linear indicial theory constitutes a generalization of this concept. In this sense, the non-linear indicial aerodynamic theory implies that the aeroelastic response of a non-linear system to an arbitrary input can be constructed by integrating a non-linear functional that involves the knowledge of the time-dependent input and the kernel response [10, 11].

4 UNSTEADY AERODYNAMIC LOADS

4.1 Incompressible flight speed regime

The circulatory component $L_c(\tau)$ of the aerodynamic lift expressed in terms of indicial Wagner’s function (also called heredity function) is

$$L_c(\tau) = -C_{l\alpha} b \rho_\infty U_\infty^2 C_L(\tau) = -C_{l\alpha} b \rho_\infty U_\infty^2 \left\{ w(0) \phi(\tau) + \int_0^\tau \frac{\partial w(\sigma)}{\partial \sigma} \phi[\tau - \sigma] d\sigma \right\} \quad (7)$$

where the downwash velocity at the 3.4-chord can be represented as

$$w(\tau) = \alpha(\tau) + \frac{1}{b} h'(\tau) + \left(\frac{1}{2} - a \right) \alpha'(\tau) \quad (8)$$

Herein, $h \equiv h(\tau)$ and $\alpha \equiv \alpha(\tau)$ are the plunging displacement (positive down) and the twist angle about the pitch axis (positive nose up), respectively. Since $M_c(\tau) = -C_{l\alpha} b \rho_\infty U_\infty^2 C_M(\tau)$ and $C_M(\tau) = ((1/2) + a) b C_L(\tau)$, the circulatory term $M_c(\tau)$ of the aerodynamic moment about the reference axis (mid-chord) can be written as

$$M_c(\tau) = \left(\frac{1}{2} + a \right) b L_c(\tau) \quad (9)$$

The aerodynamic non-circulatory components, due to the impulsive behaviour at time $\tau = 0$, can be derived using the filtering property of Dirac’s impulsive function in equation (7)

$$L_{nc1}(\tau) = -\frac{1}{2} \rho_\infty C_{l\alpha} U_\infty^2 [h''(\tau) - ab\alpha''(\tau)] \quad (10a)$$

$$L_{nc2}(\tau) = -\frac{1}{2} \rho_\infty C_{l\alpha} b U_\infty^2 \alpha'(\tau) \quad (10b)$$

$$M_{nc1}(\tau) = ab L_{nc1} \quad (11a)$$

$$M_{nc2}(\tau) = -\left(\frac{1}{2} - a \right) b L_{nc2} \quad (11b)$$

$$M_3(\tau) = \frac{1}{16} \rho_\infty C_{l\alpha} b^2 U_\infty^2 \alpha''(\tau) \quad (11c)$$

In the expressions of both the unsteady lift and moment, the coupling between the plunging and twist motions, in the sense of the involvement of time derivatives of both α and h , appears explicitly. This is referred to as the aerodynamic coupling. Note that in the incompressible flight speed regime, for infinite aspect ratio wings, the lift-curve slope $C_{l\alpha}$ assumes the value 2π , whereas in the subsonic and supersonic flowfields, $C_{l\alpha} = 2\pi/(1 - M^2)^{1/2}$ and $C_{l\alpha} = 4/(M^2 - 1)^{1/2}$, respectively [3]. The expressions in the time domain of the unsteady lift can be determined by adding equations (7) and (10), whereas the aerodynamic moment about the axis at $x = ab$, rearward from the mid-chord, by taking into account equation (7) in equation (9) as well as equations (10) considered in conjunction with equations (11).

4.2 Subsonic compressible flight speed regime

For subsonic compressible flows, there is a significant work devoted to the development of appropriate analytical expressions for the indicial functions. In this context, the circulation around the airfoil is determined by a set of four indicial functions [1–3, 13–15]. In contrast to the incompressible flow case for which the governing aerodynamic equation is of an elliptic type, for the subsonic compressible flow in which case the governing equation is of a hyperbolic type, with exception of restricted values of time, the indicial functions are not analytic. In addition, the indicial model for the incompressible flow speed features at $\tau = 0$ – an infinite value, modelled as a Dirac pulse $\delta(\tau)$, whereas the indicial model for the compressible speed aerodynamic experiences a finite value. More recently, the indicial functions in subsonic compressible flow have been derived, and their approximation and validation for any value of Mach number was carried out [4, 14]. The effect of the Mach number can be taken into account in the indicial description, and this has also been emphasized in reference [4].

Being a non-linear partial differential equation, the velocity potential equation for the compressible flowfield cannot be solved analytically, and usually the solution is obtained using finite difference numerical techniques. This is in contrast to the case of incompressible irrotational flows that is described by a linear partial differential equation (i.e. Laplace’s equation), for which analytical solutions are available. Nevertheless, various techniques can be used to determine an approximate form of the indicial response. For practical computational purposes, the indicial functions can be expressed in a generic form as

$$\phi(\tau) = \sum_{i=0}^3 A_i e^{-\beta_i \tau} \quad (12)$$

For selected Mach numbers, the related coefficients A_i and β_i are determined numerically, see e.g.

reference [1]. For compressible flight speed regimes, four indicial functions are needed. These quantities identified as $(\phi_\alpha^c; \phi_{M\alpha}^c)$ and $(\phi_q^c; \phi_{Mq}^c)$ are the lift and moment indicial functions in the compressible flow associated with the pitch angle α and the pitch rate q ($\equiv (2b/U_\infty)\dot{\alpha}$), respectively. The indicial functions are derived from an unsteady Euler code based on structured (Euler 1) and unstructured (Euler 2) grids. A finite-volume spatial discretization method is used to solve the governing equations. An Arbitrary Lagrangian–Eulerian (ALE) formulation for the Euler equations is used to calculate the flow flux with moving boundaries. More detailed information for theoretical background, numerical verifications, and applications of the present Euler codes based on a structured grid can be found in reference [16]. A comprehensive study of the unsteady aerodynamics based on the Euler equation and the application to aeroelastic calculations of two-dimensional wings is presented in reference [17].

In order to overcome the fact that the local moving grid (LMG) based on the spring analogy is not appropriate to describe the indicial pitching motion without angle-of-attack changes [3], a total moving grid (TMG) technique, which is a pure Lagrangian description, has been used [4].

In the TMG technique, a whole computational grid has a global motion according to its original flight path. For the pure plunging motion, both the TMG and the LMG techniques can be used. For the LMG technique, a step motion of the angle-of-attack change can be equivalently applied to describe the pure plunging motion. The indicial moment coefficients for the prescribed motions are computed at 1/4-chord measured from the leading edge profile.

In Fig. 2, indicial functions directly calculated by the present CFD technique are compared with analytical linear indicial functions in subsonic flow (linear asymptotic solution LAS [3]) and supersonic flow (Lomax [13]). Two different airfoils such as the NACA 0006 and NACA 0012 are considered, and both the TMG and the LMG techniques are applied. For the NACA 0006 airfoil at $M = 0.5$ and $M = 2.0$, the converged solutions by the present Euler codes with two different techniques show a very good agreement with the closed-form solutions of references [3], [13], and [14]. However, in the transonic speed range, $M = 0.8$ – 1.2 , due to the presence of non-linear effects induced by the shocks waves, some differences are experienced.

A step function used in this study is $\alpha(\tau) = \alpha_m(10\delta^3 - 15\delta^4 + 6\delta^5)$ for $\tau < \tau_0$, and $\alpha(\tau) = \alpha_m$ for

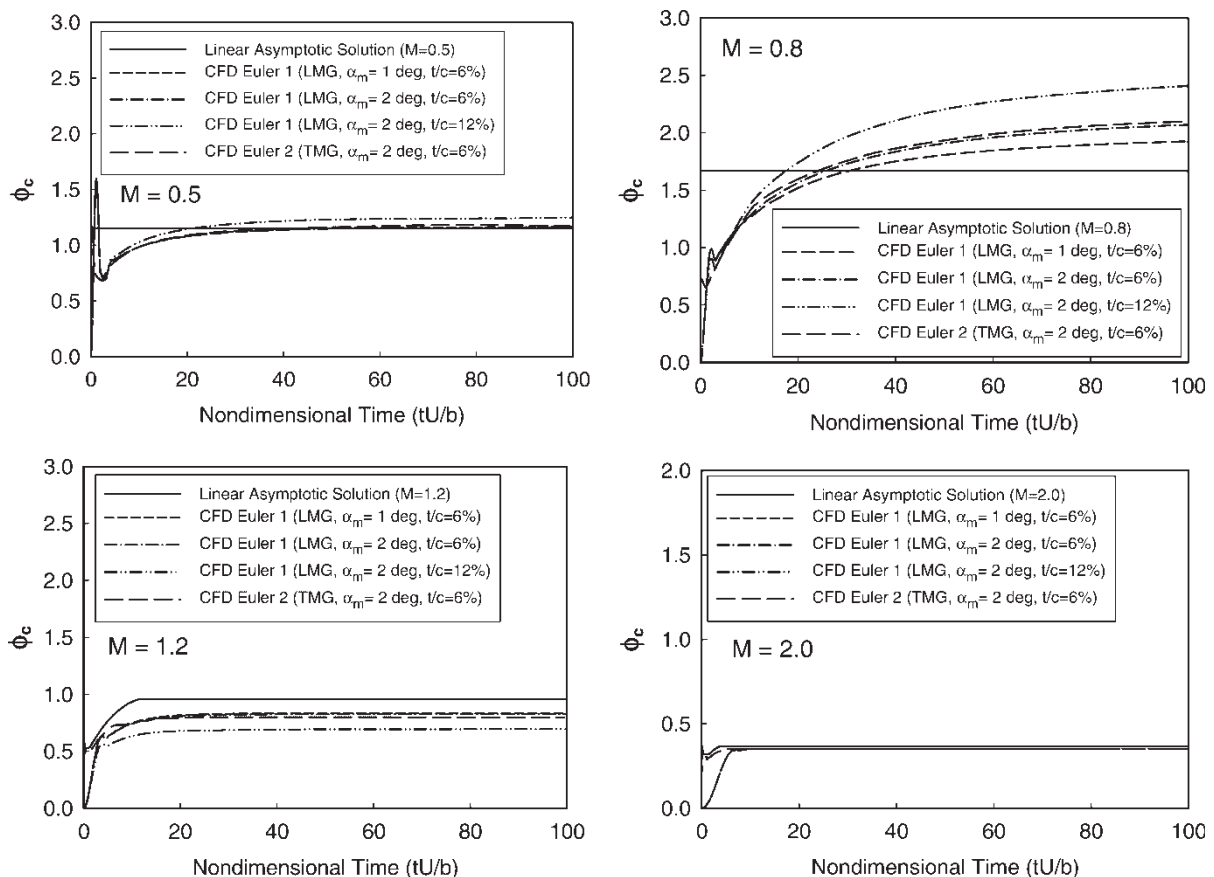


Fig. 2 Comparison of indicial functions for the plunging motion. Subsonic (LAS [4]), supersonic (Lomax [12])

$\tau \geq \tau_0$. Here, α_m denotes the amplitude angle of a step motion and $\delta = \tau/\tau_0$. For this case, two different amplitudes of the angle-of-attack change for a step motion are simulated: 1° and 2° . In general, with the exception of initial transient responses, the solutions by TMG and LMG techniques are in good agreement.

Selected indicial functions obtained by using the present method in conjunction with a non-linear curve fitting code are illustrated in Fig. 3. Herein, comparisons with the CFD Euler code, indicial function, and the ones based on the approaches presented in references [3] and [12] are presented. Although the predictions agree well with the ones based on the indicial functions by Bisplinghoff *et al.* [3] in the subsonic flight speed range (i.e. for $M < 0.8$), and with the ones by Lomax [13] in the supersonic flight speed range (i.e. for $M > 1.25$), it clearly appears that the linear model does not describe properly the transonic aerodynamics. For $0.9 < M < 1.25$, Fig. 3 reveals that the linear indicial functions cannot be applied in the transonic speed range; this fact was pointed out also in references [3], [13], and [14].

The unsteady lift \bar{L}_a^c and aerodynamic moment \bar{M}_a^c in the compressible flight speed regime evaluated at the leading edge ($x = -b$), for arbitrary plunging and pitching about the leading edge, can be expressed as

$$\bar{L}_a^c(\tau) = -C_{l\alpha} b \rho_\infty U_\infty^2 \int_{-\infty}^{\tau} \left\{ \bar{\phi}_\alpha^c[\tau - \sigma] \left(\alpha'(\sigma) + \frac{h''(\sigma)}{b} \right) - 2\bar{\phi}_q^c[\tau - \sigma] \alpha''(\sigma) \right\} d\sigma \quad (13a)$$

$$\bar{M}_a^c(\tau) = -2C_{l\alpha} b^2 \rho_\infty U_\infty^2 \int_{-\infty}^{\tau} \left\{ \bar{\phi}_{M\alpha}^c[\tau - \sigma] \times \left(\alpha'(\sigma) + \frac{h''(\sigma)}{b} \right) - 2\bar{\phi}_{Mq}^c[\tau - \sigma] \alpha''(\sigma) \right\} d\sigma \quad (13b)$$

The concept of added mass in the compressible speed range is meaningless. In order to have a unique formulation in both the incompressible and compressible speed regimes, the indicial functions in equations (13a) and (b) have to be expressed at the axis $x = ab$ rearward from the mid-chord in a form that is similar to that used in the incompressible flight speed regime. To this end, their modified expressions have to be used [1]. These are

$$\phi_\alpha^c = \bar{\phi}_\alpha^c \quad (14a)$$

$$\phi_{M\alpha}^c = \bar{\phi}_{M\alpha}^c + \left(\frac{a}{2} + \frac{1}{2} \right) \bar{\phi}_\alpha^c \quad (14b)$$

$$\phi_q^c = \bar{\phi}_q^c - \left(\frac{a}{2} + \frac{1}{2} \right) \bar{\phi}_\alpha^c \quad (14c)$$

$$\phi_{Mq}^c = \bar{\phi}_{Mq}^c + \left(\frac{a}{2} + \frac{1}{2} \right) (\bar{\phi}_q^c - \bar{\phi}_{M\alpha}^c) - \left(\frac{a}{2} + \frac{1}{2} \right)^2 \bar{\phi}_\alpha^c \quad (14d)$$

where $\bar{\phi}_\alpha^c$, $\bar{\phi}_{M\alpha}^c$, ϕ_α^c , and $\phi_{M\alpha}^c$ are the indicial lift and moment functions about the leading edge ($x = -b$) and about the axis ($x = ab$) due to a

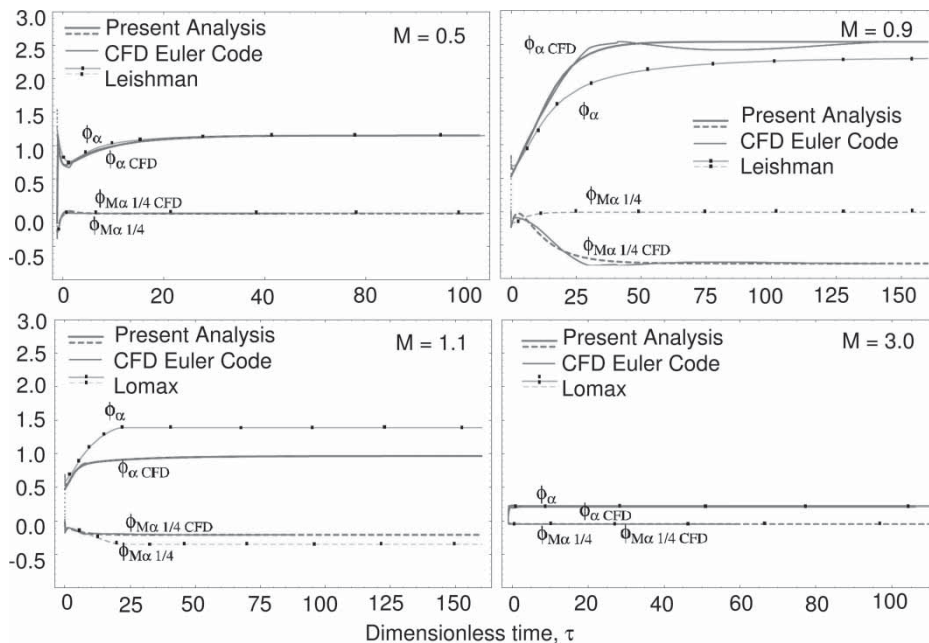


Fig. 3 Comparison of subsonic/transonic/supersonic indicial functions for plunging/pitching motions with Bisplinghoff [3], Lomax [12], and Leishman [14]

unit step change of the vertical translation velocity at the leading edge, respectively; $\bar{\phi}_q^c, \bar{\phi}_{Mq}^c; \phi_q^c, \phi_{Mq}^c$ are the indicial lift and moment functions about the leading edge ($x = -b$) and about the axis ($x = ab$) due to a unit step change of the pitching rate ($q \equiv (2b/U_\infty) \dot{\alpha} \equiv 2\alpha'$) at the leading edge. The unsteady lift L_a^c and aerodynamic moment M_a^c in the compressible flight speed regime evaluated at the axis ($x = ab$) can be expressed in the Laplace-transformed space by

$$L_a^c(s) = -C_{l\alpha} b \rho_\infty U_\infty^2 s \left\{ \Phi_\alpha^c(s) \left(\alpha_s + s \frac{h_s}{b} \right) - 2s \Phi_q^c(s) \alpha_s \right\} \quad (15a)$$

$$M_a^c(s) = -2C_{l\alpha} b^2 \rho_\infty U_\infty^2 s \left\{ \Phi_{M\alpha}^c(s) \left(\alpha_s + s \frac{h_s}{b} \right) - 2s \Phi_{Mq}^c(s) \alpha_s \right\} \quad (15b)$$

where the new indicial functions are also expressed in the Laplace-transformed space. For the condition stated in reference [12], the conversion of equations (15) to the frequency domain is obtained by replacing s with ik .

Keeping in mind the relationship between Theodorsen's function $C(k)$ and Wagner's function $\Phi(ik)$, one can define for the compressible flight speed regime an analogous of Theodorsen's function expressed in terms of the corresponding indicial functions as

$$T_\alpha^c [F_\alpha^c; G_\alpha^c] \equiv F_\alpha^c + iG_\alpha^c = ik \int_0^\infty \phi_\alpha^c(\tau) e^{-ik\tau} d\tau = ik\Phi_\alpha^c(ik) \quad (16)$$

The same relationships exist between $T_{M\alpha}^c, T_q^c, T_{Mq}^c$ and $\bar{\phi}_{M\alpha}^c, \bar{\phi}_q^c, \bar{\phi}_{Mq}^c$, respectively; $T_\alpha^c, T_{M\alpha}^c$ and T_q^c, T_{Mq}^c are the compressible analogues of Theodorsen's function $C(k)$ in plunging and pitching of the lift and moment, respectively, and can be expressed in terms of real and imaginary components as function of k . Circulatory and non-circulatory parts of the indicial functions due to α and q and their corresponding contour plots can be found in reference [1]. For the incompressible flight speed regime, the new four complex functions should reduce to Theodorsen's function. Following the developments carried out for the incompressible flight speed regime, the compressible lift and moment per unit span for plunging (L_z, M_z) and pitching (L_q, M_{zq}) about the leading edge, are expressed

similarly as

$$L_z = C_{L\alpha} \rho U_\infty^2 h'_{0c} C_\alpha \quad (17a)$$

$$L_q = 2C_{L\alpha} \rho U_\infty b^2 q_{0c} C_q \quad (17b)$$

$$M_z = 2C_{L\alpha} \rho U_\infty^2 b h'_{0c} C_{M\alpha} \quad (18a)$$

$$M_{zq} = 2C_{L\alpha} \rho U_\infty b^2 q_{0c} C_{Mq} \quad (18b)$$

In equations (17) and (18), h'_0 and q_0 are the plunging velocity and the indicial angular velocity, respectively. Combining equations (17) and (18), one obtains the expressions for the aerodynamic lift and moment, respectively.

4.3 Transonic speed regime

The transonic flight speed range is characterized by large amplitude motion induced by the shock wave that can significantly invalidate the linear assumption, usually adopted for flutter and aeroelastic response analyses of two-dimensional aircraft wings [18]. It has been demonstrated experimentally that LCO is possible to occur for airfoils in the vicinity of the transonic dip [19]. Theoretical [18] and numerical studies [20] have fully validated the presence of such complex non-linear phenomena. The lift- and moment-curve slopes associated with the plunging and pitching represented in terms of complex derivatives are in excellent agreement, with those reported in references [1] and [21]. A better agreement is reached in the quasi-steady/steady flow regimes, and a fair agreement in the unsteady regime. For small time-scale ranges, the non-linear effects arising from the shock waves may affect the computations, and these are certainly not taken into account in the purely linear approach [22]. These results are also in good agreement with the ones supplied in reference [5],

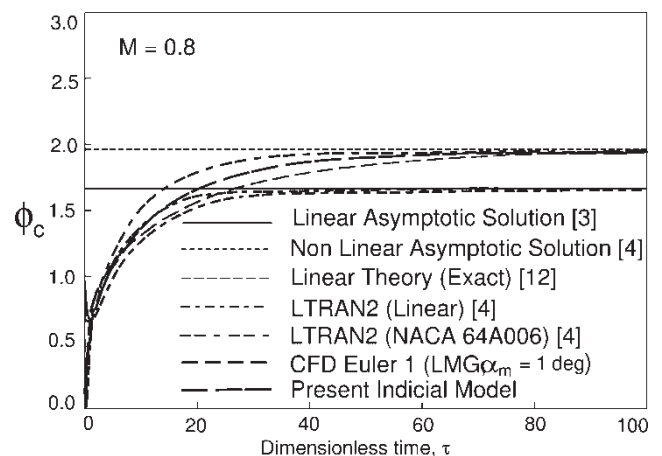


Fig. 4 Comparison of linear and non-linear indicial functions for the plunging motion at $M = 0.8$ [3, 4, 12]

which are displayed in Fig. 4. In this figure, the indicial function due to a step change in the angle of attack is presented. A time scale consistent with the low-frequency range has been selected, and the results for the exact linear theory obtained by Lomax [13], the linear and non-linear asymptotic values, as well as the linear time-integration LTRAN2 and LTRAN2 for a NACA 64A006 exhibiting non-linear effects [5] have been displayed together with those obtained using the CFD and the non-linear indicial function approach. It is shown that, after a small time scale, in which the trend of the function is dominated by the waves propagating downstream from the airfoil leading edge, the lift coefficient increases monotonically towards a common steady-state value. As indicated in reference [18], the low reduced frequency range is of primary interest in the transonic flow applications, and for this reason, special caution should be exercised.

The non-linear unsteady lift \bar{L}_a^c and aerodynamic moment \bar{M}_a^c in the compressible flight speed regime evaluated at the leading edge ($x = -b$), for arbitrary plunging and pitching about the leading edge are

$$\begin{aligned} \bar{L}_a^c(\tau) = & -C_{l\alpha} b \rho_\infty U_\infty^2 \bar{\phi}_\alpha^c[\tilde{w}(0); \tau] \left\{ \alpha(0) - \frac{h'(0)}{b} \right\} \\ & + 2C_{l\alpha} b \rho_\infty U_\infty^2 \alpha'(0) \bar{\phi}_q^c[\tilde{w}(0); \tau] \\ & - C_{l\alpha} b \rho_\infty U_\infty^2 \int_0^\tau \left\{ \bar{\phi}_\alpha^c[\tilde{w}(\zeta); \tau, \sigma] \right. \\ & \times \left(\alpha'(\sigma) + \frac{h''(\sigma)}{b} \right) \\ & \left. - 2\bar{\phi}_q^c[\tilde{w}(\zeta); \tau, \sigma] \alpha''(\sigma) \right\} d\sigma \end{aligned} \quad (19a)$$

$$\begin{aligned} \bar{M}_a^c(\tau) = & -2C_{l\alpha} b \rho_\infty U_\infty^2 \bar{\phi}_{M\alpha}^c[\tilde{w}(0); \tau] \left\{ \alpha'(0) + \frac{h''(0)}{b} \right\} \\ & + 4C_{l\alpha} b \rho_\infty U_\infty^2 \alpha'(0) \bar{\phi}_{Mq}^c[\tilde{w}(0); \tau] \\ & - 2C_{l\alpha} b^2 \rho_\infty U_\infty^2 \int_0^\tau \left\{ \bar{\phi}_{M\alpha}^c[\tilde{w}(\zeta); \tau, \sigma] \right. \\ & \times \left(\alpha'(\sigma) + \frac{h''(\sigma)}{b} \right) \\ & \left. - 2\bar{\phi}_{Mq}^c[\tilde{w}(\zeta); \tau, \sigma] \alpha''(\sigma) \right\} d\sigma \end{aligned} \quad (19b)$$

where the new non-linear indicial functions can be represented as

$$\bar{\phi}_\alpha^c[\tilde{w}(\zeta); \tau, \sigma] \equiv \lim_{\Delta\tilde{w} \rightarrow 0} \frac{\Delta \bar{C}_{L\alpha}^c(\tau)}{\Delta \tilde{w}} \quad (20a)$$

$$\bar{\phi}_q^c[\tilde{w}(\zeta); \tau, \sigma] \equiv \lim_{\Delta\tilde{w} \rightarrow 0} \frac{\Delta \bar{C}_{Lq}^c(\tau)}{\Delta \tilde{w}} \quad (20b)$$

$$\bar{\phi}_{M\alpha}^c[\tilde{w}(\zeta); \tau, \sigma] \equiv \lim_{\Delta\tilde{w} \rightarrow 0} \frac{\Delta \bar{C}_{M\alpha}^c(\tau)}{\Delta \tilde{w}} \quad (20c)$$

$$\bar{\phi}_{Mq}^c[\tilde{w}(\zeta); \tau, \sigma] \equiv \lim_{\Delta\tilde{w} \rightarrow 0} \frac{\Delta \bar{C}_{Mq}^c(\tau)}{\Delta \tilde{w}} \quad (20d)$$

Equations (20) can be adapted to the case of the arbitrary plunging and pitching about an arbitrary point, such as, for example that corresponding to the mid-chord axis.

4.4 Supersonic and hypersonic flight speed regimes

The same notations as in the subsonic compressible are used for the supersonic flight speed regime. The expressions of indicial functions specialized for $M \rightarrow 1$ provide the transonic indicial functions. These can also be expressed in terms of their real and imaginary parts. In the majority of applications, especially when Laplace's transform method is applied, the exact expressions of aerodynamic indicial functions due to Lomax [13] can be used. In Fig. 5, the real part of the aerodynamic lift coefficient for two selected Mach numbers, $M = 1.3$ and $M = 1.6$, are plotted against the reduced frequency. The present analytical predictions are in very good agreement with those based on the two-dimensional [22, 23], and three-dimensional [24] theoretical findings; however, the results, in general, overpredict the experimental ones provided in reference [25]. Moreover, as the results from Fig. 5 reveal, at high reduced frequencies and $M = 1.6$, the predictions by the proposed theory may be considered in good agreement with the experimental ones, even better than those provided by the two-dimensional and three-dimensional analytical models [22–24]. It is also evident that, when comparing with the experimental predictions, the error decreases with the increase of the Mach number. One should also stress the fact that, due to the available scattered experimental results, it is very difficult to

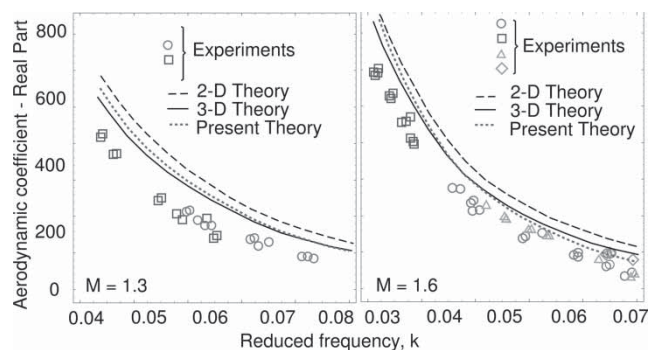


Fig. 5 Aerodynamic coefficients (real part) at $M = 1.3$ and 1.6 . Comparison with 2-D [21, 22], 3-D [23] theories and with experiments [24]

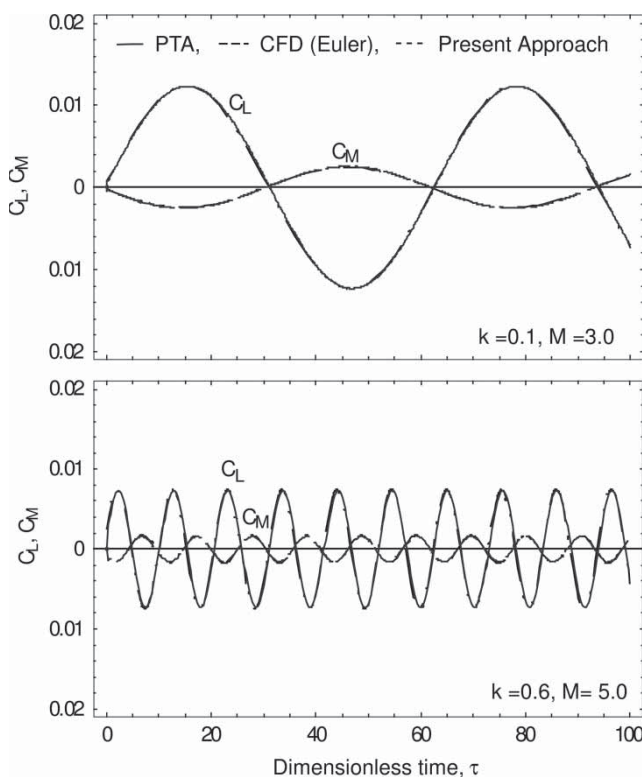


Fig. 6 Comparison of hypersonic unsteady lift and moment coefficients at $M = 3.0$ and 5.0

arrive at definitive conclusions about the agreement of the two types of predictions. The error seems also to increase when approaching the transonic flight speed regime, i.e. for Mach numbers closer to $M = 1$. This trend can be due to the fact that highly non-linear effects are induced by the shock waves. In spite of this, further analysis should be done in order to verify this approach in the transonic regime.

In the context of the hypersonic flight speed regime, it can be shown that the aeroelastic response predictions based on the use of piston theory aerodynamics [26] and exact supersonic theory aerodynamics [22] agree perfectly with those based on the indicial formulation, see [1,4]. In Fig. 6, the supersonic and hypersonic aerodynamic lift and moment coefficients for the wedge-flat-wedge-type airfoil ($t/c = 4$ per cent) at $M = 3.0$ ($k = 0.1$) and $M = 5.0$ ($k = 0.6$) obtained by non-linear PTA [4] are compared with the CFD and indicial calculations for oscillatory motion of the purely pitching airfoil. The curves are superposed at both low- and high-reduced frequencies k , and as it can be concluded, the agreement among them is excellent. It is worth remarking that the unsteady aerodynamic derivatives obtained via indicial functions are also in excellent agreement with the linear theory based on a panel method-type solution of the Possio integral equation. In reference [1], a comparison of frequency-domain

predictions based on various theories and experimental data is presented. From these results, one can conclude that the indicial function approach is an accurate one at all frequencies and Mach numbers. On the other hand, as assessed by Crouse and Leishman [27] the methodology based on indicial functions can be used for an extended range of reduced frequencies, and as presented in Nelson and Pelletier [28], also for large amplitude manoeuvres.

Finally, one should remark that the proposed approach can be extended to the aeroelastic analysis of three-dimensional lifting surfaces. Although not presented in this paper, such an approach can be carried out by extending the domain of applicability of the aerodynamic indicial function concept along the lines indicated in references [29] and [30]. The indicial function concept can produce sufficiently accurate results in predicting the two-dimensional unsteady aerodynamic loads and, as compared with direct CFD calculations, requires about four orders of magnitude with less computational time [31,32]. It is a very effective method when detailed pressure distributions are not required, in which case the CFD can provide a quite accurate solution. The main advantage resulting from the use of linear and non-linear indicial functions method stems from the numerical efficiency of the scheme compared with the harmonic, direct CFD, and time integration methods. The last methods require complete flowfield computations for each combination of motion mode, selected Mach number, and frequency. The indicial method requires flowfield computations only for each combination of modes and Mach numbers. One should indicate that the non-linear indicial function calculations require about 30 per cent more computational time when compared with the linear counterpart. However, considering its great efficiency when comparing with the direct CFD calculations, this issue becomes really marginal.

5 CONCLUSIONS

In this paper, a unified formulation of the linear and non-linear indicial aerodynamic function concept in the incompressible and compressible flight speed regimes has been presented. The indicial aerodynamic functions have been developed in conjunction with CFD codes towards determination of the unsteady aerodynamic loads for a two-DOF airfoil. As a result, these can be used towards approaching the problems of linear and non-linear aeroelastic response, flutter instability, and on studies involving stable and unstable LCO in the transonic flow speed range. The good agreements with theoretical, computational fluid dynamics, and experimental results demonstrate the capabilities and usefulness of the

indicial function concept in linear and non-linear aeroelastic problems, and their prospects for their use in aeroservoelastic analyses.

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APPENDIX

Notations

- a* dimensionless elastic axis position measured from the mid-chord, positive aft

b	half-chord length (m)	U_∞, ρ_∞	free-stream speed (m/s) and air density (kg/m^3), respectively
$C_{l\alpha}$	lift-curve slope (1/rad)		
h, α	plunging displacement (m) and the twist angle (rad) about the pitch axis, respectively	ζ	parameter denoting dependence on prior motion history
L_a, M_a	unsteady lift per unit length (N/m) and moment per unit length (N), respectively	ω, k	circular and reduced ($\equiv \omega b / U_\infty$) frequencies, respectively
q	pitch rate about the reference axis ($\equiv (2b/U_\infty)\dot{\alpha}$)	$(\cdot)(\hat{\cdot})(\check{\cdot})$	quantities associated with leading edge, mid-chord, and the 3/4-chord, respectively
t, τ	time variable (s) and its dimensionless counterpart ($\equiv U_\infty t / b$), respectively	$(\dot{\cdot}), (\dot{\cdot})'$	time derivative and its dimensionless counterpart, respectively

