

# Transmit Power Optimization for Video Transmission Over Slowly-Varying Rayleigh-Fading Channels in CDMA Systems

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**Abstract**—An optimum transmit power management scheme is proposed for wireless video service in code-division multiple-access systems. The scheme minimizes the transmit power subject to the constraint on the distortion resulted from the loss of each packet and the error propagation effect caused by the motion compensation. It implicitly controls the target bit-error rate (BER) of the video packet according to the importance of the packet. The more important the packet is, the more securely it is transmitted over noisy channel. Furthermore, the scheme adjusts the target BER as if the water filling concept is applied to the problem of minimizing the transmit power. The simulation results show that the proposed scheme considerably outperforms the conventional schemes.

**Index Terms**—Code-division multiple-access (CDMA), error concealment, H.263, power management, target bit-error rate (BER), wireless video.

## I. INTRODUCTION

OVER the past decade, there has been much research activity in the field of wireless video. Efficient rate control and error compensation schemes have been proposed for the robust video transmission over burst-error wireless channels [1], [2]. Srinivasan and Chellappa studied adaptive source-channel subband coding for wireless channels [3]. Hanzo *et al.* designed efficient wireless video phones [4], [5]. Davies *et al.* considered the practical application of wireless video and implemented a testbed [6]. In the code-division multiple-access (CDMA) system, many researchers have studied wireless video. Chang proposed a CDMA system for subband image transmission over a band-limited wireless channel [7]. In another work, Chang and Wang presented a CDMA indoor system for image transmission [8]. Iun and Khandani considered the combined source-channel coding problem for image transmission over the uplink of IS-95 [9].

Since the capacity of a CDMA system is limited by the multiple access interference (MAI), efficient power management is a vital issue. That is, if the power transmitted by each user can be reduced, the intra- and intercell interference will be decreased and the total system capacity be increased. Moreover, if a mobile's transmit power is reduced, its battery life can also be extended. It is, therefore, desirable to minimize the amount of signal power transmitted by a mobile, while

simultaneously maintaining the desired quality-of-service (QoS) level. Recently, considering the characteristics of video sequences, efficient resource and power management schemes have been studied for wireless video [10]–[16]. In particular, an optimum power control scheme has been researched based on a distortion model [12]. Considering the error propagation effects caused by motion compensation, this scheme adjusted the target bit-error rates (BERs) of image frames so that the received power consumption could be minimized. Clearly, this optimum scheme [12] provided better performance compared to the previous work. However, further considerations on the following arguments will help improve the performance.

First, although the scheme [12] optimized the power at image frame layer, optimizing the power at the group-of-block (GOB) layer would be better since the importance of each GOB may be different even in an image frame. Next, although the scheme [12] optimized the received power, the transmit power should be minimized in order to minimize the intra- and intercell interference and to maximize the battery life of each mobile terminal. In fact, almost all the researches on power optimization have focused on the transmit power. We take into account these arguments and present an optimum transmit power management scheme at GOB layer considering the Rayleigh-fading in CDMA systems. The proposed scheme adjusts the target BER as the water filling method, although it is not exactly the same as the conventional water filling in information theory.

The organization of this letter is as follows. The system model considered is described in Section II. In Section III, we formulate and solve an optimization problem to determine the optimum transmit power of each image packet. Performance comparisons and simulation results are presented in Section IV. Finally, Section V gives conclusions.

## II. SYSTEM MODEL

### A. Channel-Induced Distortion Model

To cope with channel errors, H.263 allows the insertion of synchronization words at GOB layer. Thus, H.263 can obtain resynchronization at the beginning of each GOB. When the decoder detects a corrupted GOB, it replaces the GOB with error-concealed image data using various techniques [17]. Although error concealment improves the image quality, the channel-induced distortion of a frame is propagated to the subsequent frames because of the motion compensation. Considering this interframe error propagation, many researchers have studied distortion models [2], [11], [18]–[20]. In particular, the model in [11] has considered the expected error propagation effect on the future frames. In this model, the

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effective channel-induced distortion  $D_i^C$  at the  $i$ th frame is expressed by

$$D_i^C = \frac{1 - \left( \frac{1}{iB} \sum_{k=1}^i n_k^{\text{inter}} \right)^{N-i+1}}{1 - \frac{1}{iB} \sum_{k=1}^i n_k^{\text{inter}}} \sum_{j=1}^M d_{ij} \Pr(\Psi_{ij}) \quad (1)$$

$$\stackrel{\text{def}}{=} \mu_i \sum_{j=1}^M d_{ij} \Pr(\Psi_{ij}), \quad i = 1, 2, \dots, N \quad (2)$$

where  $N$  is the period of the INTRA frame refreshment,  $M$  is the number of GOBs of a frame,  $B$  is the total number of blocks ( $16 \times 16$  pixels) of a frame,  $n_k^{\text{inter}}$  is the number of intercoded blocks of the  $k$ th frame, and  $\Psi_{ij}$  is the event that the image data of the  $j$ th GOB of the  $i$ th image frame, which will be denoted as the  $(i, j)$ th GOB, are corrupted. Furthermore,  $d_{ij}$  denotes the distortion resulted from the loss of the  $(i, j)$ th GOB. The distortion  $d_{ij}$  is given by the sum of the squared differences between the  $(i, j)$ th GOBs image data reconstructed without channel errors and the corresponding error-concealed image data in the decoder.

The parameter  $\mu_i$  represents the error propagation effect from frame  $i$  on the subsequent frames. Generally,  $\mu_i$  is a decreasing function of  $i$ . For example, if  $n_k^{\text{inter}}$  is fixed for all frames, it can be easily seen that  $\mu_i$  is a strictly decreasing function of  $i$ . This means that a frame positioned in the earlier part of an image sequence has more effect on the subsequent frames than a frame in the later part.

When INTRA macroblocks are periodically inserted rather than INTRA frames, the other distortion model [20] can be used. Then,  $\mu_i$  is fixed and given by [20]

$$\mu_i = \sum_{t=0}^{T-1} \frac{1 - \frac{t}{T}}{1 + \gamma t} \quad (3)$$

where  $T$  is the INTRA macroblock update interval and  $\gamma$  denotes the efficiency of explicit and/or implicit loop filtering to remove the introduced error [20].

If a quarter common intermediate format (QCIF) image sequence is coded in 10 frames/s (fps), the duration of a GOB is 11.1 ms on average, which is approximately the same as the duration of a radio frame or packet in WCDMA. In this letter, for simplicity, we assume that the coded image data of a GOB is packed into a packet.

### B. BER in CDMA Systems

We assume slowly varying flat Rayleigh fading. In particular, we assume that the path gains are constant for a packet duration and vary from one to another packet (quasi-static flat fading). At the  $(i, j)$ th packet, the fading coefficient  $h_{ij}$  is modeled as a low-pass filtered complex Gaussian random variable with zero mean and variance 0.5/dimension. Let  $\mathcal{P}_{ij}$  be the transmit power during the  $(i, j)$ th packet. Let  $W$ ,  $R$ , and  $I_0$  be the spreading bandwidth, the transmitted bit rate, and the sum of background noise and multiple-access interference,

respectively. Let  $P_e(i, j)$  be the target BER of the  $(i, j)$ th packet of the convolutionally coded CDMA system with the code rate  $\xi$  and the free distance  $f$ . Under the assumption of high signal-to-interference plus noise ratio (SINR), which is typical in video transmission, the transmit power  $\mathcal{P}_{ij}$  satisfying the target BER  $P_e(i, j)$  can be approximately computed by the following [12], [21], [22]:

$$P_e(i, j) \approx \zeta \exp\left(-f\xi \frac{W|h_{ij}|^2 \mathcal{P}_{ij}}{RI_0}\right) \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, M \quad (4)$$

where  $\zeta$  is the total number of nonzero information bits on all free distance paths. For the quasi-static fading channel,  $\Pr(\Psi_{ij})$  is given by

$$\Pr(\Psi_{ij}) = \sum_{k=1}^{b_{ij}} \binom{b_{ij}}{k} P_e^k (1 - P_e(i, j))^{b_{ij}-k} \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, M \quad (5)$$

where  $b_{ij}$  is the number of bits of the  $(i, j)$ th GOB.

In order to satisfy the target BER, the transmit power should be adjusted by the power control functions of IS-95 or WCDMA. Since the power control interval, e.g., 0.667 ms for WCDMA, is much shorter than the duration of a packet or a GOB, the target BER of each packet can be controlled. In real CDMA systems, however, the inner-loop power control may not be ideal. Thus, the outer loop control should be used to adjust the target SINR so that the target frame-error rate (FER) or BER can be maintained [21], [23].

### III. OPTIMUM TRANSMIT POWER CONTROL FOR WIRELESS VIDEO

To enhance video quality and to utilize power efficiently, the target BERs of video packets or GOBs are optimized so that the average transmit power consumed during the transmission of each image sequence can be minimized. Since video quality is generally evaluated by the amount of the distortion or the peak signal-to-noise ratio (PSNR), the channel-induced distortion  $\sum_{i=1}^N D_i^C$  of an image sequence is given as the constraint. We do not take into account the quantization distortion, because it is independent of the channels and it is normally controlled by various rate control schemes. The optimization problem to find the optimum transmit power can be described as follows:

$$\mathbf{P}^{\text{opt}} = \arg \min_{\mathbf{P}} \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M \mathcal{P}_{ij}$$

subject to

$$\sum_{i=1}^N D_i^C = D_{\text{TH}} \quad (6)$$

where  $D_{\text{TH}}$  is the distortion threshold and  $\mathbf{P}$  is an  $N \times M$  matrix whose  $(i, j)$ th element is  $\mathcal{P}_{ij}$ . This constrained optimization problem can be converted to an unconstrained optimization

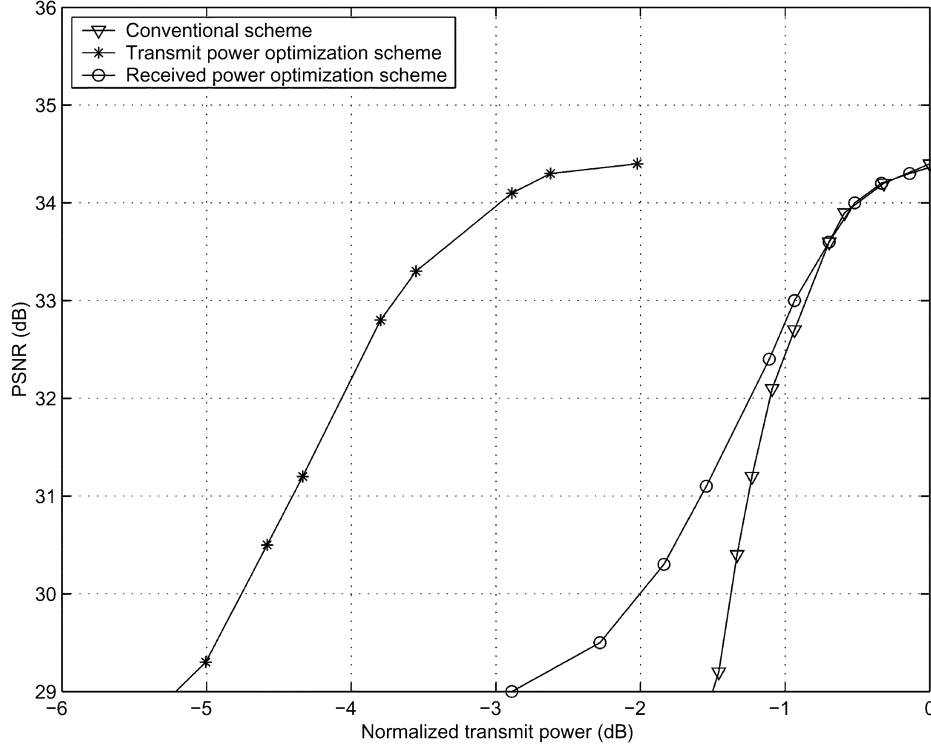


Fig. 1. Average PSNR versus the normalized power. *Mother and Daughter*.

problem using the Lagrangian method. That is, the objective is to find  $\mathbf{P}$  which minimizes

$$J = \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M \mathcal{P}_{ij} + \lambda \left\{ \sum_{i=1}^N D_i^C - D_{\text{TH}} \right\}. \quad (7)$$

To derive the optimum solution in a closed form, we introduce the same approximation as in [12]:  $(1 - P_e(i, j))^{b_{ij}}$  is expanded using the Taylor series with respect to  $P_e(i, j)$  and the higher order terms are disregarded. Then, we have  $\Pr(\Psi_{ij}) \approx b_{ij}P_e(i, j)$ . Due to this approximation,  $\Pr(\Psi_{ij})$  is slightly overestimated. However, the error induced by this approximation is negligible when  $\Pr(\Psi_{ij})$  is small. Although the approximation error increases as  $\Pr(\Psi_{ij})$  becomes larger, this error is still small for a reasonable range of GOB error probability for image transmission. For example, even for a large GOB error probability such as  $\Pr(\Psi_{ij}) = 10\%$ , the approximation error is still less than 0.5%.

The optimum solution to (6) can be obtained as

$$\mathcal{P}_{nm}^{\text{opt}} = \frac{I_0 R}{f \xi W |h_{nm}|^2} \times \ln \left\{ \frac{\zeta \mu_n d_{nm} b_{nm} |h_{nm}|^2 \sum_{i=1}^N \sum_{j=1}^M \left( \frac{1}{|h_{ij}|^2} \right)}{D_{\text{TH}}} \right\} \quad (8)$$

$$n = 1, 2, \dots, N; \quad m = 1, 2, \dots, M.$$

It is easy to show that the *Hessian* matrix of  $J$  is positive definite for all possible  $\mathbf{P}$ . This means that  $J$  is a convex function over all the possible  $\mathbf{P}$  region. Thus, (8) gives the global minimum point of  $J$ . We will refer to this power management scheme

as the *transmit power optimization scheme*. For practical use, we can replace  $D_{\text{TH}} / \sum_{i=1}^N \sum_{j=1}^M (1/|h_{ij}|^2)$  in (8) with a constant  $\tilde{D}_{\text{TH}}$ , because  $\sum_{i=1}^N \sum_{j=1}^M (1/|h_{ij}|^2)$  does not depend on specific time index  $(n, m)$ . To control the PSNR, we can adjust  $\tilde{D}_{\text{TH}}$  in a practical system.

From (4) and (8), the optimum target BER  $P_e^{\text{opt}}(n, m)$  of the  $(n, m)$ th packet is given by

$$P_e^{\text{opt}}(n, m) = \frac{D_{\text{TH}}}{\mu_n d_{nm} b_{nm} |h_{nm}|^2 \sum_{i=1}^N \sum_{j=1}^M \left( \frac{1}{|h_{ij}|^2} \right)}, \quad (9)$$

$$n = 1, 2, \dots, N; \quad m = 1, 2, \dots, M.$$

Note that  $P_e^{\text{opt}}(n, m)$  is inversely proportional to  $d_{nm}$ . This implies that a packet with a larger  $d_{nm}$  value should be more securely protected from channel errors because the packet is more important in the sense of the decoded image quality. Second,  $P_e^{\text{opt}}(n, m)$  is inversely proportional to  $\mu_n$ . As stated previously, when INTRA frames are inserted,  $\mu_n$  represents the error propagation effect of frame  $n$  on the subsequent frames and it is generally a decreasing function of  $n$ . In this case, the GOBs of a frame which is positioned in the earlier part of an INTRA refreshment period must have lower target BERs because the GOBs are more important in the sense of the decoded image quality when the interframe error propagation effect is considered. If INTRA macroblocks are inserted,  $\mu_n$  is fixed over time as stated previously. Finally,  $P_e^{\text{opt}}(n, m)$  is inversely proportional to  $|h_{nm}|^2$ . This suggests that the target BER of a packet should be determined as a lower value when the channel condition is good, and vice versa, for minimizing power consumption. This is similar to the well-known water-filling power allocation, in which more power is allocated when the channel is better in

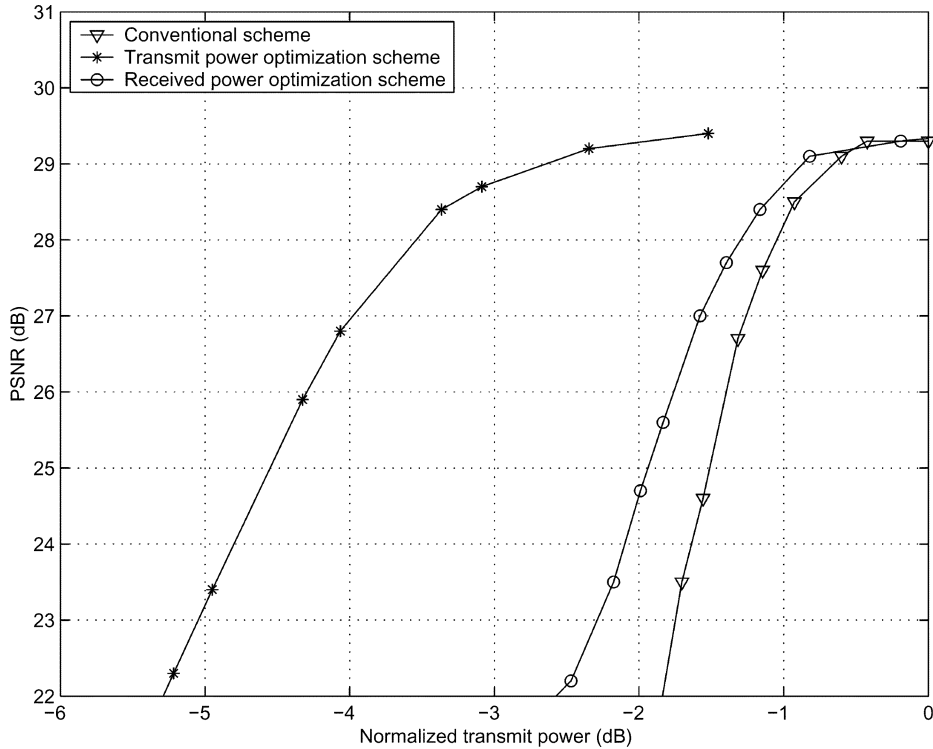


Fig. 2. Average PSNR versus the normalized power. *Foreman*.

order to maximize the capacity. For example, Goldsmith *et al.* proposed an optimum adaptive transmission scheme using the water-filling over time for power adaptation [24]. Note that the concept of the water-filling [24]–[26] is similar but not identical to ours because the aim of water filling is maximizing the capacity rather than minimizing the transmit power.

For the application of the proposed scheme, however, the information on the distortion of each packet or GOB must be transmitted from the video coding layer to the power control layer. Note that this is possible in third-generation or other future communications systems. For example, one of key design criteria in radio interface protocol of WCDMA is the separation of control plane from user plane [27]. In particular, a radio resource control (RRC) protocol entity of control plane has control interfaces with all layers of users plane. Thus, using the interfaces, the information can be transmitted from the video coding layer to the power control layer.

#### IV. PERFORMANCE COMPARISON

##### A. Previous Power Management Schemes

In this section, we consider previous power management schemes for performance comparison. We first consider the scheme which fixes the target BER. In this case, from the constraint of (6), the fixed BER  $P_e^{\text{con}}$  is given by

$$P_e^{\text{con}} = \frac{D_{\text{TH}}}{\sum_{i=1}^N \mu_i \sum_{j=1}^M b_{ij} d_{ij}}. \quad (10)$$

The transmit power is given by  $P_e^{\text{con}}$  and (4). We will refer to this scheme as the *conventional scheme*. In practice, most power

control schemes known in the literature can be classified into the conventional scheme because they try to satisfy a fixed target BER by controlling the transmit power.

Next, we consider the power management scheme which optimizes the received power at image frame layer in CDMA systems [12]. Assuming the system model adopted in this letter, the optimum target BERs of the scheme [12] can be derived as follows:

$$\bar{P}_e^{\text{opt}}(n) = \frac{D_{\text{TH}}}{\mu_n N \sum_{j=1}^M d_{nj} b_{nj}}, \quad n = 1, 2, \dots, N. \quad (11)$$

The transmit power is given by  $\bar{P}_e^{\text{opt}}(n)$  and (4). We will refer to this scheme as the *received power optimization scheme*. Note that the target BERs in (11) obtained by optimizing the received power is different from that of (9) obtained by optimizing the transmit power. In the received power optimization scheme, the GOBs in a frame possess the same target BERs. Thus, this scheme does not consider the possibility that the importance of GOBs in a frame may be different each other. Additionally, the scheme controls the target BER irrespective of the channel condition. Thus, this scheme may consume much power in order to satisfy a low target BER when the channel is very bad.

Unlike the conventional and the received power optimization schemes, the transmit power optimization scheme requires the transmitter to have the channel information. For this, the feedback channels may be needed as in the high-data-rate (HDR) system. When the time division duplexing (TDD) system is considered, which is desirable for the situation of asymmetric traffic demands, the transmitter can have the channel information without the feedback channels.

## B. Simulation Results

We evaluate the performances of the three schemes by comparing the transmit power consumptions at the same PSNR. The power consumptions are adjusted by varying the  $D_{TH}$  values in (9)–(11). Then, for the three schemes, the powers are normalized by the peak power level, where the average PSNR degradation of the conventional scheme is negligible, e.g., less than 0.1 dB. To generate the Rayleigh-fading channel, we use the well-known Jake's model assuming the pedestrian speed (4 km/h) and the carrier frequency of 2 GHz. We consider a convolutionally coded CDMA system with  $\xi = 1/2$ ,  $f = 10$ ,  $\zeta = 46$  [28]. Two well-known QCIF images are coded: a low-motion image *Mother and Daughter* at 32 kb/s and a high motion *Foreman* at 64 kb/s. The motion compensated temporal error concealment scheme is considered. We set the motion vector of the missing block to the median value of the motion vectors of the adjacent blocks. If no surrounding motion vectors are available, the motion vector is set to zero. We also assume INTRA frames are inserted every 20 frames ( $N = 20$ ). The simulation results are obtained through 100 runs.

Figs. 1 and 2 show the average PSNRs of the three schemes versus the normalized power. The proposed transmit power optimization scheme considerably outperforms the other two schemes.

## V. CONCLUSION

We have proposed an optimum transmit power control scheme for video transmission. In this scheme, the target BER of each video packet depends on the three factors. First, the BER is inversely proportional to the distortion generated by the loss of the packet. Second, this scheme adjusts the target BER considering the error propagation effect caused by the motion compensation. Thus, a packet which is more important in the sense of the decoded image quality is more securely transmitted over noisy channel, and *vice versa*. Finally, the better the channel condition is, the smaller the BER becomes. This realizes the efficient power consumption as in the water-filling method. The simulation results show that the proposed scheme considerably outperforms the previous schemes.

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