An Efficient Blind Multiuser Detection for Improper DS/CDMA Signals

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***Abstract*—This paper proposes a new linear blind multiuser detection based on widely linear (WL) signal processing. The received signal and its complex conjugate are separately filtered and the results are linearly combined. The WL maximum/minimum (max/min) mean-output-energy (MOE) receiver is derived by applying the modified cost function. It is shown that a performance gain can be attained for the improper direct-sequence code division multiple access (DS/CDMA) signal, due to the additional information contained in pseudocovariance matrix of observations. The adaptive implementation with acceptable complexity is also developed. Computer-simulation results show that a significant performance gain is obtained over the other classical methods.**

***Index Terms*—Code division multiple access (CDMA), improper signal, interference suppression, multiuser receiver, widely linear (WL) filter.**

I. Introduction

WIRELESS communication has evolved new era in line with the demand of multimedia services such as data and videos. When compared to other multiple-access techniques, direct-sequence code-division multiple-access (DS/CDMA) system has many advantages, such as robustness of multipath distortion, immunity of interuser's interceptions and jammings, and simple random access to the network [1], [2]. Thanks to these features, this technique has been considered for future third-generation (3G) wireless standards to guarantee a higher transmitting capacity and to satisfy the rising multimedia requests for services.

The capacity of DS/CDMA system is mainly limited by multiuser interference (MUI). To suppress MUI, the various multiuser detections [3]–[5] have been developed in the last two decades. The recent studies have demonstrated that the optimum multiuser detection is maximum likelihood sequence estimator (MLSE) [6]. Unfortunately, the complexity of the optimum multiuser detector is exponential in number of users and constellation size. For this reason, the recent research has been focused on designing suboptimal receivers that have low computational complexities. The most prominent results of these efforts are linear receivers, the decorrelating receiver [7], and the minimum mean-square-error (MMSE) receiver [8].

Another linear receiver based on second-order statistics (the blind minimum mean-output-energy (MOE) receiver [9]), is particularly interested in this paper. It minimizes MOE of the receiver subject to the certain constraints that guarantee no cancellation of the desired signal components. Without multipath fading or with known channel, the MOE receiver directly becomes an mmse receiver. When the multipath is present, the satisfactory performance is yielded by maximizing the output of the receiver [10].

However, most literatures about the linear multiuser detection share one common implicit hypothesis that the complex-valued received signal of DS/CDMA system is proper. On the other hand, we often encounter the improper signals of which pseudocovariance matrices should be exploited for optimal performance. The widely linear (WL) signal processing enables us to utilize the additional information in pseudocovariance matrix. The concept of the improper signal and the WL estimation has been introduced in [11], which shows that the WL mmse estimation for complex-valued signal is able to improve the performance. The systematic approach to second-order analysis of improper complex random vectors and processes is presented in [12].

In [13] and [14], WL filtering is applied to the demodulation of the DS/CDMA systems, which transmit the real-valued symbols, and significant performance gains have been reported. In addition, [15] shows that the WL mmse receiver halves the number of effective interferers and adds 3 dB to the input signal-to-noise ratio (SNR). It is also demonstrated in [16] that iterative CDMA multiuser detection enhances power gains and convergence speed by employing WL filtering. The improperness of DS/CDMA signals, informed by [13]–[15] and [17], is mainly caused by real modulation. Even when the desired user transmits the proper symbols with complex data modulation, if more than one of the interferers communicate with improper symbols, the whole DS/CDMA signal becomes improper.

In this paper, we deal with the problem of WL multiuser detection when the signal is improper. It is assumed that the multipath channel is unknown. It means that multipath components should be optimally combined with parameterized constraints, which is an analogous approach with [10]. With regard to the performance analysis, the superiority of the new scheme is demonstrated in signal-to-interference and noise ratio (SINR) aspects. The SINR of the proposed detector is compared with those of the conventional max/min MOE detector and their mmse solutions. One of the advantages of the proposed scheme is that adaptive implementation can be easily developed. In simulation results, the convergence dynamics and

Manuscript received December 1, 2004; revised April 12, 2005 and August 31, 2005. This work was supported in part by the University IT Research Center program of the government of Korea. The review of this paper was coordinated by Dr. Z. Xu.

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Digital Object Identifier 10.1109/TVT.2005.863359

steady-state performance of the proposed adaptive algorithm are also given.

The rest of the paper is organized as follows. In Section II, a baseband DS/CDMA discrete-time model and its improperness are described, and conventional max/min detector is revealed. Then, in Section III, the WL max/min receiver is derived, together with its adaptive implementation. In Section IV, simulation results are provided. Finally, the paper is concluded in Section V.

II. DS/CDMA Signal

A. System Model

Consider an asynchronous CDMA system with J users. User j transmits the information symbol with spreading sequence $\mathbf{c}_j = [c_j(0), \ldots, c_j(N-1)]^{\mathrm{T}}$, where N denotes the number of chips per symbol. Then, the jth user's nth transmitted signal at the chip rate $1/T_c$ in a baseband discrete-time model representation is given by

$$x_j(n) = \sum_{k=-\infty}^{\infty} s_j(k) c_j(n - kN) \tag{1}$$

where $s_j(k)$ represents the kth information symbol of the jth user. The sequence $s_j(k)$ is mutually independent for different js and independent identically distributed (i.i.d.) random sequences with zero mean and variances

$$E\left\{|s_j(k)|^2\right\} \triangleq \sigma_s^2 \tag{2}$$

$$E\left\{(s_j(k))^2\right\} \triangleq \rho_s^2. \tag{3}$$

If the received signal is sampled at the chip rate, the received discrete-time signal $y_j(n)$ is

$$y_j(n) = \sum_{i=-\infty}^{\infty} x_j(i) g_j(n - i - d_j) \tag{4}$$

where $g_j(n)$ is the channel impulse response sampled at the chip interval T_c, and $0 \leq d_j < N$ the transmission delay of user j in chip period. Substituting (1) into (4), we have

$$y_j(n) = \sum_{k=-\infty}^{\infty} s_j(k) h_j(n - kN - d_j) \tag{5}$$

$$h_j(n) = \sum_{i=0}^{N-1} c_j(i) g_j(n - i) \tag{6}$$

where $h_j(n)$ represents the distorted spreading sequence due to the dispersive channel $g_j(n)$. The overall received signal is

$$y(n) = \sum_{j=1}^{J} y_j(n) + v(n) \tag{7}$$

which is a superposition of the signals from all users in proper additive white Gaussian noise $v(n)$ with variance σ_v^2. Stacking up the chip rate measurement of $y(n)$ into N-vector $\mathbf{y}(k) \triangleq [y(kN), \ldots, y(kN + N - 1)]^{\mathrm{T}}$, the compact multiple input multiple output (MIMO) model, at the symbol rate, can be obtained as

$$\mathbf{y}(k) = \sum_{j=1}^{J} \sum_{l=0}^{L_h - 1} \mathbf{h}_j(l) s_j(k - l) + \mathbf{v}(k) \tag{8}$$

where $\mathbf{h}_j(l) \triangleq [h_j(lN - d_j), \ldots, h_j(lN - d_j + N - 1)]^{\mathrm{T}}$, and $\mathbf{v}(k) \triangleq [v(kN), \ldots, v(kN + N - 1)]^{\mathrm{T}}$. In (8), L_h is the maximum number of the symbols that the received signature sequences extend over. L_h can be given by

$$L_h = \max_{j=1,\ldots,J} \left\lceil \frac{N + L_g - 1 + d_j}{N} \right\rceil \tag{9}$$

where L_g is the maximum length of the channel impulse response $g_j(n)$ for all j. Using (6), the vector $\mathbf{h}_j(l)$ can be expressed in terms of the spreading sequence $c_j(n)$ and the channel impulse response $g_j(n)$ as

$$\mathbf{h}_j(l) = \mathbf{C}_j(l) \mathbf{g}_j \tag{10}$$

where $N \times L_g$ spreading code matrix $\mathbf{C}_j(l)$ is shown in (11) at the bottom of the page, and $N \times 1$ channel-impulse-response vector

$$\mathbf{g}_j = [g_j(0), g_j(1), \ldots, g_j(L_g - 1)]^{\mathrm{T}}. \tag{12}$$

A new data vector is constructed as

$$\mathbf{r}(k) \triangleq \left[\mathbf{y}^{\mathrm{T}}(k), \mathbf{y}^{\mathrm{T}}(k-1), \ldots, \mathbf{y}^{\mathrm{T}}(k - L_f + 1)\right]^{\mathrm{T}} \tag{13}$$

with the symbol rate measurement during L_f symbol periods. L_f is assumed to be $L_f \geq L_h$ so that the vector $\mathbf{r}(k)$ may contain the whole desired signal part. From (8), (NL_f)-column vector $\mathbf{r}(k)$ can be rewritten as

$$\mathbf{r}(k) = \sum_{j=1}^{J} \mathbf{H}_j \mathbf{s}_j(k) + \mathbf{w}(k) \tag{14}$$

$$\mathbf{C}_j^{(m)}(l) = \begin{bmatrix} c_j\left(lN - d_j^{(m)}\right) & c_j\left(lN - d_j^{(m)} - 1\right) & \cdots & c_j\left(lN - d_j^{(m)} - L_g + 1\right) \\ c_j\left(lN - d_j^{(m)} + 1\right) & c_j\left(lN - d_j^{(m)}\right) & \cdots & c_j\left(lN - d_j^{(m)} - L_g + 2\right) \\ \vdots & \vdots & \ddots & \vdots \\ c_j\left(lN - d_j^{(m)} + N - 1\right) & c_j\left(lN - d_j^{(m)} + N - 2\right) & \cdots & c_j\left(lN - d_j^{(m)} - L_g + N\right) \end{bmatrix} \tag{11}$$

where $\mathbf{H}_j$ is block Toeplitz matrix of dimension $(NL_f) \times (L_f + L_h - 1)$, as in (15), shown at the bottom of the page. $\mathbf{s}_j(k) \triangleq [s_j(k), s_j(k-1), \ldots, s_j(k-L_f-L_h+2)]^{\mathrm{T}}$, and $\mathbf{w}(k) \triangleq [\mathbf{v}^{\mathrm{T}}(k), \mathbf{v}^{\mathrm{T}}(k-1), \ldots, \mathbf{v}^{\mathrm{T}}(k-L_f+1)]^{\mathrm{T}}$. It is assumed without loss of generality that the first user $(j=1)$ is the user of interest, and the timing of the desired user is known through timing recovery so that $d_1 = 0$. Desired user's information symbol $s_1(k-d_f)$ may be extracted with a suitable delay d_f, which is assumed to be $d_f \leq L_h - 1$. It is seen from (14) and (15) that

$$\mathbf{r}(k) = \mathbf{h}_{d_f} s_1(k-d_f) + \mathbf{H}_{\mathrm{ISI}}\mathbf{s}_{\mathrm{ISI}}(k) + \sum_{j=2}^{J} \mathbf{H}_j \mathbf{s}_j(k) + \mathbf{w}(k) \tag{16}$$

where $\mathbf{h}_{d_f}$ is the (d_f+1)th column vector of the matrix $\mathbf{H}_1$ of (15) for user 1, $\mathbf{s}_{\mathrm{ISI}}(k)$ the vector containing all elements of $\mathbf{s}(k)$ except the (d_f+1)th element $s_1(k-d_f)$, and $\mathbf{H}_{\mathrm{ISI}}$ the matrix containing all the columns of $\mathbf{H}_1$ except $\mathbf{h}_{d_f}$. Each term of right side of (16) is interpreted desired signal, intersymbol interference (ISI), multiple-access interference (MAI), and background noise.

From (10) and (15), the vector $\mathbf{h}_{d_f}$ can be factorized as

$$\mathbf{h}_{d_f} = \mathbf{C}_1 \mathbf{g}_1 \tag{17}$$

where

$$\mathbf{h}_{d_f} = \Big[\underbrace{\mathbf{h}_1(d_f)^{\mathrm{T}}, \ldots, \mathbf{h}_1(0)^{\mathrm{T}}}_{N(d_f+1)}, \underbrace{\mathbf{0}^{\mathrm{T}}, \ldots, \mathbf{0}^{\mathrm{T}}}_{N(L_f-d_f-1)}\Big]^{\mathrm{T}} \tag{18}$$

and

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{C}_1(d_f) \\ \vdots \\ \mathbf{C}_1(0) \\ \mathbf{O}_{N(L_f-d_f-1)\times L_g} \end{bmatrix} \tag{19}$$

where $\mathbf{O}_{x\times y}$ denotes a zero matrix of dimension $x \times y$.

B. Improper Signals in DS/CDMA System

The complex random vector $\mathbf{x}$ is said to be proper [18] if its pseudocovariance matrix $\widetilde{\mathbf{R}}_{xx} \triangleq E\{\mathbf{x}\mathbf{x}^{\mathrm{T}}\}$ vanishes, i.e., $\widetilde{\mathbf{R}}_{xx} = \mathbf{0}$. In this case, the second-order moments can be described by solely the covariance matrix $\mathbf{R}_{xx} \triangleq E\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\}$. On the other hand, the improper vectors need special attention because of nonvanishing pseudocovariance matrix. For complete description of second-order behavior and the optimal performance of the complex filtering, additional knowledge of $\widetilde{\mathbf{R}}_{xx}$ is required [12].

The pseudocovariance matrix of the DS/CDMA signal $\mathbf{r}(k)$ in (14) is given by

$$\widetilde{\mathbf{R}}_{rr} \triangleq E\left\{\mathbf{r}(k)\mathbf{r}^{\mathrm{T}}(k)\right\} \tag{20}$$

$$= \sum_{j=1}^{J} \mathbf{H}_j E\left\{\mathbf{s}_j(k)\mathbf{s}_j^{\mathrm{T}}(k)\right\} \mathbf{H}_j^{\mathrm{T}} \tag{21}$$

where it vanishes if $\widetilde{\mathbf{R}}_{s_j s_j} \triangleq E\{\mathbf{s}_j(k)\mathbf{s}_j^{\mathrm{T}}(k)\} = \mathbf{0}$ for all j. It is inferred that if at least one of the users in the system employ improper symbols, the DS/CDMA signal becomes improper. Improper symbols arise from the improper data modulation, such as pulse-amplitude modulation (PAM),[1] offset quadrature amplitude modulation (OQAM), and binary phase-shift-keying (BPSK). Moreover, when the improper narrowband interference exists, the signal should be modeled as improper.

C. Review of max/min MOE Receiver [10]

The max/min MOE receiver $\mathbf{f}_C$ is obtained by minimizing MOE and maximizing the result subject to multiple constraints as follows:

$$\{\mathbf{f}_C, \mathbf{g}_C\} = \arg\max_{\mathbf{g}} \min_{\mathbf{f}} E\left\{\left|\mathbf{f}^{\mathrm{H}}\mathbf{r}(k)\right|^2\right\}$$
$$\text{subject to} \quad \mathbf{C}_1^{\mathrm{H}}\mathbf{f} = \mathbf{g}, \quad \|\mathbf{g}\| = 1 \tag{22}$$

where $\mathbf{r}(k)$ is the received signal of (14), and $\mathbf{C}_1$ is the constraining matrix given in (19). The optimal receiver is [19]

$$\mathbf{f}_C = \mathbf{R}_{rr}^{-1}\mathbf{C}_1\left(\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{C}_1\right)^{-1}\mathbf{g}_C \tag{23}$$

where $\mathbf{g}_C$ is the eigenvector corresponding to the minimum eigenvalue μ of the matrix $\mathbf{C}_1^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{C}_1$, and $\mathbf{R}_{rr}$ is the data covariance matrix defined as $\mathbf{R}_{rr} \triangleq E\{\mathbf{r}(k)\mathbf{r}^{\mathrm{H}}(k)\}$. The receiver $\mathbf{f}_C$ can be rewritten as

$$\mathbf{f}_C = \frac{1}{\mu}\mathbf{R}_{rr}^{-1}\mathbf{C}_1\mathbf{g}_C. \tag{24}$$

The SINR of the receiver $\mathbf{f}_C$ becomes

$$\mathrm{SINR}_C = \frac{1}{\frac{1}{\mu\sigma_s^2|\mathbf{g}_C^{\mathrm{H}}\mathbf{g}_1|^2} - 1}. \tag{25}$$

[1]It is also-called amplitude-shift-keying (ASK).

$$\mathbf{H}_j \triangleq \begin{bmatrix} \mathbf{h}_j(0) & \mathbf{h}_j(1) & \cdots & \mathbf{h}_j(L_h-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & & & & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_j(0) & \cdots & \mathbf{h}_j(L_h-2) & \mathbf{h}_j(L_h-1) \end{bmatrix} \tag{15}$$

Moreover, the SINR of the mmse receiver $\mathbf{f}_{C\text{mmse}} = \sigma_s^2 \mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}$ is easily shown that

$$\text{SINR}_{C\text{mmse}} = \frac{1}{\frac{1}{\sigma_s^2 \mathbf{h}_{d_f}^{\mathrm{H}} \mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}} - 1} \tag{26}$$

which is the maximum SINR obtainable with linear receiver [8]. Thus, in general

$$\text{SINR}_C \leq \text{SINR}_{C\text{mmse}}. \tag{27}$$

It is shown in [10] that the performance of this max/min blind approach adjoin that of the mmse receiver as the SNR goes high. At the same time, the parameter of the constraints converges to the desired user's channel impulse response $\mathbf{g}_1$.

III. WL MAX/MIN MOE RECEIVER

The complex received signal $\mathbf{r}(k)$ can be defined as

$$\mathbf{r}(k) = \mathbf{r}_R(k) + j\mathbf{r}_I(k), \qquad \mathbf{r}_R(k), \mathbf{r}_I(k) \in \mathbb{R}^{NL_f} \tag{28}$$

where $\mathbf{r}_R(k)$ represents the real part and $\mathbf{r}_I(k)$ the imaginary part of $\mathbf{r}(k)$. In general, the complex random vector may be treated as a two-dimensional real random vector $[\mathbf{r}_R^{\mathrm{T}}(k), \mathbf{r}_I^{\mathrm{T}}(k)]^{\mathrm{T}}$. Consider now the estimate of the desired user's symbol written as

$$\hat{s}_1(k) = \begin{bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\end{bmatrix}^{\mathrm{H}} \begin{bmatrix}\mathbf{r}_R(k)\\ \mathbf{r}_I(k)\end{bmatrix} \tag{29}$$

$$= \mathbf{f}_1^{\mathrm{H}}\mathbf{r}_R(k) + \mathbf{f}_2^{\mathrm{H}}\mathbf{r}_I(k) \tag{30}$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ are linear filters. Two vectors $[\mathbf{r}_R^{\mathrm{T}}(k), \mathbf{r}_I^{\mathrm{T}}(k)]^{\mathrm{T}}$ and $[\mathbf{r}^{\mathrm{T}}(k), \mathbf{r}^{\mathrm{H}}(k)]^{\mathrm{T}}$ can be related, using the $2NL_f \times 2NL_f$ matrix, as

$$\begin{bmatrix}\mathbf{r}_R(k)\\ \mathbf{r}_I(k)\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\mathbf{I}_{NL_f} & \mathbf{I}_{NL_f}\\ -j\mathbf{I}_{NL_f} & j\mathbf{I}_{NL_f}\end{bmatrix}\begin{bmatrix}\mathbf{r}(k)\\ \mathbf{r}^*(k)\end{bmatrix}. \tag{31}$$

Hence, the estimate $\hat{s}_1(k)$ in (29) can be rewritten as

$$\hat{s}_1(k) = \begin{bmatrix}\mathbf{f}\\ \bar{\mathbf{f}}\end{bmatrix}^{\mathrm{H}} \begin{bmatrix}\mathbf{r}(k)\\ \mathbf{r}^*(k)\end{bmatrix} \tag{32}$$

$$= \mathbf{f}^{\mathrm{H}}\mathbf{r}(k) + \bar{\mathbf{f}}^{\mathrm{H}}\mathbf{r}^*(k) \tag{33}$$

where $\mathbf{f} = (1/2)(\mathbf{f}_1 + \mathbf{f}_2)$ and $\bar{\mathbf{f}} = (1/2)(\mathbf{f}_1 - \mathbf{f}_2)$. Note that $\hat{s}_1(k)$ is linear in $[\mathbf{r}^{\mathrm{T}}(k), \mathbf{r}^{\mathrm{H}}(k)]^{\mathrm{T}}$ but not in $\mathbf{r}(k)$. For this reason, $\hat{s}_1(k)$ is called WL in $\mathbf{r}(k)$, and $[\mathbf{f}^{\mathrm{T}}, \bar{\mathbf{f}}^{\mathrm{T}}]^{\mathrm{T}}$ is called a WL filter [11].

A. Receiver Design

The problem of WL max/min MOE receiver is to find the vectors $\mathbf{f}$ and $\bar{\mathbf{f}}$ in such a way that

$$\{\mathbf{f}_M, \bar{\mathbf{f}}_M, \mathbf{g}_M, \bar{\mathbf{g}}_M\} = \arg\max_{\mathbf{g},\bar{\mathbf{g}}} \min_{\mathbf{f},\bar{\mathbf{f}}} E\left\{\left|\mathbf{f}^{\mathrm{H}}\mathbf{r}(k) + \bar{\mathbf{f}}^{\mathrm{H}}\mathbf{r}^*(k)\right|^2\right\}$$
$$\text{subject to} \quad \mathbf{C}_1^{\mathrm{H}}\mathbf{f} = \mathbf{g} \quad \text{and} \quad \mathbf{C}_1^{\mathrm{T}}\bar{\mathbf{f}} = \bar{\mathbf{g}}, \quad \|\mathbf{g}\|^2 + \|\bar{\mathbf{g}}\|^2 = 1 \tag{34}$$

or, equivalently, the following augmented problem:

$$\{\mathbf{f}_{MA}, \mathbf{g}_{MA}\} = \arg\max_{\mathbf{g}_A}, \min_{\mathbf{f}_A} E\left\{\left|\mathbf{f}_A^{\mathrm{H}}\mathbf{r}_A(k)\right|^2\right\}$$
$$\text{subject to} \quad \mathbf{C}_{1A}^{\mathrm{H}}\mathbf{f}_A = \mathbf{g}_A, \qquad \|\mathbf{g}_A\|^2 = 1 \tag{35}$$

where $\mathbf{f}_A \triangleq [\mathbf{f}^{\mathrm{T}}, \bar{\mathbf{f}}^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{r}_A(k) \triangleq [\mathbf{r}^{\mathrm{T}}(k), \mathbf{r}^{\mathrm{H}}(k)]^{\mathrm{T}}$, $\mathbf{g}_A \triangleq [\mathbf{g}^{\mathrm{T}}, \bar{\mathbf{g}}^{\mathrm{T}}]^{\mathrm{T}}$, and

$$\mathbf{C}_{1A} \triangleq \begin{bmatrix}\mathbf{C}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_1^*\end{bmatrix}. \tag{36}$$

The formulation (35) of the problem (34) enables us to yield optimal solution

$$\mathbf{f}_{MA} = \mathbf{R}_A^{-1}\mathbf{C}_{1A}\left(\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{C}_{1A}\right)^{-1}\mathbf{g}_{MA} \tag{37}$$

where $\mathbf{g}_{MA}$ represents the eigenvector corresponding to the minimum eigenvalue μ_A of the matrix $\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{C}_{1A}$, and

$$\mathbf{R}_A \triangleq E\left\{\mathbf{r}_A(k)\mathbf{r}_A^{\mathrm{H}}(k)\right\} = \begin{bmatrix}\mathbf{R}_{rr} & \widetilde{\mathbf{R}}_{rr}\\ \widetilde{\mathbf{R}}_{rr}^* & \mathbf{R}_{rr}^*\end{bmatrix} \tag{38}$$

with pseudo-covariance matrix $\widetilde{\mathbf{R}}_{rr} \triangleq E\{\mathbf{r}(k)\mathbf{r}^{\mathrm{T}}(k)\}$. Similarly to (24), the WL max/min MOE receiver $\mathbf{f}_{MA}$ can be rewritten as

$$\mathbf{f}_{MA} = \begin{bmatrix}\mathbf{f}_M\\ \bar{\mathbf{f}}_M\end{bmatrix} = \frac{1}{\mu_A}\mathbf{R}_A^{-1}\mathbf{C}_{1A}\mathbf{g}_{MA}. \tag{39}$$

From (38) and (39), the two receivers $\mathbf{f}_M$ and $\bar{\mathbf{f}}_M$ are, respectively, obtained as

$$\mathbf{f}_M = \frac{1}{\mu_A}\left(\mathbf{R}_{rr} - \widetilde{\mathbf{R}}_{rr}\widetilde{\mathbf{R}}_{rr}^{-*}\widetilde{\mathbf{R}}_{rr}^*\right)^{-1} \times \left(\mathbf{C}_1\mathbf{g}_M - \widetilde{\mathbf{R}}_{rr}\widetilde{\mathbf{R}}_{rr}^{-*}\mathbf{C}_1^*\bar{\mathbf{g}}_M\right) \tag{40}$$

and

$$\bar{\mathbf{f}}_M = \frac{1}{\mu_A}\left(\mathbf{R}_{rr}^* - \widetilde{\mathbf{R}}_{rr}^*\widetilde{\mathbf{R}}_{rr}^{-1}\widetilde{\mathbf{R}}_{rr}\right)^{-1} \times \left(\mathbf{C}_1^*\bar{\mathbf{g}}_M - \widetilde{\mathbf{R}}_{rr}^*\widetilde{\mathbf{R}}_{rr}^{-1}\mathbf{C}_1\mathbf{g}_M\right) \tag{41}$$

where $\widetilde{\mathbf{R}}_{rr}^{-*} = (\widetilde{\mathbf{R}}_{rr}^{-1})^* = (\widetilde{\mathbf{R}}_{rr}^*)^{-1}$. Unlike the conventional max/min MOE receiver (23), the receivers $\mathbf{f}_M$ and $\bar{\mathbf{f}}_M$ cannot be determined only with covariance matrix $\mathbf{R}_{rr}$ but also with pseudocovariance matrix $\widetilde{\mathbf{R}}_{rr}$.

The direct implementation of (37) is not realistic because of large computational demand of inversion procedure of the matrix $\mathbf{R}_A$ and singular value decomposition (SVD) of $\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{C}_{1A}$, as well as the need of exact knowledge of second-order statistic $\mathbf{R}_A$. The adaptive implementation without inversion of $\mathbf{R}_A$ and SVD of $\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{C}_{1A}$ is developed and presented in Section III-C.

B. SINR Analysis

In this section, the WL max/min MOE approach is compared with the conventional max/min MOE approach and with their mmse solutions. The output SINR is adopted as the performance index and defined as

$$\text{SINR} \triangleq \frac{\left|\mathbf{f}_A^{\rm H}\mathbf{h}_{d_f A}\right|^2}{\mathbf{f}_A^{\rm H}\left(\mathbf{R}_A - \mathbf{h}_{d_f A}\mathbf{h}_{d_f A}^{\rm H}\right)\mathbf{f}_A} \tag{42}$$

where

$$\mathbf{h}_{d_f A} = \begin{bmatrix} \sigma_s \mathbf{h}_{d_f} \\ \rho_s \mathbf{h}_{d_f}^* \end{bmatrix}. \tag{43}$$

The WL mmse receiver is given by [15]

$$\mathbf{f}_{M\text{mmse}} = \mathbf{R}_A^{-1}\mathbf{h}_{d_f A} \tag{44}$$

whose SINR is easily shown to be

$$\text{SINR}_{M\text{mmse}} = \frac{1}{\frac{1}{\mathbf{h}_{d_f A}^{\rm H}\mathbf{R}_A^{-1}\mathbf{h}_{d_f A}} - 1}. \tag{45}$$

Since, in general, the mmse criterion maximizes the output SINR

$$\text{SINR}_M \leq \text{SINR}_{M\text{mmse}}. \tag{46}$$

In the following, it is shown that the WL mmse receiver achieves the performance gain with improper signals.

Proposition 1: For any wide sense stationary signal $\mathbf{r}(k)$ and nonzero vector $\mathbf{h}_{d_f}$

$$\text{SINR}_{M\text{mmse}} \geq \text{SINR}_{C\text{mmse}} \tag{47}$$

where equality holds if and only if the signal $\mathbf{r}(k)$ is proper.

Proof: See Appendix. ■

Substituting (39) into (42), with some manipulations, the SINR of the receiver $\mathbf{f}_{MA}$ is given by

$$\text{SINR}_M = \frac{1}{\frac{1}{\mu_A\left|\mathbf{g}_{MA}^{\rm H}\mathbf{g}_{1A}\right|^2} - 1} \tag{48}$$

where $\mathbf{g}_{1A} = [\sigma_s \mathbf{g}_1^{\rm T}, \rho_s \mathbf{g}_1^{\rm H}]^{\rm T}$. It can be verified that the SINR of the WL MOE receiver is not smaller than that of the conventional MOE receiver for sufficiently high SNR case. It is shown in [10] that at high SNR, the channel estimation $\mathbf{g}_C$ converges to the real channel impulse response $\mathbf{g}_1/\|\mathbf{g}_1\|$ and that the SINR of MOE receiver becomes very close to that of mmse receiver. In the WL case, it can be similarly shown that $\mathbf{g}_{MA}$ converges to $\mathbf{g}_{1A}/\|\mathbf{g}_{1A}\|$. Note that Proposition 1 states that WL mmse improves the SINR of the conventional mmse receiver. From these facts, we can assure that the SINR of the WL MOE receiver is not smaller than that of the conventional MOE receiver at high SNR. In special degenerate case, the following result can be stated.

Proposition 2: When the received DS/CDMA signal is proper, the SINR of the WL max/min MOE receiver is identical to that of the conventional max/min MOE receiver, i.e.,

$$\text{SINR}_M = \text{SINR}_C. \tag{49}$$

Proof: When the received DS/CDMA signal is proper, i.e., $\widetilde{\mathbf{R}}_{rr} = \mathbf{0}$ and $\rho_s^2 = 0$, we have

$$\mathbf{C}_{1A}^{\rm H}\mathbf{R}_A^{-1}\mathbf{C}_{1A} = \begin{bmatrix} \mathbf{C}_1^{\rm H}\mathbf{R}_{rr}^{-1}\mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \left(\mathbf{C}_1^{\rm H}\mathbf{R}_{rr}^{-1}\mathbf{C}_1\right)^* \end{bmatrix}. \tag{50}$$

Reminding that $\mathbf{g}_C$ and μ are the minimum eigenvector and eigenvalue of the matrix $\mathbf{C}_1^{\rm H}\mathbf{R}_{rr}^{-1}\mathbf{C}_1$, $\mathbf{g}_{MA}$ and μ_A are

$$\mathbf{g}_{MA} = \begin{bmatrix} \mathbf{g}_C \\ \mathbf{g}_C^* \end{bmatrix} \quad \text{and} \quad \mu_A = \mu. \tag{51}$$

Then, SINR in (48)

$$\text{SINR}_M = \frac{1}{\frac{1}{\mu\sigma_s^2\left|\mathbf{g}_c^{\rm H}\mathbf{g}_1\right|^2} - 1} \tag{52}$$

which is identical to (25). ■

No SINR gain is attained with proper signals in the proposed scheme. Further, it increases the complexity of the receiver, though the WL filtering operation yields the same performance.

The relations of the SINRs can be summarized as follows:

$$\text{SINR}_C \leq \text{SINR}_{C\text{mmse}} \tag{53}$$

$$\text{SINR}_M \leq \text{SINR}_{M\text{mmse}} \tag{54}$$

$$\text{SINR}_{C\text{mmse}} \leq \text{SINR}_{M\text{mmse}} \tag{55}$$

$$\text{SINR}_C \leq \text{SINR}_M \quad \text{at high SNR}. \tag{56}$$

If the received signal $\mathbf{r}(k)$ is proper, then

$$\text{SINR}_C = \text{SINR}_M \tag{57}$$

$$\text{SINR}_{C\text{mmse}} = \text{SINR}_{M\text{mmse}}. \tag{58}$$

C. Blind-Adaptive Implementation

In this section, a recursive update equation is developed in a similar way to the recursive least squares (RLS) algorithm [20] for the least-squares estimates of

$$\mathbf{f}_{MA}(k) = \mathbf{R}_A^{-1}(k)\mathbf{C}_{1A}\left[\mathbf{C}_{1A}^{\rm H}\mathbf{R}_A^{-1}(k)\mathbf{C}_{1A}\right]^{-1}\mathbf{g}_{MA} \tag{59}$$

where $\mathbf{R}_A(k) \triangleq \sum_{i=1}^{k}\lambda^{k-i}\mathbf{r}_A(i)\mathbf{r}_A^{\rm H}(i)$ is the exponentially weighted covariance matrix, and $0 < \lambda \leq 1$ is the forgetting factor. From the matrix inversion lemma, the following recursive equation can be obtained for the inverse of the correlation matrix

$$\mathbf{R}_A^{-1}(k) = \frac{1}{\lambda}\left[\mathbf{R}_A^{-1}(k-1) - \mathbf{p}(k)\mathbf{r}_A^{\rm H}(k)\mathbf{R}_A^{-1}(k-1)\right] \tag{60}$$

where $\mathbf{p}(k)$ is the gain vector and is given by

$$\mathbf{p}(k) = \frac{\mathbf{R}_A^{-1}(k-1)\mathbf{r}_A(k)}{\lambda + \mathbf{r}_A^{\rm H}(k)\mathbf{R}_A^{-1}(k-1)\mathbf{r}_A(k)}. \tag{61}$$

Initialization

– Set the forgetting factor $\lambda \in (0,1]$ and $\delta > 0$ (small positive constant) and then, initialize the inverse correlation matrix $\mathbf{P}(0) = \delta^{-1}\mathbf{I}$ and the matrix $\mathbf{Q}(0) = [\mathbf{C}_{1A}^H \mathbf{P}(0)\mathbf{C}_{1A}]^{-1}$;

– Initialize the vector $\mathbf{g}_A(0) = [1, 0, \cdots, 0]^H$.

Filter update equations

– for $k = 1, 2, \cdots,$

$$\begin{aligned}\mathbf{p}(k) &= \frac{\mathbf{P}(k-1)\mathbf{r}_A(k)}{\lambda + \mathbf{r}_A^H(k)\mathbf{P}(k-1)\mathbf{r}_A(k)}\\ \boldsymbol{\alpha}(k) &= \frac{\mathbf{Q}(k-1)\mathbf{C}_{1A}^H\mathbf{p}(k)}{1 - \mathbf{r}_A^H(k)\mathbf{P}(k-1)\mathbf{C}_{1A}\mathbf{Q}(k-1)\mathbf{C}_{1A}^H\mathbf{p}(k)}\\ \mathbf{Q}(k) &= \lambda[\mathbf{Q}(k-1) + \boldsymbol{\alpha}(k)\mathbf{r}_A^H(k)\mathbf{P}(k-1)\mathbf{C}_{1A}\mathbf{Q}(k-1)]\\ \mathbf{P}(k) &= \frac{1}{\lambda}[\mathbf{P}(k-1) - \mathbf{p}(k)\mathbf{r}_A^H(k)\mathbf{P}(k-1)]\\ \mathbf{z}(k) &= \mathbf{Q}(k)\mathbf{g}_A(k)\\ \mathbf{g}_A(k) &= \mathbf{z}(k)/\|\mathbf{z}(k)\|_2\\ \mathbf{f}_{MA}(k) &= \mathbf{P}(k)\mathbf{C}_{1A}\mathbf{Q}(k)\mathbf{g}_A(k).\end{aligned}$$

Fig. 1. Blind-adaptive implementation of the proposed scheme.

Multiplying (60) by $\mathbf{C}_{1A}^{\mathrm{H}}$ to the left and $\mathbf{C}_{1A}$ to the right, we obtain

$$\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}(k)\mathbf{C}_{1A} = \frac{1}{\lambda}\left[\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}(k-1)\mathbf{C}_{1A} - \mathbf{C}_{1A}^{\mathrm{H}}\mathbf{p}(k)\mathbf{r}_A^{\mathrm{H}}(k)\mathbf{R}_A^{-1}(k-1)\mathbf{C}_{1A}\right]. \quad (62)$$

Applying the matrix inversion lemma to (62) once again, the recursive update equation of $\mathbf{Q}(k) \triangleq [\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{R}_A^{-1}(k)\mathbf{C}_{1A}]^{-1}$ can be obtained as

$$\mathbf{Q}(k) = \lambda\left[\mathbf{Q}(k-1) + \boldsymbol{\alpha}(k)\mathbf{r}_A^{\mathrm{H}}(k)\mathbf{R}_A^{-1}(k-1)\mathbf{C}_{1A}\mathbf{Q}(k-1)\right] \quad (63)$$

where

$$\boldsymbol{\alpha}(k) = \frac{\mathbf{Q}(k-1)\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{p}(k)}{1 - \mathbf{r}_A^{\mathrm{H}}(k)\mathbf{R}_A^{-1}(k-1)\mathbf{C}_{1A}\mathbf{Q}(k-1)\mathbf{C}_{1A}^{\mathrm{H}}\mathbf{p}(k)}. \quad (64)$$

A simple power iteration method [21, Sec. 8.2] can be used to adaptively update the vector $\mathbf{g}_A(k)$

$$\mathbf{z}(k) = \mathbf{Q}(k)\mathbf{g}_A(k) \quad (65)$$

$$\mathbf{g}_A(k) = \frac{\mathbf{z}(k)}{\|\mathbf{z}(k)\|_2}. \quad (66)$$

The adaptive algorithm is summarized in Fig. 1, and the complexity of the proposed adaptive method is $\max(O((NL_f)^2), O(L_g^2))$. The adaptive algorithms of the conventional MOE receiver are presented in [22], and their WL version of least-mean-square (LMS) algorithms are investigated in [23]. The adaptive algorithm in Fig. 1 is the WL version of an RLS algorithm for the MOE receiver, but it does not require SVD decomposition unlike the method IV of [22]. For this reason, the proposed scheme maintains the square-order complexity.

IV. Simulation Results

In this section, computer-simulation results are presented to illustrate the effectiveness of the proposed approach in comparison with the conventional max/min MOE approach. We consider a DS/CDMA system, in which each user transmits their symbols through an individual multipath channel. It is assumed that the synchronism of the desired user is known, and the transmission delays of all the interfering users are uniformly distributed over one-symbol period. The multipath channel of each user is randomly generated, omitting the user index, by

$$g(k) = \sum_{q=1}^{L_p} \alpha_q p(kT_c - \tau_q) \quad (67)$$

where τ_q is the transmission delay uniformly distributed over one-symbol interval, and α_q is the complex Gaussian random variable with zero mean and unit variance. The number of multipaths is $L_p = 10$, and $p(t)$ is the raised-cosine pulse function with roll-off factor of 0.5. The SNR at the receiver front end is defined as

$$\mathrm{SNR} \triangleq \frac{E\left\{\left\|\mathbf{h}_{d_f}s_1(k-d_f)\right\|^2\right\}}{E\left\{\|\mathbf{w}(k)\|^2\right\}} \quad (68)$$

$$= \frac{\|\mathbf{h}_{d_f}\|^2}{\sigma_v^2 N L_f}. \quad (69)$$

Unless otherwise specified, the user delay d_j is uniformly distributed within $[0, N-1]$, and the channel length L_g is ten chips. We set the receiver length in symbol period at $L_f = 3$, and the receiver delay at $d_f = 1$.

A. Verification of Analytical Results

In the set of experiments, we demonstrate the proposed method and compare it with the conventional max/min method. We also illustrate the performance of mmse receivers as performance bounds. Gold sequences are employed as spreading codes, with the spreading factor $N = 31$, and the number of

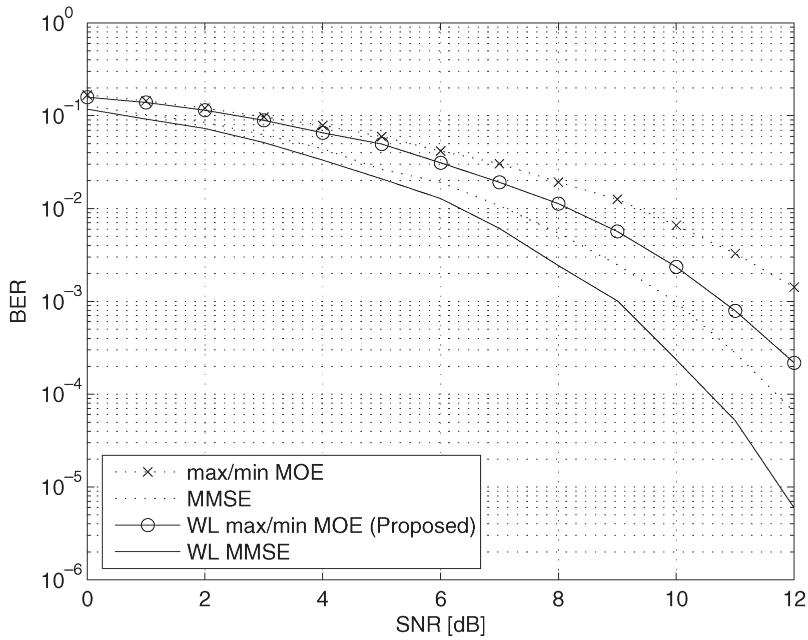

Fig. 2. BER versus input SNR when all the users transmit the BPSK symbols for $N = 31$ and $J = 20$.

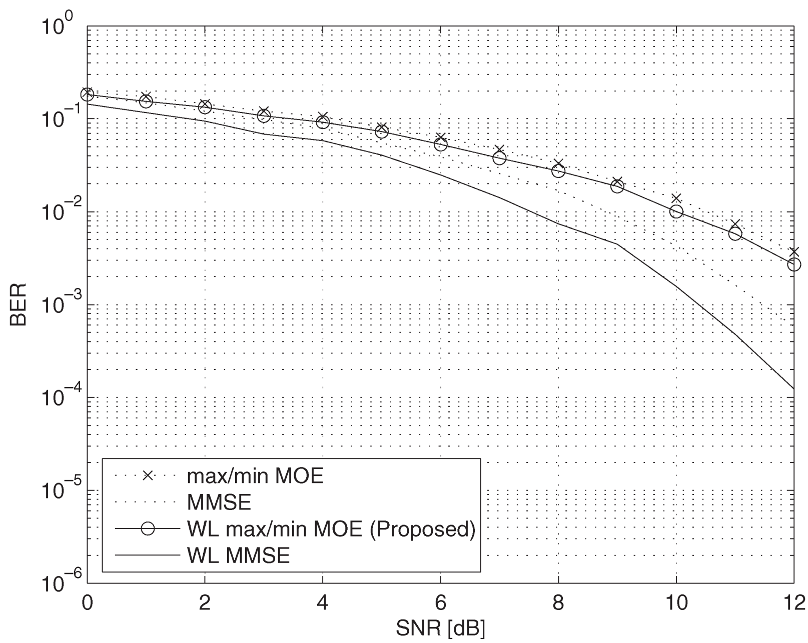

Fig. 3. BER versus input SNR when ten users, including the desired user, transmit the BPSK and the other interferers transmit the QPSK symbols for $N = 31$ and $J = 20$.

users $K = 20$. Note that most of the leading standard proposals particularly interest in the quadrature phase-shift-keying (QPSK) and in BPSK modulation [24]–[26]. To cover the various situations, we classify four cases as follows

Cases I—All Users Use BPSK: Fig. 2 illustrates the bit-error rate (BER) versus the receiver front-end SNR of four types of receivers, when all users in the system transmit the BPSK signal and DS/CDMA signal is improper. Clearly, the proposed WL max/min MOE receiver outperforms the conventional max/min MOE receiver.

Cases II—Ten Users Including Desired User Use BPSK and the Others Use QPSK: When ten users including the desired user employ BPSK data modulation and the other interferers employ QPSK data modulation, the BERs of four types of receivers are shown in Fig. 3. Although the performance difference between the proposed WL max/min MOE receiver and the conventional max/min MOE is reduced, the whole trend is sustained as in Fig. 2.

Cases III—Desired User Uses QPSK and Interferers, BPSK: In this case, the desired user transmits the QPSK signal and the interferers transmit the BPSK signals. The improperness of DS/CDMA signal results in the performance improvement. The superiority of the proposed method is prominent at high SNRs and it is shown in Fig. 4.

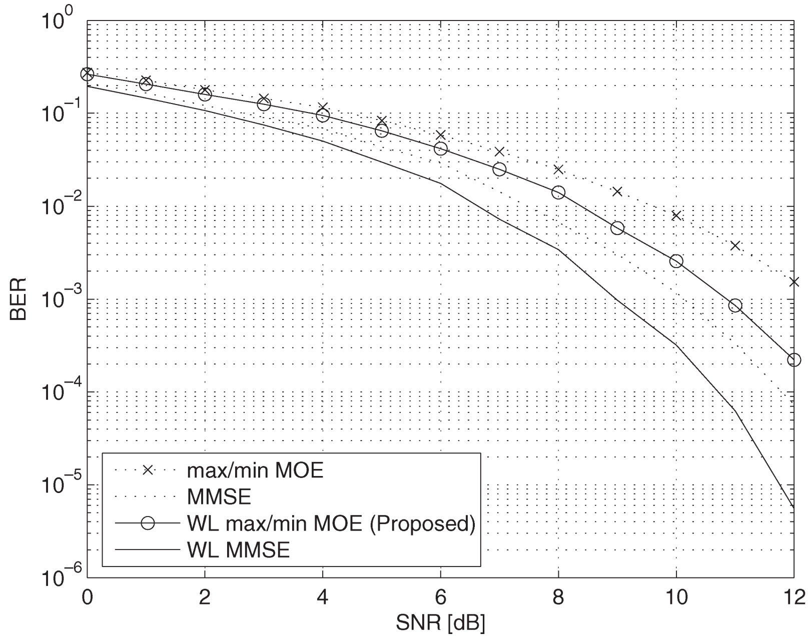

Fig. 4. BER versus input SNR when the desired user transmits the QPSK and all the interferers transmit the BPSK symbols for $N = 31$ and $J = 20$.

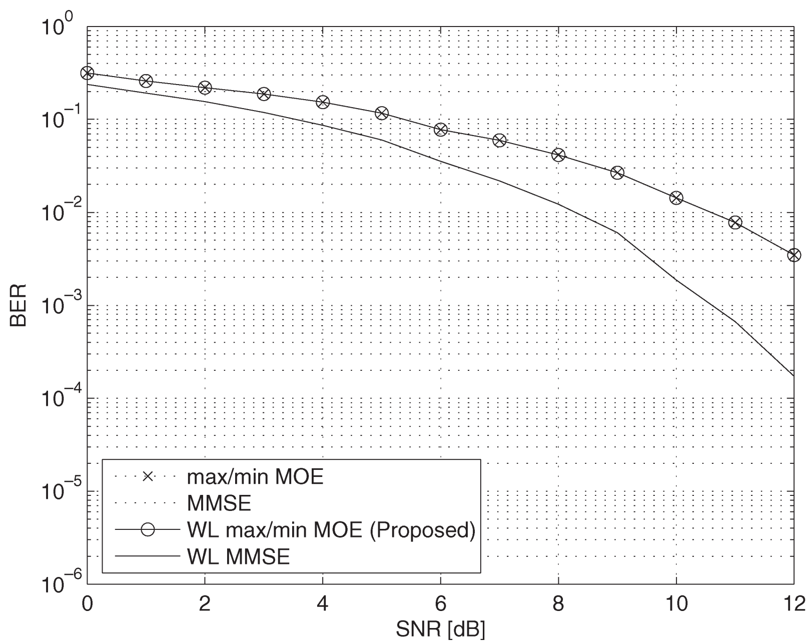

Fig. 5. BER versus input SNR when all the users transmit the QPSK symbols for $N = 31$ and $J = 20$.

Cases IV—All Users Use QPSK: When all users employ QPSK, the received DS/CDMA signal becomes proper. In this case, the SINR of the proposed method is identical to that of the conventional one as claimed in (49), and the BERs are also the same, as shown in Fig. 5.

B. Verification of Adaptive Implementation

The performance of the adaptive WL max/min MOE method is compared with that of conventional counterpart. The random spreading sequence with $N = 16$ is applied to both receivers, and the number of user is 10 ($J = 10$). All users in the system modulate the data with BPSK modulation, and the forgetting factor is taken as $\lambda = 0.998$. Fig. 6 represents the convergence dynamics at 20-dB SNR and experimentally validates the superiority of the adaptive implementation of the proposed scheme. It is seen that the proposed method has faster tracking ability and higher steady-state SINR, and significantly outperforms the conventional method.

The second example illustrates the tracking ability in dynamic loading environment. Unless otherwise specified, the environment of this simulation is identical to the first example. The random spreading sequence also applied with $N = 32$, and the number of active users in the system is assumed to

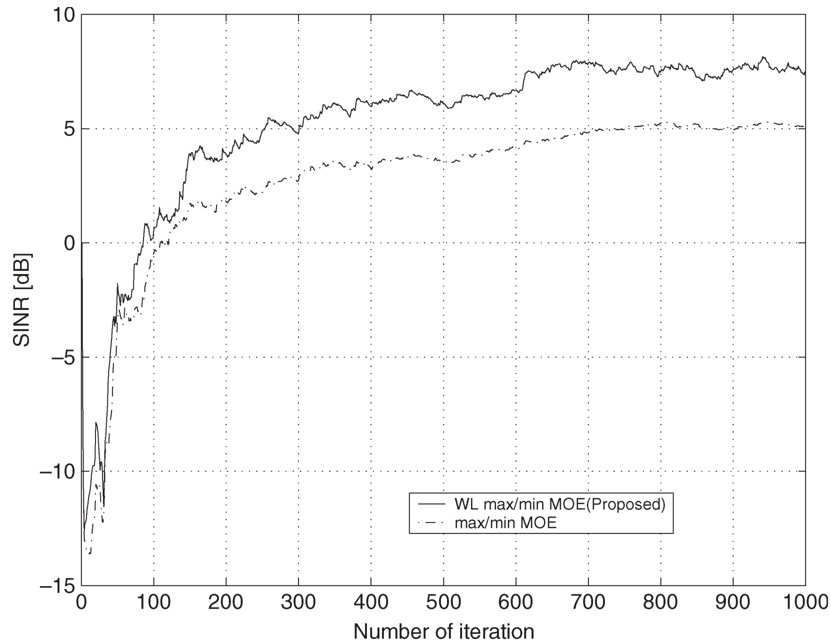

Fig. 6. Output SINR versus time of the proposed receiver and classical receivers for $N = 16$ and $J = 10$.

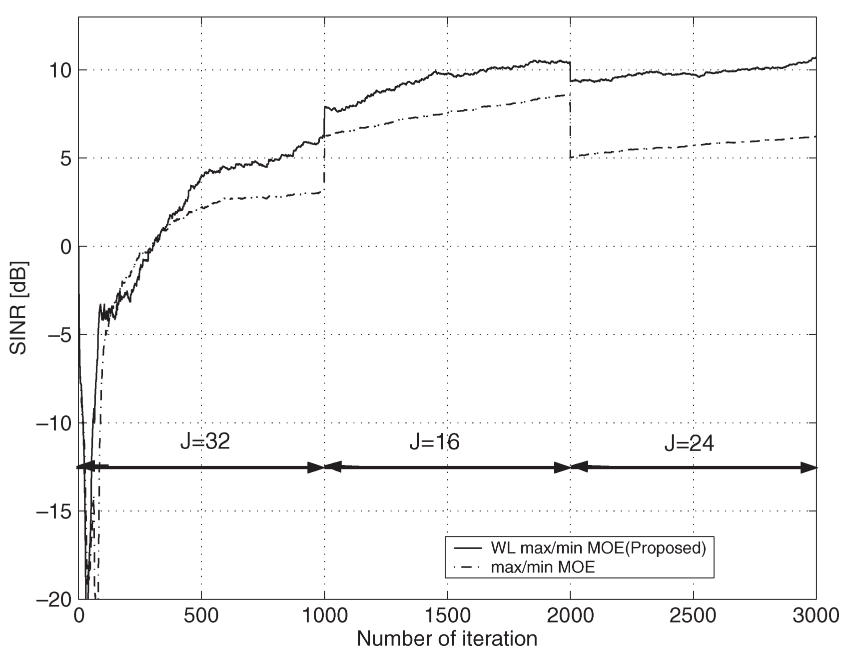

Fig. 7. Tracking ability of the WL max/min MOE algorithm and the conventional max/min MOE algorithm in dynamic loading environment.

be varying. At first, the number of active users starts with 32, which is the full-loading system, and reduces to 16 at 1000 iterations and increases to 24 at 2000 iterations. It is shown in Fig. 7 that there is a big difference between the SINR of the WL max/min MOE algorithm and that of the conventional max/min MOE algorithm.

V. Conclusion

Data-modulation formats such as PAM (ASK), OQAM, and BPSK result in improper signals. In DS/CDMA systems, if anyone of the user in the system employs such modulation format, the resulting signals become improper. An efficient blind multiuser detection scheme, based on WL signal processing, has been proposed for the improper DS/CDMA systems. The SINR of the proposed WL max/min MOE receiver is analyzed and compared with other receivers. The superiority of WL receivers comes from the additional information of the covariance matrix of the improperly received signal.

The adaptive implementation of the proposed receiver has also been considered. The adaptive algorithm will be used in practice with very stringent time requirements. It is anticipated that the proposed adaptive implementation of $\max(O((NL_f)^2), O(L_g^2))$ complexity is most probable to bring its usage into a practical regime.

APPENDIX
PROOF OF PROPOSITION 1

Note that $\mathrm{SINR}_{M\mathrm{mmse}}$ of (45) and $\mathrm{SINR}_{C\mathrm{mmse}}$ of (26) are monotonic increasing function of $\mathbf{h}_{d_f A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{h}_{d_f A}$ and $\sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}$, respectively. Therefore, the proof of

$$\mathbf{h}_{d_f A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{h}_{d_f A} \geq \sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f} \tag{70}$$

would complete the whole proof. The difference of the left side and the right side of (70) is

$$\begin{aligned}\mathbf{h}_{d_f A}^{\mathrm{H}}&\mathbf{R}_A^{-1}\mathbf{h}_{d_f A} - \sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}\\ &= \begin{bmatrix}\sigma_s\mathbf{h}_{d_f}\\ \rho_s\mathbf{h}_{d_f}^*\end{bmatrix}^{\mathrm{H}}\begin{bmatrix}\mathbf{R}_{rr} & \widetilde{\mathbf{R}}_{rr}\\ \widetilde{\mathbf{R}}_{rr}^* & \mathbf{R}_{rr}^*\end{bmatrix}^{-1}\\ &\quad\times\begin{bmatrix}\sigma_s\mathbf{h}_{d_f}\\ \rho_s\mathbf{h}_{d_f}^*\end{bmatrix} - \sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}.\end{aligned} \tag{71}$$

From the inversion of partitioned matrix [27] and with some manipulation, we have

$$\begin{aligned}\mathbf{h}_{d_f A}^{\mathrm{H}}&\mathbf{R}_A^{-1}\mathbf{h}_{d_f A} - \sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f}\\ &= \left(\sigma_s\widetilde{\mathbf{R}}_{rr}^*\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f} - \rho_s\mathbf{h}_{d_f}^*\right)^{\mathrm{H}}\left(\mathbf{R}_{rr}^* - \widetilde{\mathbf{R}}_{rr}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\widetilde{\mathbf{R}}_{rr}\right)^{-1}\\ &\quad\times\left(\sigma_s\widetilde{\mathbf{R}}_{rr}^*\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f} - \rho_s\mathbf{h}_{d_f}^*\right).\end{aligned} \tag{72}$$

Since the matrix $(\mathbf{R}_{rr}^* - \widetilde{\mathbf{R}}_{rr}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\widetilde{\mathbf{R}}_{rr})^{-1}$ is a positive matrix, we conclude that

$$\mathbf{h}_{d_f A}^{\mathrm{H}}\mathbf{R}_A^{-1}\mathbf{h}_{d_f A} \geq \sigma_s^2\mathbf{h}_{d_f}^{\mathrm{H}}\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f} \tag{73}$$

and the equality holds if and only if, from (72)

$$\sigma_s\widetilde{\mathbf{R}}_{rr}^*\mathbf{R}_{rr}^{-1}\mathbf{h}_{d_f} - \rho_s\mathbf{h}_{d_f}^* = \mathbf{0} \tag{74}$$

or, equivalently

$$\widetilde{\mathbf{R}}_{rr} = 0 \quad \text{and} \quad \rho_s = 0 \tag{75}$$

as claimed.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their helpful and constructive comments on the paper.

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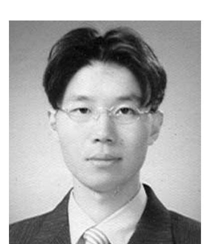

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