

# On Additional Credit Spreads Caused by Jump Risks of the Default Rate

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## Abstract

This paper studies credit spreads when the default intensity is affected by jump risks. A simple pricing model of risky bonds is derived using a reduced-form approach when there are jump risks of the factors of the default intensity, as supported by empirical evidence. Numerical analyses show that the additional credit spreads caused by jumps can be significant.

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# 1 Introduction

There are two approaches to pricing risky assets, the structural model and the reduced form model. The structural model is pioneered by Merton (1974). He uses the contingent claim methodology to value simple corporate debt which is dependent on the firm value. Thus, the pricing of risky debt reflects the structural characteristics of a firm. The reduced form model is provided by Jarrow and Turnbull (1995), Lando (1998), Madan and Unal (1998) and Duffie and Singleton (1999). The reduced form models treat defaults as unpredictable events, and determines the price for defaultable contingent claims using the observable credit spreads. In the reduced form model the intensity function must be estimated.

Recently, in relation to the reduced form approach, there have been several empirical studies illustrating which observable state variables affect the default intensity, while some theoretical papers have been presented that enrich the explanation of risky bond pricing. Madan and Unal (2000) derive a two-factor hazard rate model where the hazard rate is composed of the asset value of the firm and the stochastic interest rate. Their model has an advantage in that the structural characteristics of the firm are linked to the default rate. For modeling convenience, they use the diffusion model in firm value and state that

The jump nature of default can also be modelled by specifying a jump-diffusion process for asset values. In such a case, default can either occur on asset values diffusing to a threshold or a firm can face more than one negative jump in equity cumulating to default. For reasons of tractability, we focus on the single jump case.

This paper studies the credit spreads of risky bonds when the firm that issues the bonds has

jump risks. To obtain the results, we extend the two-factor hazard rate model of Madan and Unal (2000) by allowing the firm value to have accumulated jumps and the size of the jump to be random. By assuming that jumps are triggered by a Poisson process and the size of jump is log-normally distributed, we obtain the analytical additional credit spreads caused by jumps.

In modeling jump risks, we use the jump-diffusion model in firm value as in Ahn (1992), Merton (1976), and Zhou (2001). There is empirical evidence (e.g., Bates (1996), Jarrow and Rosenfeld (1984), Jorion (1988), and Kon (1984)) on jumps in asset values. In addition to empirical studies, many authors obtain more flexible features in pricing risky bonds by adopting the jump-diffusion model.

Mason and Bhattacharya (1981) provide risky bond pricing under a pure jump process. They show that the value of a safety covenant under a pure jump is different from that under a pure diffusion.

Duffie and Lando (2001) use the jump-diffusion framework to study credit spreads with incomplete information. By using a jump-diffusion model, they show that the shape of the term structure of credit spreads provides some indication of the quality of information.

Based on the structural approach, the Zhou (2001) model provides the price of a risky bond when the default of a firm can occur suddenly from an unpredictable jump in firm values by using jump-diffusion evolution. He obtains several results under the jump-diffusion process of the firm. Like Duffie and Lando (2001), one of the results is that the non-zero credit spreads on short maturity can be obtained. This is not allowed under a pure-diffusion framework because of the continuity of the diffusion process.

Contrary to Zhou's structural approach, we use the reduced-form approach incorporating

properties of the structural approach. We show that additional credit spreads caused by jump risk can be nontrivial. In general, we show that the credit spreads of higher-rated firms are affected more by jump risk. Among the jump risk components, i.e., average jump size, jump frequency, and jump size volatility, the effect of the average jump size is largest, and jump frequency is the next largest.

This model can be easily applied to the model of Janosi, Jarrow and Yildirim (2001) and the Jarrow (2001) and Jarrow and Turnbull (2000) models in which the factors of the default intensity are the spot interest rate and the market index (e.g., S&P 500).<sup>1</sup>

The organization of this paper is as follows: In Section 2 we set up the framework of credit risk and obtain the pricing formula for a risky bond. In Section 3 we compare the credit spread of our extended model to that of Madan and Unal (2000) by using the analytical solution for risky bonds obtained in Section 2. Section 3 describes the impact of jump risks on different rated firms' credit spreads. Section 4 concludes and summarizes the paper.

## 2 Model

In this section we provide a simple reduced-form model where the default intensity has accumulated jump components. Suppose that there are no arbitrage opportunities in the economy represented by a probability space  $(\Omega, \mathcal{F}_t, F, Q)$ , where  $F \equiv \{\mathcal{F}_t\}_{0 \leq t \leq T}$ . This implies that there exists an equivalent martingale measure  $Q$  with respect to the physical probability measure  $P$ . Under the equivalent martingale measure  $Q$ , all money market scaled asset prices are

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<sup>1</sup>Jarrow and Rosenfeld (1984) show the discontinuity of the market portfolio. In other words, the discontinuity of the market portfolio in daily price changes is observed empirically.

martingales. We use a valuation scheme under the equivalent martingale measure  $Q$ .

Consider a firm issuing a risky discount bond  $D$  with maturity  $T$ . Let  $\tau$  be the default time of the firm. As in Lando (1998), we assume that the default time is considered as the jump time of a Cox process<sup>2</sup>. The intensity process  $\lambda$  is a function of a state variable  $X_t$ . The intensity process  $\lambda(X_t)$  represents the approximate default probability of the firm, that is,  $P(\tau \leq t + \Delta t | \tau \geq t) = \lambda(X_t)\Delta t$ . Let  $\delta(\tau)$  denote the recovery rate of this bond. If a default occurs, then the bondholders receive  $\delta(\tau)$ . Otherwise, the bondholders receive the face value of this bond. The time  $t$  price of the risky bond is given by

$$D(t, T) = E_t^Q \left[ e^{-\int_t^T r(s)ds} \{ I_{\{\tau \geq T\}} + \delta(\tau) I_{\{\tau \leq T\}} \} \right]. \quad (1)$$

In this framework, we need to specify the recovery rate  $\delta(\tau)$ , the intensity process  $\lambda(X_t)$  and the spot interest rate  $r(t)$ .

## 2.1 When the firm value contains jumps

As in Madan and Unal (2000), the spot interest rate and the firm value are assumed to be two state variables determining the default intensity process. Following Duffie and Lando (2001), and Zhou (2001), we assume that the dynamics of firm value are governed by a jump-diffusion process.

**Assumption 1.** *The evolution of the firm value is given by*

$$\frac{dV^J}{V^J} = [r - \nu E(\Pi - 1)]dt + \sigma_V dW + (\Pi - 1)dY, \quad (2)$$

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<sup>2</sup>This is called a doubly stochastic Poisson process, which is a Poisson process under conditioning of the state variables.

where  $Y$  is a Poisson process with intensity  $\nu$ , and  $Y$  is independent of the jump size  $\Pi$ , the standard Brownian motion  $W$  under the equivalent martingale measure  $Q$ .

As in Ahn (1992), Merton (1976), and Zhou (2001), the firm value has a log-normal jump diffusion process as follows:<sup>3</sup> each  $\Pi_i$  is independently log-normally distributed with

$$\ln \Pi_i \sim N(\mu_\pi, \sigma_\pi^2) \quad \forall i. \quad (3)$$

This gives  $\phi \equiv E(\Pi - 1) = \exp(\mu_\pi + \frac{1}{2}\sigma_\pi^2) - 1$ . Let  $V^J$  denote the firm value with jumps and  $V$  denote the firm value without jumps and also note that zero intensity ( $\nu$ ) reduces this model into the model of Madan and Unal (2000).

**Assumption 2.** *The dynamic of spot interest rate is given by*

$$dr = \kappa(\theta - r)dt + \sigma dz, \quad (4)$$

where  $\kappa, \theta$  and  $\sigma$  are constants,<sup>4</sup> and  $z$  is a standard Brownian motion that is independent of  $Y$  and  $\Pi$ . The covariance between  $dz$  and  $dW$  is  $\rho dt$ .

Using results in Vasicek (1977), the price of a default-free discount bond is

$$P(t, T) = E_t^Q[e^{-\int_t^T r(u)du}] = e^{-\mu_R + \frac{1}{2}\sigma_R^2}, \quad (5)$$

where

$$\begin{aligned} \mu_R &\equiv E_t^Q\left[\int_t^T r(s)ds\right] = \theta(T-t) + \frac{r(t) - \theta}{\kappa}[1 - e^{-\kappa(T-t)}], \\ \sigma_R^2 &\equiv \text{Var}_t^Q\left[\int_t^T r(s)ds\right] = \frac{\sigma^2}{2\kappa^3}[2\kappa(T-t) + 4e^{-\kappa(T-t)} - 3 - e^{-2\kappa(T-t)}]. \end{aligned}$$

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<sup>3</sup>Mason and Bhattacharya (1981) adopt a pure jump process with jump amplitude following a binomial distribution.

<sup>4</sup>As in Jarrow and Turnbull (2000), we can suppose that  $\theta$  is a deterministic function of time  $t$ . For simplicity, in this paper the long-term interest rate is a constant.

Since this is followed by a Gaussian process, the interest rate may be negative. The probability of having a negative interest rate is very small.

**Assumption 3.** *The default intensity<sup>5</sup> consists of the market index and the spot interest rate:*

$$\lambda(t) = \max[a - b \ln V^J(t) + cr(t), 0] \quad (6)$$

If the market index has no jumps, then the default intensity is the same as that of Madan and Unal (2000).

We use the recovery rate as a fractional market value, as in Jarrow (2001).

**Assumption 4.** *If a default occurs, the bondholders receive a fractional market value  $(1 - L(\tau))D(\tau-, T)$ , where  $\tau-$  is an instant before the default and  $L(t)$  is a fractional loss. We assume that  $L(t)$  is a constant.*

From Duffie and Singleton (1999) and Jarrow (2001), we obtain the price of a risky discount bond when the firm value follows continuous sample paths.

$$\begin{aligned} D(t, T) &= E_t^Q \left[ e^{-\int_t^T r(s) + \lambda(s) L ds} \right] \\ &= E_t^Q \left[ e^{-\int_t^T r(s) + \{a - b \ln V(t) + cr(s)\} L ds} \right]. \end{aligned} \quad (7)$$

When compared with the price of a risky bond without accumulated jumps,

$$\ln V^J(u) = \ln V(u) + \psi(u), \quad (8)$$

where

$$\psi(u) = \sum_{i=N_t+1}^{N_u} \ln \Pi_i - \int_t^u \nu \phi ds. \quad (9)$$

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<sup>5</sup>For analytical tractability, we omit the maximum operator as in Janosi, Jarrow and Yildirim (2002).

From Duffie and Singleton (1999) and the above assumptions, we obtain the price of a risky discount bond when the firm value has jumps.

$$\begin{aligned}
D(t, T) &= E_t^Q [e^{-\int_t^T r(s) + \lambda(s) L ds}] \\
&= E_t^Q [e^{-\int_t^T r(s) + \{a - b \ln V(s) - b\psi(s) + cr(s)\} L ds}] \\
&= E_t^Q [e^{-\int_t^T r(s) + \{a - b \ln V(s) + cr(s)\} L ds}] E_t^Q [e^{-\int_t^T (-b\psi(s)) L ds}]. \tag{10}
\end{aligned}$$

Using results in Madan and Unal (2001), we can obtain the first term of the price of a risky discount bond.<sup>6</sup>

Then we focus on the second term, that is, the part caused by jumps:

$$\varphi \equiv E_t^Q [e^{-\int_t^T (-b\psi(s)) L ds}]. \tag{11}$$

**Proposition 1.** *The value of a risky discount bond with cumulative jump components is*

$$D(t, T) = P(t, T) D_0(t, T) \varphi,$$

where

$$\varphi = \exp \left( \nu \left\{ \int_0^{T-t} e^{cL\mu_\pi x + \frac{1}{2} c^2 L^2 \sigma_\pi^2 x^2} dx - (T-t) - \frac{cL}{2} (e^{\mu_\pi + \frac{1}{2} \sigma_\pi^2} - 1) (T-t)^2 \right\} \right), \tag{12}$$

and  $P(t, T) D_0(t, T)$  is provided in proposition 1 by Madan and Unal.

Since the credit spread  $\chi(t, T)$  is given by

$$\chi(t, T) = -\frac{1}{T-t} \ln \frac{D(t, T)}{P(t, T)}, \tag{13}$$

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<sup>6</sup>See proposition 1 in Madan and Unal (2000). We can obtain the formula of the diffusion component by replacing the coefficients  $a, b$ , and  $c$  in Madan and Unal by  $aL, bL$ , and  $cL$ , respectively.



the additional credit spread is as follows:

$$-\frac{1}{T-t} \ln \varphi = -\frac{1}{T-t} [\nu \{ \int_0^{T-t} e^{cL\mu_\pi x + \frac{1}{2}c^2L^2\sigma_\pi^2 x^2} dx - (T-t) - \frac{cL}{2}(e^{\mu_\pi + \frac{1}{2}\sigma_\pi^2} - 1)(T-t)^2 \}]. \quad (14)$$

**Proof.** The difference of our formula from the one by Madan and Unal (2000) is that the jump component is contained in the firm value. We concentrate on the jump part  $\varphi$ . Since  $Y$ ,  $\{\Pi_i\}_i$  are mutually independent of  $W(t)$ ,  $z(t)$ ,

$$\varphi \equiv E_t^Q [e^{-\int_t^T -b\{\sum_{i=N_t+1}^{N_s} \ln \Pi_i - \nu \phi(s-t)\} L ds}].$$

We need to solve

$$\begin{aligned} & E_t^Q [\exp(bL \int_t^T \sum_{i=N_t+1}^{N_s} \ln \Pi_i ds)] \\ &= E_t^Q [\exp(bL \int_0^{T-t} \sum_{i=1}^{N_s} \ln \Pi_i ds)]. \end{aligned} \quad (15)$$

The exponent part of (15) is

$$\begin{aligned} bL \int_0^{T-t} \sum_{i=1}^{N_s} \ln \Pi_i ds &= bL \int_0^{T-t} \sum_{i=1}^{\infty} \ln \Pi_i I_{\{N_s \geq i\}} ds \\ &= bL \sum_{i=1}^{\infty} \ln \Pi_i \int_0^{T-t} I_{\{N_s \geq i\}} ds \\ &= bL \sum_{i=1}^{\infty} \ln \Pi_i \max [(T-t) - T_i, 0], \end{aligned} \quad (16)$$

where  $T_i$  is the time when the  $i$ th jump occurs and  $I$  is the indicator function.

Equation (15) becomes

$$\begin{aligned} & E_t^Q \left[ \exp(bL \sum_{i=1}^{\infty} \ln \Pi_i \max ((T-t) - T_i, 0)) \right] \\ &= \sum_{n=0}^{\infty} P(N_{T-t} = n) E_t^Q \left[ \exp(bL \sum_{i=1}^{\infty} \ln \Pi_i \max ((T-t) - T_i, 0)) | N_{T-t} = n \right]. \end{aligned} \quad (17)$$

The value of the above expectation is

$$\begin{aligned}
& E_t^Q \left[ \exp \left( bL \sum_{i=1}^{\infty} \ln \Pi_i \max((T-t) - T_i, 0) \right) \middle| N_{T-t} = n \right] \\
&= E_t^Q \left[ \exp \left( bL \sum_{i=1}^n \ln \Pi_i ((T-t) - U_{(i)}) \right) \right] \\
&= E_t^Q \left[ \exp \left( bL \sum_{i=1}^n \ln \Pi_i ((T-t) - U_i) \right) \right], \tag{18}
\end{aligned}$$

where  $U_{(1)}, U_{(2)}, \dots, U_{(n)}$  are the order statistics of  $U_1, U_2, \dots, U_n$  which are uniformly distributed.<sup>7</sup>

In particular, suppose that  $\{\ln \Pi_i\}_i$  are independently normally distributed with mean  $\mu_\pi$  and variance  $\sigma_\pi^2$ .

$$\begin{aligned}
& E_t^Q \left[ \exp \left( bL \sum_{i=1}^n \ln \Pi_i ((T-t) - U_i) \right) \right] \\
&= \frac{1}{(T-t)^n} \int_0^{T-t} \cdots \int_0^{T-t} E_t^Q \left[ \exp \left( bL \sum_{i=1}^n x_i \ln \Pi_i \right) \right] dx_1 \cdots dx_n \\
&= \frac{1}{(T-t)^n} \int_0^{T-t} \cdots \int_0^{T-t} E_t^Q \left[ \exp \left( bL \mu_\pi \sum_{i=1}^n x_i + \frac{1}{2} b^2 L^2 \sigma_\pi^2 \sum_{i=1}^n x_i^2 \right) \right] dx_1 \cdots dx_n \\
&= \left( \frac{1}{T-t} \int_0^{T-t} \exp \left( bL \mu_\pi x + \frac{1}{2} b^2 L^2 \sigma_\pi^2 x^2 \right) dx \right)^n, \tag{19}
\end{aligned}$$

We present the jump component:

$$\begin{aligned}
& E_t^Q \left[ \exp \left( bL \int_t^T \sum_{i=N_t+1}^{N_s} y_i ds \right) \right] \\
&= \sum_{n=0}^{\infty} \exp(-\nu(T-t)) \frac{(\nu(T-t))^n}{n!} \left( \frac{1}{T-t} \int_0^{T-t} \exp \left( bL \mu_\pi x + \frac{1}{2} b^2 L^2 \sigma_\pi^2 x^2 \right) dx \right)^n \\
&= \exp(-\nu(T-t)) \exp \left( \nu(T-t) \left( \frac{1}{T-t} \int_0^{T-t} \exp \left( bL \mu_\pi x + \frac{1}{2} b^2 L^2 \sigma_\pi^2 x^2 \right) dx \right) \right) \\
&= \exp \left( \nu \left( \int_0^{T-t} \exp \left( bL \mu_\pi x + \frac{1}{2} b^2 L^2 \sigma_\pi^2 x^2 \right) dx \right) - (T-t) \right). \tag{20}
\end{aligned}$$

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<sup>7</sup>Conditioned on the Poisson, the time of the jump is uniformly distributed, for details, see Ross (1995).

Finally,

$$\varphi = \exp \left( \nu \left\{ \int_0^{T-t} e^{bL\mu_\pi x + \frac{1}{2}b^2L^2\sigma_\pi^2 x^2} dx - (T-t) - \frac{bL}{2}(e^{\mu_\pi + \frac{1}{2}\sigma_\pi^2} - 1)(T-t)^2 \right\} \right). \quad (21)$$

This completes the proof.  $\square$

In the risky bond pricing formula, we can obtain the analytical solution of the additional credit spreads caused by jumps.

We can extend the two-factor hazard rate model of Janosi, Jarrow and Yildirim (2002) by using above results. The different aspects are the following: First, the market index  $M$  is assumed to have jump components as in assumption 1:

$$\frac{dM^J}{M^J} = [r - \nu E(\Pi - 1)]dt + \sigma dW + (\Pi - 1)dY. \quad (22)$$

Second, the default intensity consists of the market index and the spot interest rate as follows:

$$\lambda(t) = \max[a + br(t) + c(W(t) + \sum_{i=1}^{N_t} \ln \Pi_i - \nu\phi t), 0]. \quad (23)$$

The characteristics of this additional spread are analyzed in the next section.

### 3 Numerical Analysis and Implications

As stated previously, we will illustrate the impact of the accumulated jumps on the credit spreads. In order to compare the numerical results in Madan and Unal (2000) with ours, we examine the credit spreads only when the asset value of the firm contains discontinuities. First, the differences of credit spreads with jump risks and without jump risks are examined. In Figure 1, the additional credit spreads required by the jumps are illustrated. In order to choose reasonable parameters determining the high or low rating of firms, we extract the value

of  $b$  from the parameters used in Madan and Unal (2000). They extracted the parameters by calibrating two credit spread curves (two rating categories, AA1-AA2, and B3) reported by Bloomberg. If a firm is rated as AA1-AA2, the value of the coefficient  $b$  is 0.0334. In the case of a B3-rated firm, the value of the coefficient  $b$  is 0.0078.

We also choose the jump parameters ( $\nu$ ,  $\mu_\pi$ , and  $\sigma_\pi$ ) as those used in Zhou (2001). As in Zhou (2001), the variance of the firm value is given by

$$\frac{Var(d \ln V)}{dt} = \sigma_V^2 + \nu \sigma_\pi^2. \quad (24)$$

Based on Ingersoll (1987), the equity volatility is about 30%. Following Zhou (2001), we use a firm value variance of 0.035, implying 30% equity volatility per year.

As the coefficient  $b$  of  $\ln V(t)$  in equation (6) becomes larger, greater additional credit spreads are required. This is consistent with the trend that for the same jumps (the same mean and intensity of jumps) a high-rated firm may face a greater sudden drop in values when jumps occur, than a low-rated firm. To obtain the pure effect of jumps on the credit spreads, a zero recovery rate is assumed. The values of parameters are set as the jump frequency  $\nu = 1$ , the mean of the jump size  $\mu_\pi = 0.4$  and the standard deviation of the jump size  $\sigma_\pi = 0.15$ . From (24) the volatility of firm value,  $\sigma_V$  is 0.1. Figure 1 illustrates the additional credit spreads by the jumps of asset value for a high-rated firm and a low-rated firm. In the case of the high-rated firm, the required additional spreads range from 35 basis points for a 2-year maturity bond to 146 basis points for a 10-year maturity bond. Smaller additional spreads are required on the low-rated firm than on the high-rated firm. In the case of the low-rated firm, the additional credit spread for the 10-year maturity bond is about 40 basis points. When the size of jumps is controlled, the high-rated firm may face a greater drop in value than the low-

rated firm when jumps occur. This requires larger additional credit spreads for the high-rated firm.

Table 1 compares the effect of the jump intensity ( $\nu$ ) on the additional credit spreads caused by jumps with that of the volatility of jump size ( $\sigma_\pi$ ), after controlling the size of the variance of the firm value given in equation (24). We assume that the volatility of the diffusion term in equation (2),  $\sigma_V$  is 0.1, and consider the high-rated firm case. We look at the various combinations of  $\nu$  and  $\sigma_\pi$  that make the variance of the firm value,  $Var(d \ln V)/dt$ , 0.035. Table 1 shows that a combination of larger jump intensity and smaller jump size volatility requires larger additional credit spreads than a combination of smaller jump intensity and larger jump size volatility, regardless of maturity. For example, the additional credit spread for a 5-year discount bond is around 70 basis points in the case of  $\nu = 0.8$  and  $\sigma_\pi = 0.17$ , while it is around 29 basis points in the case of  $\nu = 0.2$  and  $\sigma_\pi = 0.35$ . This shows that larger credit spreads are required when we have frequent jumps and smaller jump size volatility than when we have infrequent jumps and larger jump size volatility.

Table 2 compares the effect of jump intensity ( $\nu$ ) on the additional credit spreads with that of the average jump size ( $\mu_\pi$ ), after controlling the average jump effect on the firm value,  $\nu E(\Pi - 1)$ . We assume that  $\nu E(\Pi - 1) = 0.11$ , and consider the high-rated firm case. We look at the various combinations of  $\nu$  and  $\mu_\pi$  that guarantee  $\nu E(\Pi - 1) = 0.11$  in table 2. The table shows that a combination of larger jumps size and smaller jump intensity requires larger additional credit spreads than a combination of smaller jump size and larger jump, regardless of maturity, For example, the additional credit spread for a 5-year discount bond is around 45 basis points in the case of  $\nu = 0.05$  and  $\mu_\pi = 1.2$ , while it is around 13 basis points in the case of  $\nu = 0.8$  and  $\mu_\pi = 0.125$ . This shows that larger credit spreads are required when we have

infrequent jumps and larger average jump size than when we have frequent jumps and smaller average jump size.

In sum, numerical analysis shows that the credit spreads of higher-rated firms are affected more by jump risk. Among the jump risk components, i.e., average jump size, jump frequency, and jump size volatility, the effect of the average jump size is largest, and jump frequency is the next largest.

## 4 Conclusions

This paper explores the additional credit spreads when the default intensity is affected by jump risks. To analyze the magnitude of the spreads, this paper derives a simple pricing model of risky bonds using a reduced-form approach when there exist jump risks of the factors of the default intensity. Numerical examples show that the additional credit spreads caused by jumps can be significant. For example, for the same jump size and frequency, the accumulated jumps of asset value of a high-rated firm require much greater credit spreads than those of a low-rated firm. In addition, it is shown that the effect of the average jump size is largest among the jump risk components.

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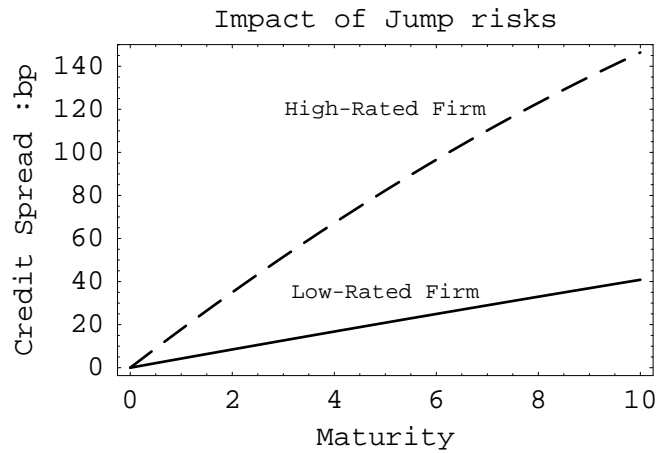
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Figure 1: **Additional Credit Spreads caused by Jumps: Comparison of a High Rated Firm and a Low Rated Firm.**



The dashed curve describes the credit spread required additionally when the asset value of the high-rated firm jumps ( $b = 0.0334$  from Madan and Unal (2000)). The solid curve illustrates the credit spread required additionally when the asset value of the low-rated firm jumps ( $b = 0.0078$  from Madan and Unal (2000)). The parameters of jump frequency, the mean of jump size and the volatility of jump size are  $\nu = 1.0$ ,  $\mu_{\pi} = 0.4$ ,  $\sigma_{\pi} = 0.15$ , respectively.

Table 1: **Additional Credit Spreads caused by Jumps with respect to default intensity and jump size volatility**

Maturity	$\nu = 0.2/\sigma_\pi = 0.35$	$\nu = 0.5/\sigma_\pi = 0.22$	$\nu = 0.8/\sigma_\pi = 0.17$
1	6.1883	12.7790	15.1560
2	12.1600	25.0810	43.7220
3	17.9100	36.8970	41.1860
5	28.7290	59.0460	69.9080
7	38.6110	79.1660	93.6490
10	51.5940	105.4000	124.5300

Table 1 shows the additional credit spreads when the variance of firm is constant,  $Var(d\ln V)/dt = 0.035$ . When the volatility of the diffusion term,  $\sigma_V$  is 0.1, each combination of  $\nu$  and  $\sigma_\pi$  in the table provides the same value for the variance of firm as 0.035.

Table 2: **Additional Credit Spreads caused by Jumps with respect to default intensity and average jump size**

Maturity	$\nu = 0.8/\mu_\pi = 0.125$	$\nu = 0.5/\mu_\pi = 0.2$	$\nu = 0.1/\mu_\pi = 0.77$	$\nu = 0.05/\mu_\pi = 1.2$
1	2.7509	2.8894	6.8179	9.5516
2	5.3871	5.6611	13.4010	18.8210
3	7.9078	8.3139	19.7440	27.8000
5	12.5990	13.2590	31.6920	44.8480
7	16.8190	17.7160	42.6180	60.6190
10	22.2490	23.4690	56.9970	81.6940

Table 2 provides the additional credit spreads when the additional term in the drift term caused by jumps is constant, that is,  $\nu E[\Pi - 1] = 0.11$ . Each combination of  $\nu$  and  $\sigma_\pi$  in the table guarantees  $\nu E[\Pi - 1] = 0.11$ .