IDENTIFICATION OF ANISOTROPY AND ASYMMETRY IN ROTOR SYSTEM FROM DIRECTIONAL FREQUENCY RESPONSE ESTIMATES

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ABSTRACT

Anisotropy and asymmetry of rotor systems are important factors to determine dynamic characteristics including the stability and response of rotors. Directional frequency response functions (dFRFs) are known as a powerful tool for detecting the presence and degree of anisotropy or asymmetry. However when a rotor system possesses both anisotropy and asymmetry, since the differential equation of motion is characterized by periodically time-varying coefficients, it is difficult to perform the conventional modal analysis and thus to estimate dFRFs accurately. In this paper, an estimation method, based on a modal analysis method developed for rotor systems with both anisotropy and asymmetry, is newly developed. This method essentially introduces a multi-input/single-output relation for estimation of dFRFs in the complex domain, as a finite degree-of-freedom time-varying equation is transformed into an infinite degree-of-freedom time-invariant one, employing the modulate coordinate. Finally a finite element model update method is applied to identify the anisotropy and asymmetry degree of rotor systems using estimated dFRFs. The effectiveness for the estimation method is demonstrated by numerical examples and experiments.

Keywords: Directional frequency response functions, spectral estimation, asymmetric rotor, anisotropic stator, periodically time-varying system, identification of asymmetry

INTRODUCTION

As the industrial technology advances, many power plants tend to become large in size and capacity. Among others, rotating machines are still the major element that converts energy and transmits power. As the operation of such critical machines requires more safety and reliability, the early detection of any probable malfunctions in the machines becomes more beneficial. The faults or malfunctions in a rotating machine are normally caused by the presence of anisotropy and asymmetry.

Rotating machinery can be classified into two subsystems: one is the rotating part such as shafts and disks, and the other is stationary part such as bearings and foundations. Non-axi-symmetric mechanical properties, if exists, of the rotating and stationary parts may be referred to asymmetry and anisotropy, respectively [1]. Asymmetric properties may cause instability near critical speeds and anisotropic properties may give rise to backward whirling, which should be avoided in order not to cause any fatigue failures in the rotating shaft. Asymmetry and anisotropy may be intentionally introduced in the design stage of machines, but they are mainly caused by machine faults. For example, development of a transverse crack on the rotating shaft increases asymmetry in shaft stiffness and failure of the stationary bearing elements induces anisotropy in bearing coefficients. Therefore, it is paramount important to identify the asymmetry and anisotropy of rotating machines in operation for early detection of machine faults and malfunctions such that the safe and reliable operation is guaranteed.

Numerous identification methods of rotor systems have been developed in recent several decades. In particular, many investigations have attempted to detect growth of a crack from change of the natural frequency, response, sub-critical speed and singular behavior [2,3]. But they were successful only when the asymmetry caused by the crack growth is significantly large, which is not of a practical use, because the analysis method heavily relied on measurement or operational responses. On the other hand, there have been some attempts for active identification methods to excite rotor systems in operation. Nordmann [4] obtained frequency response functions (FRFs) of a rotor by impact hammer excitation. Musynska [5] performed modal testing of a rotor by using an asynchronous harmonic excitation device. Rogers and Ewins [6] identified modal parameters of a rotor by using a conventional electro-dynamic shaker. But most of previous modal testing methods developed for rotors, except [5], failed in capturing the directivity nature of modes that are important properties of rotating component, primarily because of employing real coordinates in formulation. To accurately identify modal properties associated with the forward and backward modes, Lee [7] and Lee and Joh [8] developed the complex modal testing theory, introducing the concept of normal and reverse directional frequency response functions (dFRFs). They demonstrated that the reverse dFRFs are a good indicator to detect the presence and degree of anisotropy or asymmetry when rotor systems have asymmetry or anisotropy, not both. Recently Lee, et al. [9] developed the modal analysis method of asymmetric rotor with anisotropic stator using the modulated coordinates.

The objectives of this paper are to identify asymmetry and anisotropy of rotor systems employing the modal analysis method for general rotors with asymmetry and anisotropy. First of all, a new estimation method of directional frequency response functions is developed by expanding an existing estimation method. And the asymmetry and anisotropy are identified through a model update method based on the estimated directional frequency response functions.

1. DIRECTIONAL FREQUENCY RESPONSE FUNCTIONS (dFRFS) [9]

Using the stationary coordinates, the equation of motion for a general rotor system with asymmetric rotors and anisotropic stators can be written, at the rotational speed of Ω, as

$$\begin{aligned}\mathbf{M_f}\ddot{\mathbf{p}}(t)+\mathbf{M_b}\ddot{\bar{\mathbf{p}}}(t)+\mathbf{M_r}\ddot{\bar{\mathbf{p}}}(t)e^{j2\Omega t}+\mathbf{C_f}\dot{\mathbf{p}}(t)+\mathbf{C_b}\dot{\bar{\mathbf{p}}}(t)+\mathbf{C_r}\dot{\bar{\mathbf{p}}}(t)e^{j2\Omega t}\\ +\mathbf{K_f}\mathbf{p}(t)+\mathbf{K_b}\bar{\mathbf{p}}(t)+\mathbf{K_r}\bar{\mathbf{p}}(t)e^{j2\Omega t}=\mathbf{g}(t)\end{aligned} \tag{1}$$

$$\mathbf{p}(t)=\mathbf{y}(t)+j\mathbf{z}(t) \qquad \bar{\mathbf{p}}(t)=\mathbf{y}(t)-j\mathbf{z}(t)$$

$$\mathbf{g}(t)=\mathbf{f}_y(t)+j\mathbf{f}_z(t) \qquad \bar{\mathbf{g}}(t)=\mathbf{f}_y(t)-j\mathbf{f}_z(t)$$

where $\mathbf{M}_i$, $\mathbf{C}_i$ and $\mathbf{K}_i$ denote the complex valued $N \times N$ generalized mass, damping (including gyroscopic effect) and stiffness matrices, respectively. The subscript f, b and r refer to the mean value and the deviatoric values for anisotropy and asymmetry, respectively. Each component of the complex displacement and force vectors, $\mathbf{p}(t)$ and $\mathbf{g}(t)$, $\bar{\mathbf{p}}(t)$ and $\bar{\mathbf{g}}(t)$ being their complex conjugates, describes the planar motions in the two-dimensional vector space.

Since equation (1) is linear, but periodically time-varying, formulation of the conventional eigenvalue problem, developed for linear time-invariant systems, and thus construction of the frequency response functions are not straightforward. Now let us introduce the modulated coordinate defined by

$$\mathbf{p}_n(t)\equiv\mathbf{p}(t)e^{j2n\Omega t}, \quad \mathbf{g}_n(t)\equiv\mathbf{g}(t)e^{j2n\Omega t}, \quad \bar{\mathbf{p}}_n(t)\equiv\overline{\mathbf{p}(t)e^{j2n\Omega t}}=\bar{\mathbf{p}}(t)e^{-j2n\Omega t}, \bar{\mathbf{g}}_n(t)\equiv\bar{\mathbf{g}}(t)e^{-j2n\Omega t}$$

$$\mathbf{p}_{(\infty)} = \left\{ \cdots \quad \mathbf{p}_{-1}^T(t) \quad \bar{\mathbf{p}}_0^T(t) \quad \mathbf{p}_0^T(t) \quad \bar{\mathbf{p}}_{-1}^T(t) \quad \cdots \right\}^T$$
$$\mathbf{g}_{(\infty)} = \left\{ \cdots \quad \mathbf{g}_{-1}^T(t) \quad \bar{\mathbf{g}}_0^T(t) \quad \mathbf{g}_0^T(t) \quad \bar{\mathbf{g}}_{-1}^T(t) \quad \cdots \right\}^T \tag{2}$$

where n is an integer. By substituting equation (2) into equation (1), we can derive an infinite order linear time-invariant equation of motion for general rotors as

$$\mathbf{M}_{(\infty)}\ddot{\mathbf{p}}_{(\infty)}(t) + \mathbf{C}_{(\infty)}\dot{\mathbf{p}}_{(\infty)}(t) + \mathbf{K}_{(\infty)}\mathbf{p}_{(\infty)}(t) = \mathbf{g}_{(\infty)}(t) \tag{3}$$

$$\mathbf{M}_{(\infty)} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & \bar{\mathbf{M}}_{\mathbf{f}} & \bar{\mathbf{M}}_{\mathbf{b}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{M}_{\mathbf{b}} & \mathbf{M}_{\mathbf{f}} & \mathbf{M}_{\mathbf{r}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \bar{\mathbf{M}}_{\mathbf{r}} & \bar{\mathbf{M}}_{\mathbf{f}} & \bar{\mathbf{M}}_{\mathbf{b}} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{M}_{\mathbf{b}} & \mathbf{M}_{\mathbf{f}} & \mathbf{M}_{\mathbf{r}} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{M}}_{\mathbf{r}} & \bar{\mathbf{M}}_{\mathbf{f}} & \bar{\mathbf{M}}_{\mathbf{b}} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{\mathbf{b}} & \mathbf{M}_{\mathbf{f}} & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \mathbf{C}_{(\infty)} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & \bar{\mathbf{C}}_{\mathbf{f};1} & \bar{\mathbf{C}}_{\mathbf{b};1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{C}_{\mathbf{b};-1} & \mathbf{C}_{\mathbf{f};-1} & \mathbf{C}_{\mathbf{r};0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \bar{\mathbf{C}}_{\mathbf{r};1} & \bar{\mathbf{C}}_{\mathbf{f};0} & \bar{\mathbf{C}}_{\mathbf{b};0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{b};0} & \mathbf{C}_{\mathbf{f};0} & \mathbf{C}_{\mathbf{r};1} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{C}}_{\mathbf{r};0} & \bar{\mathbf{C}}_{\mathbf{f};-1} & \bar{\mathbf{C}}_{\mathbf{b};-1} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathbf{b};1} & \mathbf{C}_{\mathbf{f};1} & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$\mathbf{K}_{(\infty)} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & \bar{\mathbf{K}}_{\mathbf{f};1} & \bar{\mathbf{K}}_{\mathbf{b};1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{K}_{\mathbf{b};-1} & \mathbf{K}_{\mathbf{f};-1} & \mathbf{K}_{\mathbf{r};0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \bar{\mathbf{K}}_{\mathbf{r};1} & \bar{\mathbf{K}}_{\mathbf{f};0} & \bar{\mathbf{K}}_{\mathbf{b};0} & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{K}_{\mathbf{b};0} & \mathbf{K}_{\mathbf{f};0} & \mathbf{K}_{\mathbf{r};1} & \mathbf{0} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{K}}_{\mathbf{r};0} & \bar{\mathbf{K}}_{\mathbf{f};-1} & \bar{\mathbf{K}}_{\mathbf{b};-1} & \cdots \\ \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{\mathbf{b};1} & \mathbf{K}_{\mathbf{f};1} & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$\begin{aligned} \mathbf{C}_{i;n} &= \mathbf{C}_i - j4n\Omega\mathbf{M}_i \\ \mathbf{K}_{i;n} &= \mathbf{K}_i - j2n\Omega\mathbf{C}_i - 4n^2\Omega^2\mathbf{M}_i \\ i &= r, b, f \qquad n = 0, \pm 1, \pm 2, \ldots \end{aligned} \tag{4}$$

Fourier transform of equation (3) then reduces to

$$\mathbf{D}_{(\infty)}(j\omega)\mathbf{P}_{(\infty)}(j\omega) = \mathbf{G}_{(\infty)}(j\omega) \tag{5a}$$

or equivalently

$$\mathbf{P}_{(\infty)}(j\omega) = \mathbf{H}_{(\infty)}(j\omega)\mathbf{G}_{(\infty)}(j\omega) \tag{5b}$$

where

$$\mathbf{P}_{(\infty)}(j\omega) = \left\{ \cdots \quad \mathbf{P}_{-1}^T(j\omega) \quad \bar{\mathbf{P}}_0^T(j\omega) \quad \mathbf{P}_0^T(j\omega) \quad \bar{\mathbf{P}}_{-1}^T(j\omega) \quad \cdots \right\}^T \quad \mathbf{G}_{(\infty)}(j\omega) = \left\{ \cdots \quad \mathbf{G}_{-1}^T(j\omega) \quad \bar{\mathbf{G}}_0^T(j\omega) \quad \mathbf{G}_0^T(j\omega) \quad \bar{\mathbf{G}}_{-1}^T(j\omega) \quad \cdots \right\}^T$$

$$\mathbf{H}_{(\infty)}(j\omega) = \mathbf{D}_{(\infty)}^{-1}(j\omega) = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \iddots \\ \cdots & \mathbf{H}_{g_{-1}p_{-1}} & \mathbf{H}_{\bar{g}_0 p_{-1}} & \mathbf{H}_{g_0 p_{-1}} & \mathbf{H}_{\bar{g}_{-1} p_{-1}} & \cdots \\ \cdots & \mathbf{H}_{g_{-1}\bar{p}_0} & \mathbf{H}_{\bar{g}_0 \bar{p}_0} & \mathbf{H}_{g_0 \bar{p}_0} & \mathbf{H}_{\bar{g}_{-1} \bar{p}_0} & \cdots \\ \cdots & \mathbf{H}_{g_{-1}p_0} & \mathbf{H}_{\bar{g}_0 p_0} & \mathbf{H}_{g_0 p_0} & \mathbf{H}_{\bar{g}_{-1} p_0} & \cdots \\ \cdots & \mathbf{H}_{g_{-1}\bar{p}_{-1}} & \mathbf{H}_{\bar{g}_0 \bar{p}_{-1}} & \mathbf{H}_{g_0 \bar{p}_{-1}} & \mathbf{H}_{\bar{g}_{-1} \bar{p}_{-1}} & \cdots \\ \iddots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Here, $\mathbf{D}_{(\infty)}(j\omega) = (j\omega)^2\mathbf{M}_{(\infty)} + j\omega\mathbf{C}_{(\infty)} + \mathbf{K}_{(\infty)}$ is the dynamic stiffness matrix defined in the frequency domain and $\mathbf{H}_{(\infty)}(j\omega)$ is the $\infty \times \infty$ directional frequency response matrix (dFRM).

It is found that $\mathbf{H}_{g_0 p_0}(j\omega)$, $\mathbf{H}_{\bar{g}_0 p_0}(j\omega)$ and $\mathbf{H}_{\bar{g}_{-1} p_0}(j\omega)$ are sensitive to the isotropy, anisotropy and asymmetry, respectively. For notational convenience, $\mathbf{H}_{g_0 p_0}(j\omega)$, $\mathbf{H}_{\bar{g}_0 p_0}(j\omega)$ and $\mathbf{H}_{\bar{g}_{-1} p_0}(j\omega)$ are often denoted by $\mathbf{H}_{gp}(j\omega)$, $\mathbf{H}_{\hat{g}p}(j\omega)$ and $\mathbf{H}_{\bar{g}p}(j\omega)$, respectively.

2. ESTIMATION OF DIRECTIONAL FREQUENCY RESPONSE FUNCTIONS (dFRFS)

Note that only a single $N \times \infty$ row or $\infty \times N$ column matrix in the dFRM of infinite dimension is independent of others, representing all properties of rotor systems including the isotropy, asymmetry and anisotropy [9]. Thus the single representative matrix equation associated with the physically measured response vector $\mathbf{P}_0(j\omega)$ which can be written as

$$\mathbf{P}_0(j\omega) = \cdots + \mathbf{H}_{g_{-1}p_0}(j\omega)\mathbf{G}_{-1}(j\omega) + \mathbf{H}_{\bar{g}_0 p_0}(j\omega)\bar{\mathbf{G}}_0(j\omega) + \mathbf{H}_{g_0 p_0}(j\omega)\mathbf{G}_0(j\omega) + \mathbf{H}_{\bar{g}_{-1}p_0}(j\omega)\bar{\mathbf{G}}_{-1}(j\omega) + \cdots \qquad (6)$$

From equation (6), we can model a general rotor system with anisotropy and asymmetry as a multi-input single-output (MISO) vector system shown in Figure 1. Since the modulation coordinate vectors, $\mathbf{g}_n(t)$ and $\bar{\mathbf{g}}_n(t)$, are defined by equation (2), each input vector can be obtained from Fourier transform of the actual measured input $\mathbf{g}_0(t)$, modulated with an integer multiple of the rotational speed Ω. To estimate the $N \times N$ dFRMs cross spectral densities between the inputs and output can be calculated as

$$\begin{aligned}
\mathbf{S}_{g_{-1}p_0} &= \frac{1}{T}E[\mathbf{G}_{-1}^{*}\mathbf{P}_0^{'}] = \cdots + \mathbf{H}^{'}_{g_{-1}p_0}\mathbf{S}_{g_{-1}g_{-1}} + \mathbf{H}^{'}_{\bar{g}_0 p_0}\mathbf{S}_{g_{-1}\bar{g}_0} + \mathbf{H}^{'}_{g_0 p_0}\mathbf{S}_{g_{-1}g_0} + \mathbf{H}^{'}_{\bar{g}_{-1}p_0}\mathbf{S}_{g_{-1}\bar{g}_{-1}} + \cdots \\
\mathbf{S}_{\bar{g}_0 p_0} &= \frac{1}{T}E[\bar{\mathbf{G}}_0^{*}\mathbf{P}_0^{'}] = \cdots + \mathbf{H}^{'}_{g_{-1}p_0}\mathbf{S}_{\bar{g}_0 g_{-1}} + \mathbf{H}^{'}_{\bar{g}_0 p_0}\mathbf{S}_{\bar{g}_0\bar{g}_0} + \mathbf{H}^{'}_{g_0 p_0}\mathbf{S}_{\bar{g}_0 g_0} + \mathbf{H}^{'}_{\bar{g}_{-1}p_0}\mathbf{S}_{\bar{g}_0\bar{g}_{-1}} + \cdots \\
\mathbf{S}_{g_0 p_0} &= \frac{1}{T}E[\mathbf{G}_0^{*}\mathbf{P}_0^{'}] = \cdots + \mathbf{H}^{'}_{g_{-1}p_0}\mathbf{S}_{g_0 g_{-1}} + \mathbf{H}^{'}_{\bar{g}_0 p_0}\mathbf{S}_{g_0\bar{g}_0} + \mathbf{H}^{'}_{g_0 p_0}\mathbf{S}_{g_0 g_0} + \mathbf{H}^{'}_{\bar{g}_{-1}p_0}\mathbf{S}_{g_0\bar{g}_{-1}} + \cdots \\
\mathbf{S}_{\bar{g}_{-1}p_0} &= \frac{1}{T}E[\bar{\mathbf{G}}_{-1}^{*}\mathbf{P}_0^{'}] = \cdots + \mathbf{H}^{'}_{g_{-1}p_0}\mathbf{S}_{\bar{g}_{-1}g_{-1}} + \mathbf{H}^{'}_{\bar{g}_0 p_0}\mathbf{S}_{\bar{g}_{-1}\bar{g}_0} + \mathbf{H}^{'}_{g_0 p_0}\mathbf{S}_{\bar{g}_{-1}g_0} + \mathbf{H}^{'}_{\bar{g}_{-1}p_0}\mathbf{S}_{\bar{g}_{-1}\bar{g}_{-1}} + \cdots
\end{aligned} \qquad (7)$$

where T is the period of one sample record, E stands for expectation and the superscript * and ` denote the conjugate and transpose, respectively. Although other inputs are calculated from the actual measured single input $\mathbf{g}_0(t)$, cross correlations among inputs are theoretically zero, because of the nature of frequency modulation. But they are not practically zero, due to finite time length signal processing. Considering non-vanishing cross correlations among inputs, equation (7) can be written as

$$\begin{Bmatrix} \vdots \\ \mathbf{S}_{g_{-1}p_0} \\ \mathbf{S}_{\bar{g}_0 p_0} \\ \mathbf{S}_{g_0 p_0} \\ \mathbf{S}_{\bar{g}_{-1}p_0} \\ \vdots \end{Bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & ⋰ \\ \cdots & \mathbf{S}_{g_{-1}g_{-1}} & \mathbf{S}_{g_{-1}\bar{g}_0} & \mathbf{S}_{g_{-1}g_0} & \mathbf{S}_{g_{-1}\bar{g}_{-1}} & \cdots \\ \cdots & \mathbf{S}_{\bar{g}_0 g_{-1}} & \mathbf{S}_{\bar{g}_0\bar{g}_0} & \mathbf{S}_{\bar{g}_0 g_0} & \mathbf{S}_{\bar{g}_0\bar{g}_{-1}} & \cdots \\ \cdots & \mathbf{S}_{g_0 g_{-1}} & \mathbf{S}_{g_0\bar{g}_0} & \mathbf{S}_{g_0 g_0} & \mathbf{S}_{g_0\bar{g}_{-1}} & \cdots \\ \cdots & \mathbf{S}_{\bar{g}_{-1}g_{-1}} & \mathbf{S}_{\bar{g}_{-1}\bar{g}_0} & \mathbf{S}_{\bar{g}_{-1}g_0} & \mathbf{S}_{\bar{g}_{-1}\bar{g}_{-1}} & \cdots \\ ⋰ & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{Bmatrix} \vdots \\ \mathbf{H}^{'}_{g_{-1}p_0} \\ \mathbf{H}^{'}_{\bar{g}_0 p_0} \\ \mathbf{H}^{'}_{g_0 p_0} \\ \mathbf{H}^{'}_{\bar{g}_{-1}p_0} \\ \vdots \end{Bmatrix} \qquad (8)$$

Now we can estimate the $N \times N$ dFRMs from the above equation (8).

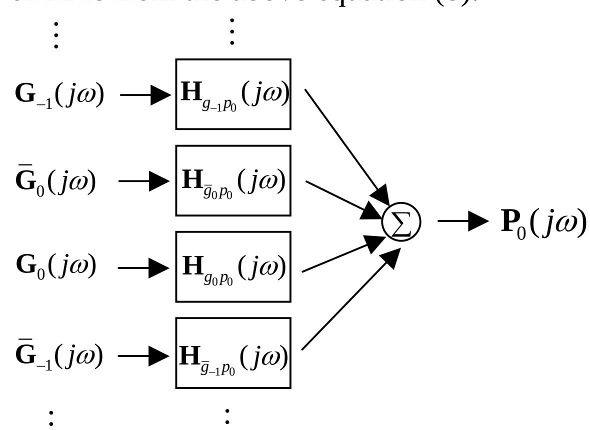


Fig. 1. Complex multi-input / complex single-output model of general rotor systems

3. LEAST SQUARE MODEL UPDATE METHOD

Similarly to stationary structures, the modal parameters such as natural frequency, modal damping and modal residue for rotating structures can be well identified from the measured dFRFs [7]. In order to further identify the physical parameters such as the degree of anisotropy and asymmetry, development of a curve-fitting method in accordance with the change in the physical parameter of interest is required. In this study, a least square model update method based on the sensitivity matrix is used to identify those parameters, which minimizes the difference between the analytical and measured FRFs with respect to the parameters changes, i.e. the sensitivity [10]. Let us first define the difference between the measured and analytical dynamic stiffness matrices (DSMs) given by

$$\left[\Delta\mathbf{D}(j\omega)\right] = \left[\mathbf{D}_x(j\omega)\right] - \left[\mathbf{D}_a(j\omega)\right] \tag{9}$$

where subscripts a and x mean the analytical and measured quantities, respectively. Since DSM is the inversion of frequency response matrix (FRM), it holds, for the jth input,

$$\left\{\mathbf{H}_x(j\omega)\right\}_j^T \left[\mathbf{D}_x(j\omega)\right] = \left\{\mathbf{H}_a(j\omega)\right\}_j^T \left[\mathbf{D}_a(j\omega)\right] = \left\{\mathbf{I}\right\}_j^T \tag{10}$$

where $\{\mathbf{I}\}$ is $N \times N$ identity matrix and the subscript j indicates j^{th} column vector. Substituting equation (9) into equation (10), and, post-multiplying $\left[\mathbf{H}_a(j\omega)\right]$, we obtain

$$\left\{\mathbf{H}_x(j\omega)\right\}_j^T \left[\Delta\mathbf{D}(j\omega)\right]\left[\mathbf{H}_a(j\omega)\right] = \left\{\mathbf{H}_a(j\omega)\right\}_j^T - \left\{\mathbf{H}_x(j\omega)\right\}_j^T \tag{11}$$

where $\left[\Delta\mathbf{D}(j\omega)\right]$ can be expanded by Talyor's series with respect to the physical parameter p, as

$$\left[\Delta\mathbf{D}(j\omega)\right] = \sum_{i=1}^{N_p} \frac{\partial\left[\mathbf{D}(j\omega)\right]}{\partial p_i}\Delta p_i \tag{12}$$

Substituting equation (12) into equation (11), we obtain

$$\left\{\Delta\mathbf{P}\right\}^T \left[\mathbf{S}_j(j\omega)\right] = \left\{\Delta\mathbf{H}(j\omega)\right\}^T \tag{13}$$

where $\left[\mathbf{S}_j\right] = \begin{bmatrix} \left\{\mathbf{H}_x\right\}_j^T \dfrac{\partial[\mathbf{D}]}{\partial p_1}\left[\mathbf{H}_a\right] \\ \left\{\mathbf{H}_x\right\}_j^T \dfrac{\partial[\mathbf{D}]}{\partial p_2}\left[\mathbf{H}_a\right] \\ \vdots \\ \left\{\mathbf{H}_x\right\}_j^T \dfrac{\partial[\mathbf{D}]}{\partial p_{N_p}}\left[\mathbf{H}_a\right] \end{bmatrix}$, $\left\{\Delta\mathbf{P}\right\} = \begin{Bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_{N_p} \end{Bmatrix}$ and $\left\{\Delta\mathbf{H}(j\omega)\right\} = \left\{\mathbf{H}_a(j\omega)\right\}_j^T - \left\{\mathbf{H}_x(j\omega)\right\}_j^T$

From equation (13), we can calculate the parameter change Δp using the least square scheme by taking more data than the number of parameters to be identified.

4. IDENTIFICATION OF ANISOTROPY AND ASYMMETRY

Each of dFRFs has its own unique sensitivity properties. $\mathbf{H}_{gp}(j\omega)$ is dominated by the isotropic property, while $\mathbf{H}_{\hat{g}p}(j\omega)$ and $\mathbf{H}_{\bar{g}p}(j\omega)$ are strongly influenced in magnitude by the degree of anisotropy and asymmetry, respectively. The cross coupled effect, if not completely zero, is negligible small in magnitude with the order of anisotropy times asymmetry, which makes the identification of the isotropy, anisotropy and asymmetry independent of each other.

As an illustrative example, consider a simple (Jeffcott) general rotor system with a known mass m, where the equation motion, neglecting both asymmetry and anisotropy, is given

$$m\ddot{p}(t) + c\dot{p}(t) + k_{eq}\,p(t) = g(t) \tag{14a}$$

or, the dFRF is given by

$$H_{gp}(j\omega)=\frac{P(j\omega)}{G(j\omega)}=\frac{1}{k_{eq}-m\omega^2+jc\omega} \tag{14b}$$

We can then easily identify the damping (c) and equivalent (k_{eq}) from $H_{gp}(j\omega)$ from the isotropy rotor model (14a). In the presence of anisotropy, equation (14) is modified to become

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{p}(t) \\ \ddot{\bar{p}}(t) \end{Bmatrix}+\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\begin{Bmatrix} \dot{p}(t) \\ \dot{\bar{p}}(t) \end{Bmatrix}+\begin{bmatrix} k_{eq} & \Delta_{eq}k_{eq} \\ \Delta_{eq}k_{eq} & k_{eq} \end{bmatrix}\begin{Bmatrix} p(t) \\ \bar{p}(t) \end{Bmatrix}=\begin{bmatrix} p(t) \\ \bar{p}(t) \end{bmatrix} \tag{15a}$$

and

$$\begin{aligned} H_{gp}(j\omega)&=\frac{P(j\omega)}{G(j\omega)}=\frac{k_{eq}-m\omega^2+jc\omega}{\left(k_{eq}-m\omega^2+jc\omega\right)^2-\left(\Delta_{eq}k_{eq}\right)^2}\cong\frac{1}{k_{eq}-m\omega^2+jc\omega} \\ H_{\hat{g}p}(j\omega)&=\frac{P(j\omega)}{\hat{G}(j\omega)}=\frac{-\Delta_{eq}k_{eq}}{\left(k_{eq}-m\omega^2+jc\omega\right)^2-\left(\Delta_{eq}k_{eq}\right)^2}\cong\frac{-\Delta_{eq}k_{eq}}{\left(k_{eq}-m\omega^2+jc\omega\right)^2} \end{aligned} \tag{15b}$$

From the above relations (15b) and the least square update method, we can obtain the equivalent anisotropy (Δ_{eq}). Similarly, in the presence of asymmetry, equation (14) is modified to become

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{p}(t) \\ \ddot{\tilde{p}}(t) \end{Bmatrix}+\begin{bmatrix} c & 0 \\ 0 & c-j4\Omega m \end{bmatrix}\begin{Bmatrix} \dot{p}(t) \\ \dot{\tilde{p}}(t) \end{Bmatrix}+\begin{bmatrix} k_{eq} & \delta_{eq}k_{eq} \\ \delta_{eq}k_{eq} & k_{eq}-j2\Omega c-4\Omega^2 m \end{bmatrix}\begin{Bmatrix} p(t) \\ \tilde{p}(t) \end{Bmatrix}=\begin{bmatrix} p(t) \\ \tilde{p}(t) \end{bmatrix} \tag{16a}$$

and

$$\begin{aligned} H_{gp}(j\omega)&=\frac{P(j\omega)}{G(j\omega)}=\frac{k_{eq}-m\left(\omega-2\Omega\right)^2+jc\left(\omega-2\Omega\right)}{\left(k_{eq}-m\omega^2+jc\omega\right)\left(k_{eq}-m\left(\omega-2\Omega\right)^2+jc\left(\omega-2\Omega\right)\right)-\left(\delta_{eq}k_{eq}\right)^2}\cong\frac{1}{k_{eq}-m\omega^2+jc\omega} \\ H_{\tilde{g}p}(j\omega)&=\frac{P(j\omega)}{\tilde{G}(j\omega)}=\frac{-\delta_{eq}k_{eq}}{\left(k_{eq}-m\omega^2+jc\omega\right)\left(k_{eq}-m\left(\omega-2\Omega\right)^2+jc\left(\omega-2\Omega\right)\right)-\left(\delta_{eq}k_{eq}\right)^2} \\ &\cong\frac{-\delta_{eq}k_{eq}}{\left(k_{eq}-m\omega^2+jc\omega\right)\left(k_{eq}-m\left(\omega-2\Omega\right)^2+jc\left(\omega-2\Omega\right)\right)} \end{aligned} \tag{16b}$$

The equivalent asymmetry (δ_{eq}) can now be identified from equation (16a) and the model update method. In order to calculate the physical parameter from the equivalent mathematical parameters, we need a more refined model for the system. For the case when the physical model is given as in Figure 2, where the shaft has asymmetric stiffness and the bearing has anisotropic stiffness, we obtain the relations

$$\Delta_{eq}=\frac{\Delta k_s}{(1-\Delta)(1+\Delta)k_b+k_s} \qquad \delta_{eq}=\frac{\delta k_b}{(1-\delta)(1+\delta)k_s+k_b} \tag{17}$$

where k_s and k_b are the shaft stiffness and bearing stiffness, respectively.

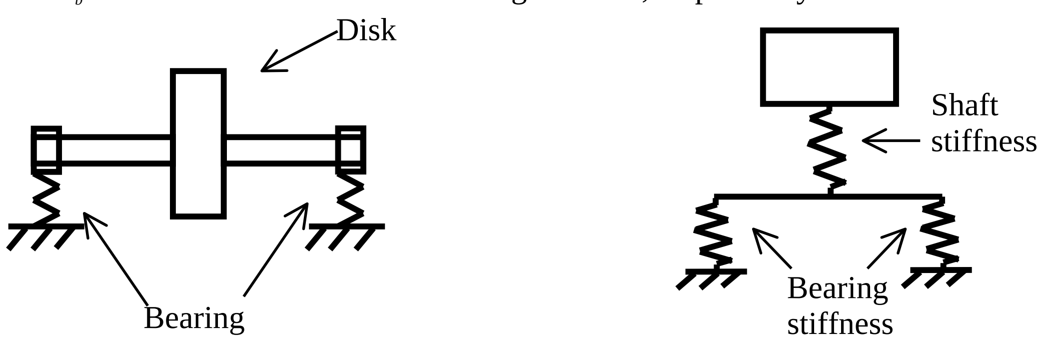


Fig. 2. Single degree-of-freedom model of simple rotor systems

5. EXPERIMENTAL RESULT

Figure 3 shows the experimental set-up, where a disk is located in the center of a shaft supported at ends by self-aligning bearing to represent a Jeffcott rotor system. Stiffness asymmetry due to the rectangular cross-sectioned part is introduced to the shaft near the disk location. The rotor is driven by a DC motor, and an encoder is used to obtain the reference signal and the rotational speed. Two pairs of electro-magnets excite the rotor at the disk location and the excitation forces are estimated by using Hall effect sensors. A pair of non-contacting type proximity probes, placed perpendicular to each other, measure the displacements of the rotor. The specifications of the test rotor are given in Table 1. Band-limited signals up to 200(*Hz*) are fed into four electro-magnets to generate the y and z directional forces through power amplifiers. For data processing, the sampling frequency is 400 (*Hz*), the number of data is 2048 and the number of averaging for spectral estimates is 100. A computer with DSP board generates the excitation signals and acquires the excitation force and rotor displacement data.

Figure 4 shows the estimated dFRFs and directional multiple coherence function (dMCOH). Note that, as the number of inputs taken for data processing increases, the estimation results are significantly improved, particularly near the modes associated with the presence of asymmetry. It is also clearly demonstrated by the fact that the dMCOH almost converges to 1 over the frequency range of interest, for the number of inputs up to 6. Figure 5 compares the estimated dFRFs by using the 6 inputs/single output model and the regenerated dFRFs from least square model update method based on the anisotropic or asymmetric rotor model. Note that the regenerated dFRF for $\mathbf{H}_{gp}(j\omega)$ based on the anisotropic rotor model lacks the modulated modes and the complex conjugate modes that are assoicated with the system asymmetry. The discrepancy between the estimated and regenerated results in $\mathbf{H}_{\hat{g}p}(j\omega)$ is believed to be due to nonlinearity in the bearing property, including clearance. The inserted Table in Figure 5 compares theoretical and identified parameter values: equivalent stiffness of the test rotor, bearing stiffness anisotropy and shaft stiffness asymmetry. The degree of anisotropy or asymmetry is a non-dimensional parameter, which is the ratio of the deviatoric to the mean stiffness. Theoretical values of asymmetry and anisotropy can be obtained from the load-deflection relation of the simply-supported beam. It can be concluded that the identification results are quite satisfactory for all parameters of interest.

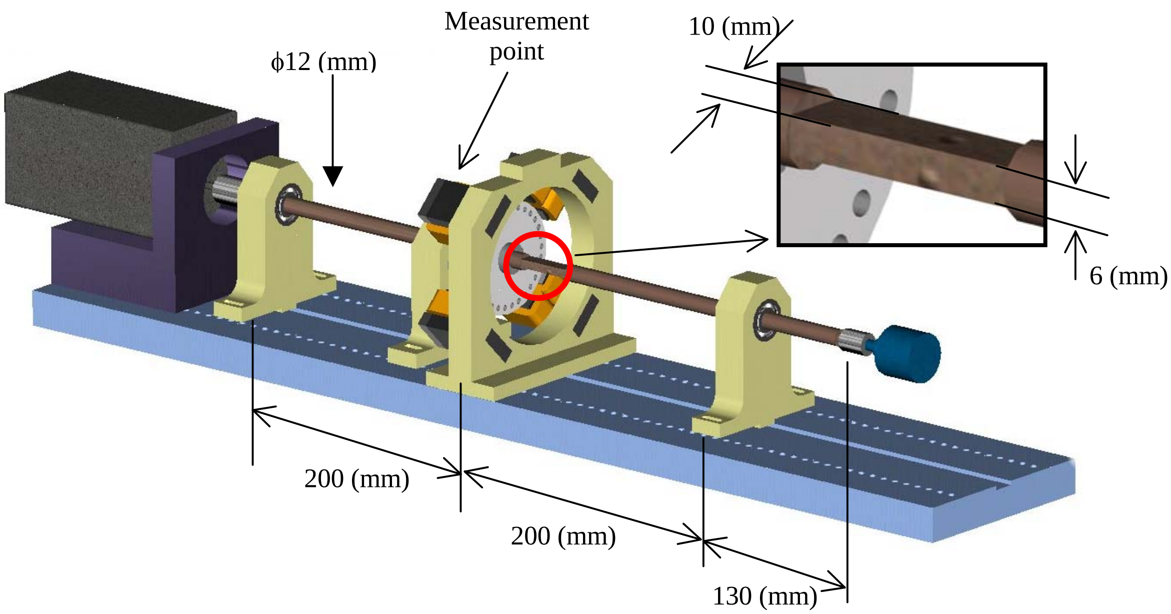


Fig. 3. Experimental set-up and asymmetry

Disk	Mass	Polar moment of inertia	Diametrical moment of inertia
	0.742 (*kg*)	580×10^{-6} ($kg\,m^2$)	315×10^{-6} ($kg\,m^2$)
Shaft	Mass	Length	Diameter
	0.452 (*kg*)	530 (*mm*)	12 (*mm*)
Critical speed		3300 (rpm)	
Rotational speed		600 (rpm)	

Table 1 Specifications of the test rotor system

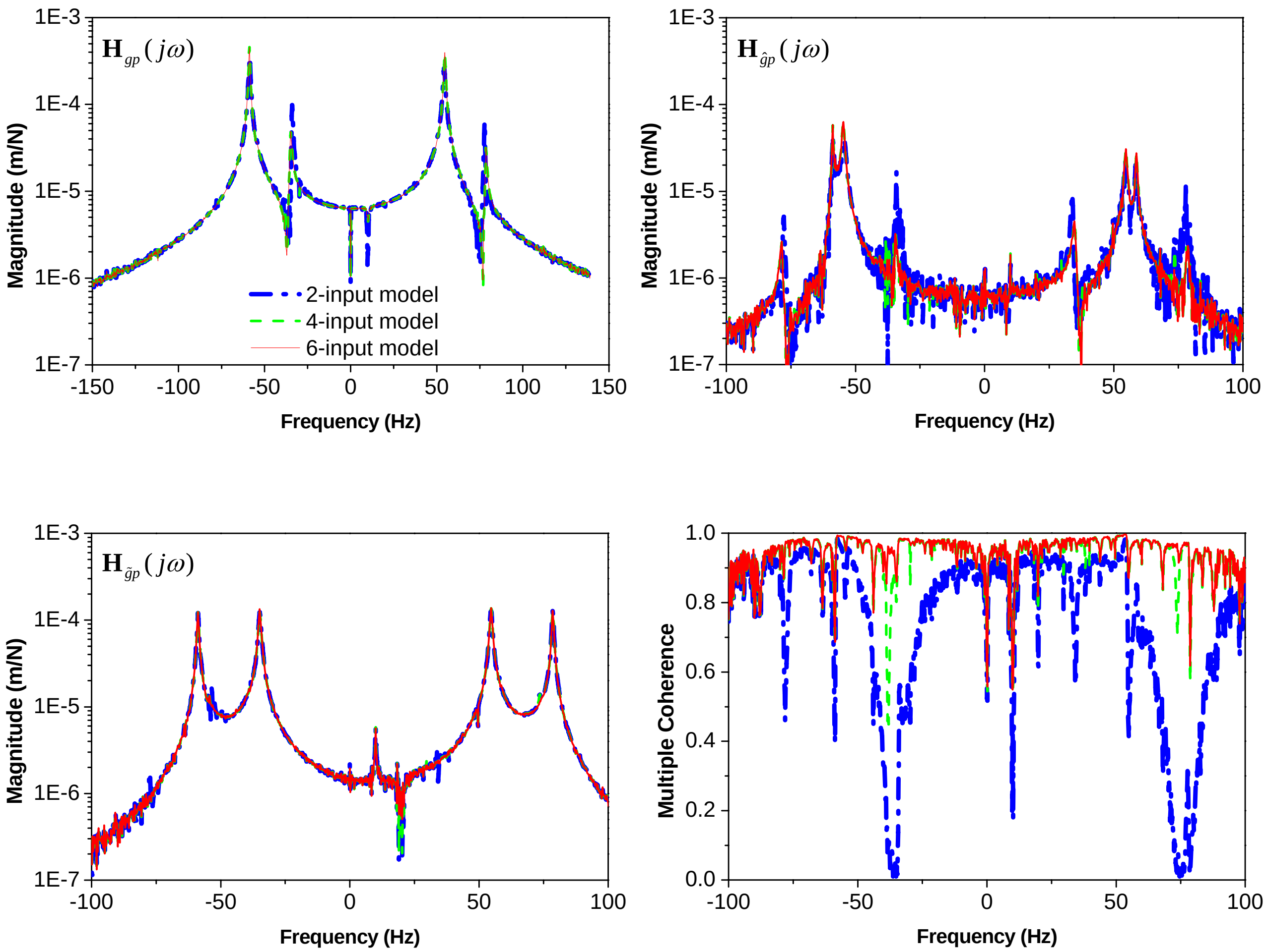
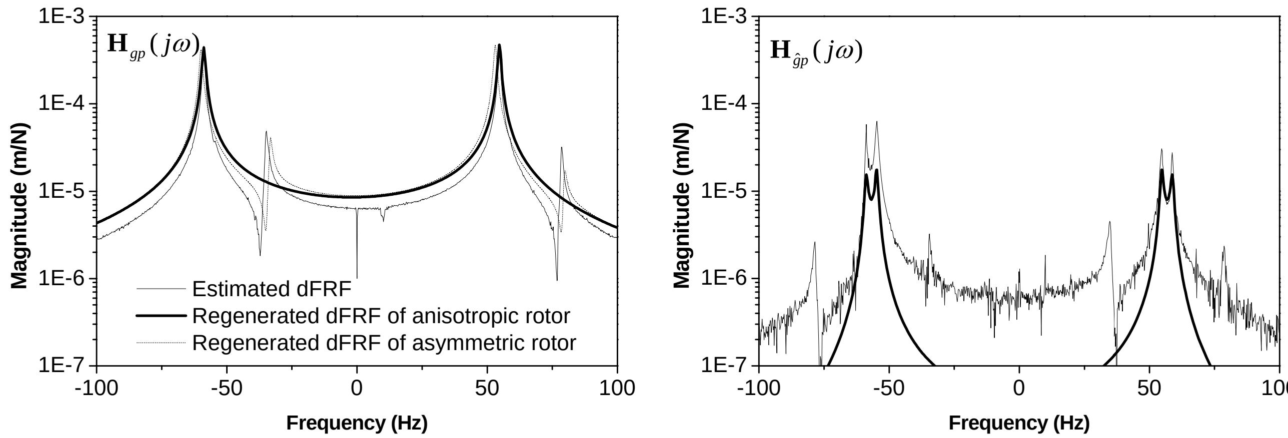
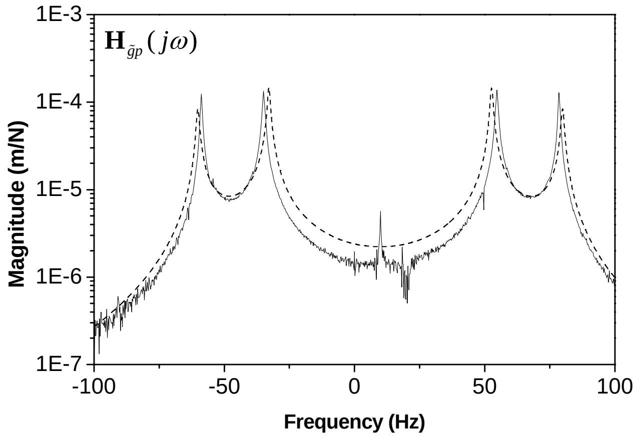


Fig. 4. Estimation results in accordance with the number of input





	Theoretical value	Identified value
Stiffness	0.112(MN/m)	0.113(MN/m)
Anisotropy	0.006	0.0059
Asymmetry	0.19	0.23

Fig. 5. Estimated and regenerated dFRFs and identification results

CONCLUSIONS

In this paper, a new estimation method of directional frequency response functions(dFRFs) for general rotors is proposed. Unlike conventional methods that are limited in application to anisotropic and asymmetric rotor systems. The proposed method is to estimate dFRFs for general rotor systems with both asymmetry and anisotropy. It essentially converts the time-invariant linear single input/single output relation, which is of typical with general rotor systems, into the time-invariant multiple (infinite number, in theory) input/single output relation in the frequency domain. The estimated multiple coherence function suggested that practically the six input/single output model is sufficient to estimate the dFRFs with fair accuracy. And the identification of anisotropy and asymmetry from estimated dFRFs is also performed with a test rotor in the laboratory, using a model update method. It is concluded that the identification results are quite satisfactory for all parameters of interest.

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