An Integrated Approach Using Change-Point Detection and Artificial Neural Networks for Interest Rates Forecasting

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Abstract

This article suggests integrated neural network models for the interest rate forecasting using change-point detection. The basic concept of proposed model is to obtain intervals divided by change point, to identify them as change-point groups, and to involve them in interest rate forecasting. The proposed models consist of three stages. The first stage is to detect successive change points in interest rate dataset. The second stage is to forecast change-point group with data mining classifiers. The final stage is to forecast the desired output with BPN. Based on this structure, we propose three integrated neural network models in terms of data mining classifier: (1) multivariate discriminant analysis (MDA)-supported neural network model, (2) case based reasoning (CBR)-supported neural network model and (3) backpropagation neural networks (BPN)-supported neural network model. Subsequently, we compare these models with a neural network model alone and, in addition, determine which of three classifiers (MDA, CBR and BPN) can perform better. This article is then to examine the predictability of integrated neural network models for interest rate forecasting using change-point detection.

Key Words: Change-point detection; Pettitt test; Artificial neural networks; Integrated systems; Interest rates

1. Introduction

Interest rates are one of the most closely watched variables in the economy. Their movements are reported almost daily by the news media since they directly affect our everyday lives and have important consequences for the economy. There exist extensive studies in this area using statistical approaches, such as term structure models, vector autoregressive (VAR) models, autoregressive conditionally heteroskedastic (ARCH) - generalized autoregressive conditionally heteroskedastic (GARCH) models and other time series analysis approaches.

Currently, several studies have demonstrated that artificial intelligence approaches, such as fuzzy theory (Ju et al., 1997) and neural networks (Deboeck and Cader, 1994; Hong and Han, 1996), can be alternative methodologies for chaotic interest rate data (Larrain, 1991; Peter, 1991; Jaditz and Sayers, 1995). Previous work in the interest rate forecasting has tended to emphasize statistical techniques and artificial intelligent (AI) techniques in isolation over the past decades. However, an integrated approach, which makes full use of statistical approaches and AI techniques, offers the promise of increasing performance over each method alone (Chatfield, 1993). This article explores the ways in which such technologies may be combined synergistically, and illustrates the approach through the use of MDA, CBR and BPN as a data mining classifier. Up to date, it has been proposed that the integrated neural network model combining two or more models have a potential to achieve a high predictive performance in interest rate forecasting (Kim and Kim, 1996; Kim and Noh, 1997).

In general, interest rate data are more fluctuated sensitively by government's monetary policy than other financial data (Gordon and Leeper, 1994; Strongin, 1995; Bernanke and Mihov, 1995; Christiano et al., 1996; Leeper et al, 1996; Bagliano and Favero). Especially, banks play a very important role in determining the supply of money: Much regulation of these financial intermediaries is intended to improve its control. One crucial regulation is reserve requirements, which make it obligatory for all depository institutions to keep a certain fraction of their deposits in accounts with the Federal Reserve System, the central bank in the United States (Mishkin, 1995). It is supposed that government take an intentional action to control the currency flow which has direct influence upon interest rates. Therefore, we can conjecture that the movement of interest rates has a series of change points occurred by the planned monetary policy of government.

Based on these inherent characteristics in interest rates, we propose three integrated neural network models in terms of data mining classifier: (1) MDA-supported neural network model, (2) CBR-supported neural network model and (3) BPN-supported neural network model. Subsequently, we compare these models with a neural network alone, and determine which of three classifiers (MDA, CBR and BPN) can

perform better. This article is then to examine the predictability of the integrated neural network models for interest rate forecasting using change-point detection and to compare the performance of several data mining classifiers.

The applied study performed in this article consists of the Treasury bill rate of 1 year's maturity in the U.S. from Jan., 1961 to May, 1999. Input variable selection is based on the causal model of interest rates presented by the econometricians. To explore the predictability, we divided the interest data into the training data over one period and the testing data over the other period. The predictability of interest rates is examined using the metrics of the root mean squared error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE).

In section 2, we outline the development of change-point detection and its application to the financial economics. Section 3 describes the proposed integrated neural network model details through the various data mining classifiers. Section 4 and 5 report the processes and the results of applied study. Finally, the concluding remarks are presented in Section 6.

2. Change-Point Detection

2.1. Application of Change-Point Detection for the Financial Economics

Financial analysts and econometricians have frequently used piecewise-linear models which also include change-point models. They are known as models with structural breaks in the economics literature. In these models, the parameters are assumed to shift - typically once - during a fixed sample period and the goal is to estimate the two sets of parameters as well as the change point or structural break.

This technique has been applied to macroeconomic time series. The first study in this field are conducted by Rappoport and Reichlin (1989) and Perron (1989, 1990). From then on, several statistics have been developed which work well in a change-point framework, all of which are considered in the context of breaking trend variables (Banerjee et al., 1992; Christiano, 1992; Zivot and Andrews, 1992; Perron, 1995; Vogelsang and Perron, 1995). In those cases where only a shift in the mean is present, the statistics proposed in the papers of Perron (1990) or Perron and Vogelsang (1992) stand out.

In spite of the significant advances by these works, we should bear in mind that some variables do not show just one change point. Rather, it is common for them to exhibit the presence of multiple change points. Thus, it seems advisable to introduce a large number of change points in the specifications of the models that allow us to obtain the abovementioned statistics. For example, Lumsdaine and Papell (1997) have considered the presence of two or more change points in trend variables. Based on this fact, we also assume the Treasury bill rates have two or more change points in our research model.

There are no artificial intelligence models for financial applications to consider the change-point

detection problems. Most of the previous research has a focus on the finding of unknown change points for the past, not to forecast for the future (Wolkenhauer and Edmunds, 1997; Li and Yu, 1999). Our model finds change points in the learning phase and forecasts change points in the testing phase. It is demonstrated that the introduction of change points to our model will make the predictability of interest rate greatly improve.

In this article, a series of change points will be detected by Pettitt test, a nonparametric change-point detection method since nonparametric statistical property is a suitable match for a neural network model that is a kind of nonparametric method (White, 1992).

2.2. Pettitt Test

The tests (Pettitt, 1979, 1980a, 1980b) assume that the observations form an ordered sequence, and that initially the distribution of responses has one median and at some point there is a shift in the median of the distribution. The alternative hypothesis could be one-tailed or two-tailed, but no prediction is made about the direction of change: H_0 is the hypothesis that there is no change in the location parameter (i.e. the median) of the sequence of observations, and H_1 is the hypothesis that there is a change in the location parameter of the sequence.

The change-point test has two kinds. One is appropriate when the data are binary and are observations with some binomial process (Pettitt, 1980a). Another test assumes that the data are continuous (Pettitt, 1979). The logic of the tests is similar although the computational formulas are different. We use the continuous type since we forecast the real value of interest rates. Pettitt test is explained as follows.

First, each of the observations $X_1, X_2, ..., X_N$ must be ranked from 1 to N. Let r_i be the rank associated with the observation X_i . Then at each place j in the series, we calculate

$$K_{m,n} = \max |2W_j - j(N+1)|, \quad j = 1, 2, ..., N-1$$
 (1)

where
$$W_j = \sum_{i=1}^{j} r_i$$
, $(N-m=n)$, m and n are the

number of observations before and after the change point j respectively.

Whether this value of $K_{m,n}$ is larger than we would expect under H_0 can be tested by referring to a table of the sampling distribution of W_i , the sum of ranks. If W exceeds the tabled value of W at the appropriate significance level, we may reject H_0 that there is no change in distribution.

If N becomes large, W is approximately normally distributed with mean m(N+1)/2 and variance mn(N+1)/12 under the assumption of no change in distribution. Thus, when the series is long, the test for change may be done and tested using the standard normal distribution table by transforming W

into a
$$Z$$
:

$$Z = \frac{W + h - m(N+1)/2}{\sqrt{mn(N+1)/12}}$$
 (2)

where h = -0.5 if W > m(N+1)/2 and h = +0.5 if W < m(N+1)/2.

Pettitt test detects a possible change point in the time sequence distribution. Once the change point is detected through the test, then the dataset is divided into two intervals. The intervals before and after the change point form homogeneous groups which take heterogeneous characteristics each other.

3. Model Specification

The statistical technique, data mining classifiers and neural network learning methods have been integrated to forecast the Treasury bill rate of 1 year's maturity in the U.S. The advantages of combining multiple techniques to yield synergism for discovery and prediction have been widely recognized (Gottman, 1981; Kaufman et al., 1991). The proposed models are determined by the kind of data mining classifier which is applied to the second stage of model.

In this section, we discuss the architecture and the characteristics of our research model to involve the change-point detection and the BPN. Based on Pettitt test, the proposed model consists of three stages: (1) change-point detection (CPD) stage, (2) change-point assisted group detection (CPGD) stage and (3) output forecasting neural network (OFNN) stage.

3.1. CPD stage: Construction and analysis on homogeneous groups

Pettitt test is a method to find a change-point in longitudinal data (Pettitt, 1979). It is known that interest rates at time t are more important than fundamental economic variables in determining interest rates at time t+1 (Larrain, 1991). Thus, we apply Pettitt test to Treasury bill rates at time t to generate a forecast for t+1 in the leaning phase. We, first of all, have to decide the number of change point. If change point is assumed to occur just one in given dataset, only the first step will be perform. Otherwise, all of three steps will be performed successively. The interval made by this process is defined as the significant interval, labeled SI, which is identified with a homogeneous group.

Step 1: Find a change point in $1 \sim N$ intervals by Pettitt test. If r_1 is a change point, $1 \sim r_1$ intervals are regarded as SI_1 and $(r_1 + 1) \sim N$ intervals are regarded as SI_2 . Otherwise, it is concluded that there does not exist a change point for $1 \sim N$ intervals. $(1 \le r_1 \le N)$

Step 2: Find a change point in $1 \sim r_1$ intervals by Pettitt test. If r_2 is a change point, $1 \sim r_2$ intervals

are regarded as SI_{11} and $(r_2 + 1) \sim r_1$ intervals are regarded as SI_{12} . Otherwise, $1 \sim r_1$ intervals are regarded as SI_1 like Step 1. $(1 \le r_2 \le r_1)$

Find a change point in $(r_1+1) \sim N$ intervals by Pettitt test. If r_3 is a change point, $(r_1+1) \sim r_3$ intervals are regarded as SI_{21} and $(r_3+1) \sim N$ intervals are regarded as SI_{22} . Otherwise, $(r_1+1) \sim N$ intervals are regarded as SI_{23} is like Step 1. $(r_1 \leq r_3 \leq N)$

Step 3: By applying the same procedure of Step 1 and 2 to subsamples, we can obtain several significant intervals under the dichotomy if we need five or more significant intervals.

This process plays a role of clustering which constructs groups for the learning phase as well as maintains time sequence. In this point, CPD stage is distinguished from other clustering methods such as the k-means nearest neighbor method and the hierarchical clustering method. They classify data samples by the Euclidean distance between cases without considering time sequence. In addition, we analyze the characteristic of groups according to descriptive statistics including the mean and the variance, and also observe the density plot of groups since the classification accuracy is highly sensitive to the density of the sample (Wang, 1995).

3.2. CPGD stage: Forecast the group with data mining classifier

The significant intervals by CPD stage are grouped to detect the regularities hidden in them and to represent the homogeneous characteristics of them. Such groups represent a set of meaningful trends encompassing the significant intervals. Since those trends help to find regularity among the related output values more clearly, the neural network model can have a better ability of generalization for the unknown data. This is indeed a very useful point for sample design. In general, the error for forecasting may be reduced by making the subsampling units within groups homogeneous and the variation between groups heterogeneous (Cochran, 1977). After the appropriate groups hidden in the significant intervals are detected by CPD stage, various classifiers (MDA, CBR and BPN) are applied to the input data samples at time twith group outputs for t+1. In this sense, CPGD is a model that is trained to find an appropriate group for each given sample.

3.3. OFNN stage: Forecast the desired output with BPN OFNN is built by applying the BPN model to each group. OFNN is a mapping function between the input sample and corresponding desired output (i.e. Treasury bill rate). Once OFNN is built, then the sample can be used to forecast the Treasury bill rate.

According to the kind of classifier used in the

CPGD stage, we propose three integrated neural network models: (1) MDA-supported neural network model, (2) CBR-supported neural network model and (3) BPN-supported neural network model.

4. Data and Variables

In this article, input variables are selected based on the Fisher's hypothesis, which is the theory under which nominal interest rates (i.e. monthly US Treasury bills of 1 year's maturity) consist of expected real interest rates and anticipated inflation as follows:

Nominal Interest Rates = Expected Real Interest Rates + Anticipated Inflation

Many econometricians have conducted the research upon this Fisher-type interest rate equation (Mundell, 1963; Tobin, 1965; Darby, 1975; Feldstein, 1976; Tanzi, 1980; Makin, 1983). They have explained the impact of anticipated inflation on nominal interest rates. Moreover, they have investigated the relation of money surprise and real GNP growth for the Fisher-type interest rate equation. These relationships can be summarized in Figure 1. In Figure 1, it is shown that the straight line has more causal effects than the stitch line.

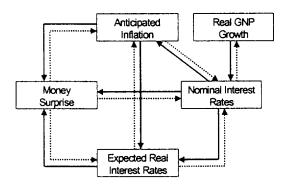


Figure 1. The economic model under the Fisher-type interest rate equation

The input data sets in this study consist of monthly rate of change figures. Given the data sequence d_1, d_2, \dots, d_t , we form the rate of change at time t+1 by dividing the first difference at that time by the datum at time t:

$$\frac{d_{t+1} - d_t}{d_t} \tag{3}$$

The variables included in this model are anticipated inflation and expected real interest rates to involve money surprise and real GNP growth in Figure 1. The rate of change of consumer price index is used as a measure for anticipated inflation while expected real interest rate is calculated as the difference between the nominal interest rates and the anticipated inflation at time t according to the Fisher-type interest rate equation. M2 and industrial production index are added to input variables as a measure for money surprise and

real GNP growth respectively to keep the property of anticipated inflation and expected real interest rates.

The learning phase involves observations from January 1961 to August 1991 while the testing phase runs from September 1991 to May 1999. The lists of variables used in this study are summarized in Table 1. The interest rate data are presented in Figure 2. Figure 2 show that the movement of interest rates is highly fluctuated during the last forty years.

Table 1 Description of Variables.

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Variable Name	Description	Attribute
TBILL	Treasury Bill with 1 year's maturity	Output
M2	Money Stock	Input
CPI	Consumer Price Index	Input
ERIR	Expected Real Interest Rates	Input
IPI	Industrial Production Index	Input

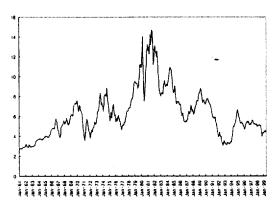


Figure 2. Yield of U.S. Treasury bills with a maturity of 1 year from Jan. 1960 to May 1999

The study employed two neural network models. One model, labeled Pure_NN, involve input variables at time t to generate a forecast for t+1. The input variable is M2, CPI, ERIR and IPI. The second type has two-step forecasting models which consist of three stages mentioned in section 3. The first step is the CPGD stage that forecasts the change-point group while the next step is the OFNN stage that forecasts the desired output. The classifiers used in the model were as shown in Table 2. For validation, four learning models were also compared.

Table 2. Models and their associated classifiers for US
Treasury bill rate forecasting.

Model	Classifier used in the model	
Pure NN	None	
CBR_NN	Case Based Reasoning	
BPN NN	Backpropagation Neural Network	
MDA_NN	Multivariate Discriminant Analysis	

5. Empirical Results

The Pettitt analysis is applied to the interest rate dataset. Since the interest dataset is about forty years long, it is considered that there exist two or more change points. Therefore, we obtain 4 significant intervals as like the result of Table 3. Table 3 also presents descriptive statistics including the mean and the variance. Group 1 is the stable interval that has low variance. Group 2 and 3 are more fluctuated intervals than Group 1 in term of the variance. Group 4 is highly fluctuated. Skewness and kurtosis show that four groups have similar attributes in the distribution. Figure 3 depicts the density plot for each group. By Figure 3, Group 2 and 4 are considered to have the similar distribution in terms of scale parameter, the variance.

Table 3. Period and descriptive statistics of groups for

the learning phase, Jan. 1961 - Aug. 1991					
	Group	Group	Group	Group	
	1	2	3	4	
Periods	61/1 -	12/65 -	03/73 -	06/78 -	
	11/65	02/73	05/78	08/91	
Minimum	2.720	3.600	4.640	5.260	
Maximum	4.230	7.610	8.880	14.700	
Range	1.510	4.010	4.240	9.440	
Mean	3.378	5.419	6.507	8.654	
Variance	0.219	0.938	1.008	5.240	
Standard	0.468	0.969	1.004	2.289	
Deviation	0.400	0.909	1.004	2.289	
Skewness	0.147	0.496	0.363	0.781	
Kurtosis	-1.544	-0.361	-0.575	-0.135	

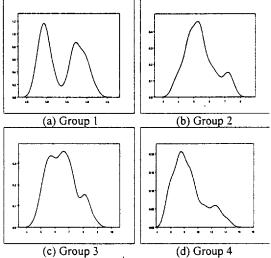


Figure 3. Density plot of four homogeneous groups for TBILL

To highlight the performance due to various models, the actual values of Treasury bill rates and their predicted values are shown in Figure 4. The predicted values of pure BPN model (i.e. PURE_NN) and CBR-supported neural network model get apart from the actual values in some intervals. Numerical values for the performance metrics by predictive model are given in Table 4. Figure 5 presents histograms of RMSE,

MAE and MAPE of predictions for each learning model. According to RMSE, MAE and MAPE, the outcomes indicate that BPN-supported neural network model and MDA-supported neural network model are superior to the pure BPN model and CBR-supported neural network model.

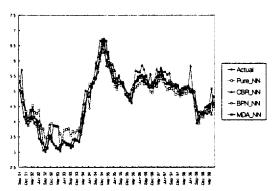


Figure 4. Actual vs predicted values due to various models for TBILL

Table 4. Performance results in the case of US Treasury bill rate forecasting based on the root mean squared error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE)

Model	RMSE	MAE	MAPE
Pure NN	0.0973	0.2506	5.969%
CBR_NN	0.1015	0.2489	5.553%
BPN NN	0.0584	0.1745	3.746%
MDA NN	0.0481	0.1733	3.784%

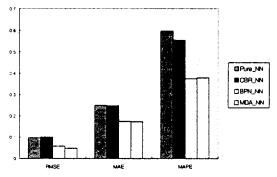


Figure 5. Histogram of RMSE, MAE and MAPE resulting from forecasts of TBILL

Our approach to integration involves a multistrategy technique which may be called second-order learning. CBR has provided the good results with the second order learning process and the integrated approach (Kim and Noh, 1997; Kim and Joo, 1997). However, CBR did not perform well in this study. In the second-order learning, the forecast from the superior method is selected on a case-by-case basis to determine the output of overall model. In other words, the second step (BPN in this study) serves as a metalevel process to determine which of three elementary modules (CBR, BPN and MDA in this study) perform better. In this point, CBR is not a good metalevel predictive method. Thus, we will choose

BPN or MDA as the elementary module for real application of model.

We use the pairwise t-test to examine whether there exist the differences in the predicted values of models according to the absolute percentage error (APE). This metric is chosen since it is commonly used (Carbone and Armstrong, 1982) and is highly robust (Armstrong and Collopy, 1992; Makridakis, 1993). Since the forecasts are not statistically independent and not always normally distributed, we compare the forecast's APEs using the pairwise t-test. Where sample sizes are reasonably large, this test is robust to the distribution of the data, to nonhomogeneity of variances, and to statistical dependence (Iman and Conover, 1983). Table 5 shows t-values and p-values when the prediction accuracies of the left-vertical methods are compared with those for the righthorizontal methods. Mostly, the neural network models using change-point detection perform significantly better than the pure BPN model at a 1% significant level except the analysis of CBR-supported neural network model. Therefore, our research model is demonstrated to obtain the improved performance through the change-point detection approach.

The neural network models using change-point detection turn out to have a high potential in interest rate forecasting. This is attributable to the fact that it categorizes the input data samples into homogeneous group and extracts regularities from each homogeneous group. Therefore, the neural network models using change-point detection can cope with the noise or irregularities more efficiently than the pure BPN model. In addition, BPN and MDA perform very well as a tool in interest rates forecasting.

Table 5. Pairwise t-tests for the differences in residuals for US interest rate prediction based on the absolute percentage error (APE) with the significance level in

parentheses.					
Model	BPN NN	CBR_NN	Pure NN		
MDA_NN	-0.14 (0.882)	3.29 (0.001)*	3.75 (0.000)*		
BPN_NN		3.25 (0.001)*	3.43 (0.000)*		
CBR_NN			0.72 (0.467)		

^{*} Significant at 1%

6. Concluding Remarks

This article has suggested the integrated neural network models in the interest rate forecasting using change-point detection. The basic concept of proposed model is to obtain significant intervals by change-point detection, to identify them as change-point groups, and to involve them in interest rate forecasting. We propose integrated neural network models which consist of three stages. In the first stage, we conduct the nonparametric statistical test for the change-point detection to construct the homogeneous groups. In the second stage, we apply several kinds of classifiers to

forecast the change-point group. In the final stage, we apply BPN to forecast the desired output.

The neural network models using change-point detection perform significantly better than the pure BPN model at a 1% significant level except the analysis of CBR-supported neural network model. Experimental results showed that the neural network models using change-point detection outperform the pure BPN model significantly, which implies the high potential of involving the change-point detection in the model. BPN and MDA performed very well as data mining classifiers while CBR did not. Our integrated neural network models are demonstrated to be useful intelligent data analysis methods with the concept of change-point detection. In conclusion, we have shown that the proposed models improve the predictability of interest rates significantly.

The proposed model has the promising possibilities to improve the performance if further studies are to focus on the various approaches in the construction and the prediction of change-point group. In final stage of the model, other intelligent approaches can be used to forecast the final output besides BPN. In addition, the proposed models may be applied to other chaotic time series data, such as stock market prediction and exchange rate prediction. By the extension of these points, future research are expected to provide more improved neural network models with superior performances.

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