OPTIMAL RESOURCE ALLOCATION FOR SUCCESSFUL NEW PRODUCT DEVELOPMENT PROJECTS

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ABSTRACT

Risk management is an important activity for successful project completion. New product development (NPD) projects, especially, can be affected by risk factors due to intrinsic environmental uncertainties. In order to complete projects successfully, resources need to be allocated to systematic risk-mitigating efforts for controlling various sources of uncertainty. In this paper, we employ the optimal control theory to identify and analyze key decision variables for successful NPD under a game-theoretic situation.

KEYWORDS

Risk Management, Project Management, Optimal Control Theory, Game Theory

1. Introduction

Risk management, non-financial as well as financial, is essential in project management. For effective risk management, a systematic risk-managing scheme is necessary, especially when the company faces complex and uncertain environments during its NPD process. In this paper, we focus on risk management for resource allocating activities of NPD projects. To find out optimal resource allocating strategy, an optimal control model is developed under a game-theoretic situation.

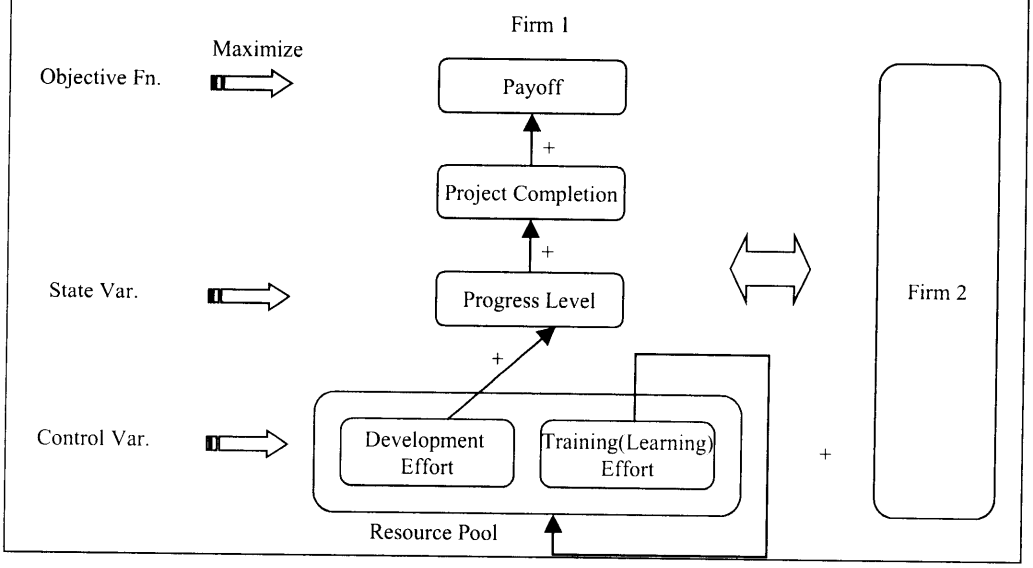
In the next section, we review the literature relevant to our research. Section 3 develops a mathematical model to solve the decision problem for new product development projects. Analytical results are provided in Section 4. Finally, we suggest critical managerial implications along with future research direction.

2. Literature Review

In this section, we review papers relevant to our research, especially focused on risk management in NPD. Abetti and Stuart [1] presented three dimensions of product risk and guidelines for evaluating and controlling the risk inherent in new product development projects. Ahmadi and Wang [2] suggested that the development risks be managed and controlled in design processes. They found out that iterative risk management in the design process results in better project performance.

Resource management has been thought as an essential part of successful project management [4, 5]. Ash [3] argued that learning is an important part of human resource in resource constrained development projects. While there is relative richness of research in the production and operations management literature, little has been done using the optimal control theory. But, we believe the theory can be easily adapted to NPD. Finally, the game-theoretic situation has been widely studied by economists, but little has been done in the project management literature with rare examples such as Aloysius [8] and Gerchak and Parlar [9].

Figure 1. Overall research context.



3. Research Model

Before developing the optimal control model, we first describe the research context. Two firms are competing to develop a new product. The winning firm receives all of the positive payoffs and the other receives nothing. In order to develop a new product, each firm needs to carry on development efforts that lead to project progression. Neither firm *a priori* knows the total amount of development efforts needed to complete the project. Each firm can allocate resources into two types of activities, *direct development* and *training*. Training activities are not directly related with the project completion, but contribute to the knowledge accumulation that enlarges the resource pool. Overall research context is in Figure 1. The primary research question is how to optimally allocate resources, *to either direct development or training activities*, to maximize the payoffs under various uncertain situations. We suggest the following propositions.

Proposition 1. Training activity can enlarge the resource pool, which results in more development effort and high probability to win a project.
Proposition 2. When there are less resource available and more uncertainty in project completion, training activity is more preferred.
Proposition 3. When there is more time delay between training and performance increase, it is less likely to devote resources to the training activity.

In order to verify above propositions, we employ an optimal control model. Before developing the optimal control model, we list basic notations to be used in our model.

P : Present value of the winner's payoff that commences at project completion
$p_i(t)$: Progression level for each firm that leads project completion
$d_i(t)$: Development efforts for each firm
$l_i(t)$: Training (learning) efforts for each firm
$R_i(t)$: Available resources for each firm
a_i : Efficiency of development effort for each firm
b_i : Efficiency of training effort for each firm
γ_i : Cost coefficient of developing effort for each firm
δ_i : Cost coefficient of training effort for each firm
λ_i : Organizational capability to project completion for each firm

We assume that the probability of successful development at t follows an exponential distribution as follows.

$$F_i(p_i(t)) = 1 - e^{-\lambda_i p_i(t)}, \quad F_i(0) = 0, \quad i = 1,2$$

Base Model

First, consider only the fundamental parts of the model, excluding the training effect in Figure 1. The problem in this model is finding optimal pattern of development effort for each player. The cost of developing at rate $d_i(t)$ is assumed to be $d_i^2/2$ for the computational efficiency. Cost coefficient is also neglected for the sake of simplification. So the problem faced by each firm can be written as,

$$\textit{Maximize} \quad \int_0^T [P(1-F_j(p_j))\frac{dF_i(p_i)}{dt} - e^{-rt}(1-F_i(p_i))(1-F_j(p_j))\frac{d_i^2}{2}]dt, \quad i=1,2$$

The first term of the integrand is due to the project payoffs and the second due to incurring costs. So we can generate an optimal control model for player i as follows.

$$\textit{Maximize} \quad \int_0^T [P(1-F_j(p_j))\frac{dF_i(p_i)}{dt} - e^{-rt}(1-F_i(p_i))(1-F_j(p_j))\frac{d_i^2}{2}]dt, \quad i=1,2$$

$$\textit{Subject to} \quad p_i'(t) = a_i d_i(t).$$

We assume that available resources are enough to progress the project.

Extended Model

In this model, training effect in Figure 1 is added to the base model. Each firm must exhaust its available resources by allocating the resources to either development or training activities. We assume that the costs of development and training are $\gamma_i d_i^2/2$ and $\delta_i l_i^2/2$, respectively. So, the problem can be modeled as follows.

$$\textit{Maximize} \quad \int_0^T [P(1-F_j(p_j))\frac{dF_i(p_i)}{dt} - e^{-rt}(1-F_i(p_i))(1-F_j(p_j))\frac{\gamma_i d_i^2 + \delta_i l_i^2}{2}]dt, \quad i=1,2$$

$$\begin{aligned} \textit{Subject to.} \quad & p_i'(t) = a_i d_i(t) \\ & l_i(t) + d_i(t) = R_i(t) \\ & R_i'(t) = b_i l_i(t) \\ & R_i(0) = R_{i0} \end{aligned}$$

4. Result

The optimal control model developed in the previous section is solved by formal optimal control theory. Problem solving procedure is shown in Appendix from the perspective of Firm 1. In the case of base model, the optimal dynamics of developing activity is derived as follows.

$$d_1^*(t) = a_1 J_{p_1}^{\ 1}(e^{rt} e^{\lambda_1 p_1 + \lambda_2 p_2}) = \frac{2a_1 a_2^2 P \lambda_1 \lambda_2^2 e^{rt}}{2a_2^3 \lambda_2^2 - a_1^2 \lambda_1^2 (e^{a_2^2 P \lambda_2^2 (e^{rt} - e^{rT})/r} - a_2)}$$

It says that the optimal pattern of development efforts is independent of the progression level.

For the extended model, a closed-form solution cannot be derived as depicted in Appendix. But, we can describe the relationship between the solution of the base model and that of the extended model as follows.

$$d_1^{**}(t) = \frac{\delta_1}{\gamma_1 + \delta_1} R_1(t) + \frac{a_1 J_{p_1}^{\ 1}}{\gamma_1 + \delta_1}(e^{rt} e^{\lambda_1 p_1 + \lambda_2 p_2}) \approx \frac{\delta_1}{\gamma_1 + \delta_1} R_1(t) + \frac{1}{\gamma_1 + \delta_1} d_1^{*}(t)$$

$$l_1^{**}(t) = R_1(t) - d_1^{**}(t) = \frac{\gamma_1}{\gamma_1 + \delta_1} R_1(t) - \frac{1}{\gamma_1 + \delta_1} d_1^{*}(t)$$

In the first equation, $J_{p_1}^{\ 1}$ is not the same as that in the base model (see Appendix). But, if we assume that the cost term is relatively smaller than the profit term, the equation can hold approximately.

The above two equations indicate that the optimal solution must be changed from that in the base model. In other words, by introducing the training effect, optimal dynamics of developing effort should be changed. According to the above two equations, both development and training efforts increase as time passes because the resource pool itself is increasing. It means that a firm can allocate more resources to the development efforts than in the base model. This training effect is strengthened as the cost coefficient of development efforts is increased and that of training efforts is decreased. These explanations ensure Proposition 1.

In order to verify other propositions, we need to analyze solutions numerically. Computer simulation could be suitable to this analysis. We leave it for the future work.

5. Conclusion

In this paper, we modeled firm's new product development processes under the game-theoretic situation. We proposed two NPD models, *base* and *extended*, and derived solutions by applying the optimal control theory. The major implication of this paper is that when there is a training effect for NPD, the optimal dynamics of development efforts could be very different from the base model case. It highlights the importance of the training effects in NPD projects. Although we cannot verify all of the propositions, other managerial implications can be obtained by numerical analysis: it is an important direction for future research. .

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APPENDIX

Problem Solving Procedure for the Base Model

A) $J^1(d_1,d_2)=\int_0^T[P(1-F_2(p_2))\frac{dF_1(p_1)}{dt}-e^{-rt}(1-F_1(p_1))(1-F_2(p_2))\frac{d_1^2}{2}]dt$ □

By letting $dv=(\frac{dF_1(p_1)}{dt})dt$, $u=1-F_2(p_2)$, □ can be rearranged as,

$$J^1(d_1,d_2)=P(1-F_2(p_2))F_1(p_1)+P\int_0^T\frac{dF_2(p_2)}{dt}F_1(p_1)dt-\int_0^T e^{-rt}(1-F_1(p_1))(1-F_2(p_2))\frac{d_1^2}{2}dt$$ □

Substituting $F_i(p_i)=1-e^{-\lambda_i p_i}$ and $\frac{dF_i(p_i)}{dt}=a_i\lambda_i d_i e^{-\lambda_i p_i}$ into □ gives

$$J^1(d_1,d_2)=\int_0^T[P(1-e^{-\lambda_1 p_1})e^{-\lambda_2 p_2}a_2\lambda_2 d_2-e^{-rt}(e^{-\lambda_1 p_1-\lambda_2 p_2})\frac{d_1^2}{2}]dt+P[e^{-\lambda_2 p_2(T)}-e^{-\lambda_1 p_1(T)-\lambda_2 p_2(T)}]$$ □

Last term in □ can be regarded as salvage value.

B) Now we can use HJB to find feedback Nash equilibrium as following

$-J_t(t,x)=\max_u[f(t,x,u)+J_x(t,x)g(t,x,u)]$ → HJB Equation. So, this problem can be treated as,

$$-J_t^1(t,p_1,p_2)=\max_{d_1(t,p_1,p_2)}[P(1-e^{-\lambda_1 p_1})e^{-\lambda_2 p_2}a_2\lambda_2\overline{d_2}-e^{-rt}e^{-\lambda_1 p_1-\lambda_2 p_2}\frac{d_1^2}{2}+a_1 J_{p_1}^1 d_1+a_2 J_{p_2}^1\overline{d_2}]$$ □

The first order condition for □ is $-e^{-rt}e^{-\lambda_1 p_1-\lambda_2 p_2}d_1+a_1 J_{p_1}^1=0$ → $d_1=a_1 J_{p_1}^1(e^{rt}e^{\lambda_1 p_1+\lambda_2 p_2})$ □

Likewise, for player 2, → $d_2=a_2 J_{p_2}^2(e^{rt}e^{\lambda_1 p_1+\lambda_2 p_2})$

Substituting for d_1 and d_2 back into □ gives

$$J_t^1(t,p_1,p_2)+\frac{a_1^2}{2}(J_{p_1}^1)^2(e^{rt}e^{\lambda_1 p_1+\lambda_2 p_2})+a_2^2 J_{p_2}^1 J_{p_2}^2 e^{rt}e^{\lambda_1 p_1+\lambda_2 p_2}+a_2^2 P J_{p_2}^2 e^{rt}\lambda_2(e^{\lambda_1 p_1}-1)=0$$ □

C) From salvage term in □, we propose

$J^i(t,p_1,p_2)=b(t)e^{-\lambda_i p_i}+k(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$, $i\neq j$, $i=1,2$ □

From □ it follows that $J_{p_1}^1=-\lambda_1 k(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-1

$J_{p_2}^1=-\lambda_2 b(t)e^{-\lambda_2 p_2}-\lambda_2 k(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-2

$J_{p_1}^2=-\lambda_1 b(t)e^{-\lambda_1 p_1}-\lambda_1 k(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-3

$J_{p_2}^2=-\lambda_2 k(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-4

$J_t^1=b'(t)e^{-\lambda_2 p_2}+k'(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-5

$J_t^2=b'(t)e^{-\lambda_1 p_1}+k'(t)e^{-\lambda_1 p_1-\lambda_2 p_2}$ □-6

Substituting □-1 through □-6 into □ gives $\{b'(t)+a_2^2\lambda_2^2 b(t)k(t)e^{rt}-a_2^2 P\lambda_2^2 k(t)e^{rt}\}e^{-\lambda_2 p_2}$

$$+\{k'(t)+\frac{a_1^2\lambda_1^2 k^2(t)e^{rt}}{2}+a_2^2\lambda_2^2 k^2(t)e^{rt}+a_2^2 P\lambda_2^2 k(t)e^{rt}\}e^{-\lambda_1 p_1-\lambda_2 p_2}=0$$ □

In order to satisfy □, following two equations must be hold

$b'(t)+a_2^2\lambda_2^2 b(t)k(t)e^{rt}-a_2^2 P\lambda_2^2 k(t)e^{rt}=0$ □

$k'(t)+\frac{a_1^2\lambda_1^2 k^2(t)e^{rt}}{2}+a_2^2\lambda_2^2 k^2(t)e^{rt}+a_2^2 P\lambda_2^2 k(t)e^{rt}=0$ □

From □ and □, $J^1(T,p_1(T),p_2(T))=P[e^{-\lambda_2 p_2(T)}-e^{-\lambda_1 p_1(T)-\lambda_2 p_2(T)}]$

It follows that $b(T)=P$, $k(T)=-P$ □

D) Rearranging □ gives

$$k'(t)+(k(t)+P)(a_2{}^2\lambda_2{}^2k(t)e^{rt})+\frac{a_1{}^2\lambda_1{}^2k^2(t)e^{rt}}{2}=0 \qquad □$$

Equation □ is known as Bernoulli equation and can be easily solved by letting $k(t)=-\frac{1}{q(t)}$. Substituting it into □ and multiplying through by $q^2(t)$ gives $q'(t)-a_2{}^2Pq(t)\lambda_2{}^2e^{rt}+(a_2{}^2\lambda_2{}^2+\frac{a_1{}^2\lambda_1{}^2}{2})e^{rt}=0$ □

A solution to □ is derived as $q(t)=c_0e^{a_2{}^2P\lambda_2{}^2e^{rt}/r}+\frac{a_2{}^2\lambda_2{}^2+a_1{}^2\lambda_1{}^2/2}{a_2{}^2P\lambda_2{}^2}$.

Recall □, it follow that $q(t)=-\frac{a_1{}^2\lambda_1{}^2}{2a_2{}^2P\lambda_2{}^2}e^{a_2{}^2P\lambda_2{}^2(e^{rt}-e^{rT})/r}+\frac{a_2{}^2\lambda_2{}^2+a_1{}^2\lambda_1{}^2/2}{a_2P\lambda_2{}^2}$

So $k(t)$ is derived as $k(t)=-\frac{1}{q(t)}=\frac{2a_2{}^2P\lambda_2{}^2}{a_1{}^2\lambda_1{}^2(e^{a_2{}^2P\lambda_2{}^2(e^{rt}-e^{rT})/r}-a_2)-2a_2{}^3\lambda_2{}^2}$ (16)

Combining □, □-1, (16) gives $d_1{}^*(t)=\frac{2a_1a_2{}^2P\lambda_1\lambda_2{}^2e^{rt}}{2a_2{}^3\lambda_2{}^2-a_1{}^2\lambda_1{}^2(e^{a_2{}^2P\lambda_2{}^2(e^{rt}-e^{rT})/r}-a_2)}$.

Problem Solving Procedure for Extended Model

A) $J^1(d_1,d_2,l_1,l_2)=\int_0^T[P(1-F_2(p_2))\frac{dF_1(p_1)}{dt}-e^{-rt}(1-F_1(p_1))(1-F_2(p_2))\frac{\gamma_1d_1{}^2+\delta_1l_1{}^2}{2}]dt$ □

By letting $dv=(\frac{dF_1(p_1)}{dt})dt$, $u=1-F_2(p_2)$, □ can be rearranged as,

$$J^1(d_1,d_2,l_1,l_2)=P(1-F_2(p_2))F_1(p_1)+P\int_0^T\frac{dF_2(p_2)}{dt}F_1(p_1)dt$$
$$-\int_0^T e^{-rt}(1-F_1(p_1))(1-F_2(p_2))\frac{\gamma_1d_1{}^2+\delta_1l_1{}^2}{2}dt \qquad □$$

Substituting $F_i(p_i)=1-e^{-\lambda_ip_i}$, $\frac{dF_i(p_i)}{dt}=a_i\lambda_id_ie^{-\lambda_ip_i}$ and $l_i(t)=R_i(t)-d_i(t)$ into □ gives

$$J^1(d_1,d_2,R_1,R_2)=\int_0^T[P(1-e^{-\lambda_1p_1})e^{-\lambda_2p_2}a_2\lambda_2d_2-e^{-rt}(e^{-\lambda_1p_1-\lambda_2p_2})\frac{\gamma_1d_1{}^2+\delta_1(R_1-d_1)^2}{2}]dt$$
$$+P[e^{-\lambda_2p_2(T)}-e^{-\lambda_1p_1(T)-\lambda_2p_2(T)}] \qquad □$$

Last term in □ can be regarded as salvage value

B) Now we can use HJB to find feedback Nash equilibrium as following

$$-J_t{}^1(t,p_1,p_2,R_1,R_2)=\max_{d_1(t,p_1,p_2,R_1)}[P(1-e^{-\lambda_1p_1})e^{-\lambda_2p_2}a_2\lambda_2\overline{d_2}-e^{-rt}e^{-\lambda_1p_1-\lambda_2p_2}\frac{\gamma_1d_1{}^2+\delta_1(R_1-d_1)^2}{2}$$
$$+a_1J_{p_1}{}^1d_1+a_2J_{p_2}{}^1\overline{d_2}+b_1J_{R_1}{}^1(R_1-d_1)+b_2J_{R_2}{}^1(R_2-\overline{d_2})] \qquad □$$

The first order condition for □ is $-e^{-rt}e^{-\lambda_1p_1-\lambda_2p_2}(\gamma_1d_1-\delta_1(R_1-d_1))+a_1J_{p_1}{}^1-b_1J_{R_1}{}^1=0$

$$\rightarrow\ d_1=\frac{\delta_1}{\gamma_1+\delta_1}R_1+\frac{a_1J_{p_1}{}^1-b_1J_{R_1}{}^1}{(\gamma_1+\delta_1)e^{-rt}e^{-\lambda_1p_1-\lambda_2p_2}} \qquad □$$

Likewise, for player 2, $\rightarrow\ d_2=\frac{\delta_2}{\gamma_2+\delta_2}R_2+\frac{a_2J_{p_2}{}^2-b_2J_{R_2}{}^2}{(\gamma_2+\delta_2)e^{-rt}e^{-\lambda_1p_1-\lambda_2p_2}}$ □

C) As in the base case, we propose

$J^i(t,p_1,p_2)=b(t)e^{-\lambda_jp_j}+k(t)e^{-\lambda_1p_1-\lambda_2p_2}$, $i\neq j$, $i=1,2$ □

From □ it follows that $J_{p_1}{}^1=-\lambda_1k(t)e^{-\lambda_1p_1-\lambda_2p_2}$ □-1

$J_{p_2}{}^1=-\lambda_2b(t)e^{-\lambda_2p_2}-\lambda_2k(t)e^{-\lambda_1p_1-\lambda_2p_2}$ □-2

$$J_{p_1}{}^2 = -\lambda_1 b(t) e^{-\lambda_1 p_1} - \lambda_1 k(t) e^{-\lambda_1 p_1 - \lambda_2 p_2}$$ -3

$$J_{p_2}{}^2 = -\lambda_2 k(t) e^{-\lambda_1 p_1 - \lambda_2 p_2}$$ -4

$$J_t{}^1 = b'(t) e^{-\lambda_2 p_2} + k'(t) e^{-\lambda_1 p_1 - \lambda_2 p_2}$$ -5

$$J_t{}^2 = b'(t) e^{-\lambda_1 p_1} + k'(t) e^{-\lambda_1 p_1 - \lambda_2 p_2}$$ -6

$$J_{R_1}{}^1 = J_{R_2}{}^1 = J_{R_1}{}^2 = J_{R_2}{}^2 = 0$$ -7

Substituting , , -1 through -7 into and rearranging gives

$$\{b'(t) + \frac{a_2\lambda_2(a_2\lambda_2 k(t)e^{rt} - \delta_2 R_2)}{\gamma_2 + \delta_2} b(t) + \frac{a_2\lambda_2 P(\delta_2 R_2 - a_2\lambda_2 k(t)e^{rt})}{\gamma_2 + \delta_2}\} e^{-\lambda_2 p_2}$$
$$+ \{k'(t) + (\frac{\lambda_2{}^2 e^{rt}}{(\gamma_2 + \delta_2)} + \frac{a_1{}^2\lambda_1{}^2 e^{rt}}{2(\gamma_1 + \delta_1)}) k^2(t) + (\frac{Pa_2{}^2\lambda_2{}^2 e^{rt} - a_2\lambda_2\delta_2 R_2}{\gamma_2 + \delta_2} - \frac{a_1\lambda_1\delta_1 R_1}{\gamma_1 + \delta_1}) k(t)$$
$$- (\frac{a_2\lambda_2\delta_2 R_2 P}{\gamma_2 + \delta_2} + \frac{\delta_1\gamma_1 R_1{}^2 e^{-rt}}{2(\gamma_1 + \delta_1)})\} e^{-\lambda_1 p_1 - \lambda_2 p_2} = 0$$

In order to satisfy , following two equations must be hold

$$b'(t) + \frac{a_2\lambda_2(a_2\lambda_2 k(t)e^{rt} - \delta_2 R_2)}{\gamma_2 + \delta_2} b(t) + \frac{a_2\lambda_2 P e^{rt}(\delta_2 R_2 - a_2\lambda_2 k(t))}{\gamma_2 + \delta_2} = 0$$

$$k'(t) + (\frac{a_2{}^2\lambda_2{}^2 e^{rt}}{(\gamma_2 + \delta_2)} + \frac{a_1{}^2\lambda_1{}^2 e^{rt}}{2(\gamma_1 + \delta_1)}) k^2(t) + (\frac{a_2\lambda_2(Pa_2\lambda_2 e^{rt} - \delta_2 R_2)}{\gamma_2 + \delta_2} - \frac{a_1\lambda_1\delta_1 R_1}{\gamma_1 + \delta_1}) k(t)$$
$$- (\frac{a_2\lambda_2\delta_2 R_2 P}{\gamma_2 + \delta_2} + \frac{\delta_1\gamma_1 R_1{}^2 e^{-rt}}{2(\gamma_1 + \delta_1)}) = 0$$

But, equation is NP-complete and we cannot solve it directly.