MULTI-DIMENSIONAL VIBRATION POWER PATH ANALYSIS WITH ROTATIONAL TERMS INCLUDED : APPLICATION TO A COMPRESSOR

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Abstract

Vibration generated by an excitation source is in general transmitted to a receiver through multiple paths, i.e., multiple points in multiple directions. Therefore, a systematic approach is required to achieve the vibration isolation in the most effective way. The method of vibration power provides a good solution for this purpose. The reason is that the power transmission along each path can be easily represented as a ratio to the total power transmission and furthermore the vibration power transmitted into the receiver structure is closely linked to the noise radiation from the structure over a high frequency range.

The idea of the multi-dimensional vibration power path analysis is simple to understand and formulate but rather complicated to apply in practice. For an accurate estimation of the power flow especially over a high frequency range, rotational motions should be taken into consideration together with translational motions at the points on the receiver structure where isolators are mounted. In practice, however, the power transmissions related to the rotational terms are often neglected mainly due to difficulties in instrumentation. In this paper, formulas and mechanical quantities necessary for the path analysis with the rotational terms included based on the vibration power will be reviewed and some experimental results for a commercial air conditioner compressor system will be shown.

1. INTRODUCTION

Various types of isolation techniques were developed in the literature to reduce vibration/force transmitted onto receiver structures from excitation sources. Regarding noise radiations in the high frequency range from flexible receiver structures, the concept of power flow was employed[1]. Although, however, the mathematical formulations to estimate the vibration power transmitted from the vibratory machine to the receiver structures have been almost completed, their applications to practical systems still encounter difficulties mainly due to instrumental limitations especially in rotational motions. It is known that contribution of the power transmission by the rotational terms to the total power should be taken into consideration as the frequency range of interest goes up.

Koh and White showed that the coupling mobility between the translational and the rotational terms can give rise to cancellation of the vibration power and illustrated that the vibration power can be minimized by adjusting the moment excitation[2].

A recent research by Swanson showed that both over- and under- estimation of the power transmission can be significant especially in the high frequency range when rotational stiffness of the isolator is not taken into account[3]. Moorhouse and Gibbs estimated the rotational power transmission for compressors in situ by employing two matched accelerometers at a given connection point[4]. Petersson claimed that the moment becomes important even at low frequencies when translational motion is constrained in the proximity of a discontinuity[5]. Lee and Kim showed the effects of the rotational terms by the decomposition of total power flow into the power transmissions related to pure translational motions, pure rotational motions and coupling between translational and rotational motions respectively [6].

In this paper, mathematical formulas necessary for the vibration power approximation with the rotational terms included will be reviewed in several different forms according to mechanical and measurable quantities in practical situations. In addition, we will apply the concept of power flow to investigate the vibration transmission path analysis for an air conditioner compressor system. Some of the experimental results will be shown as well.

2.REPRESENTATION OF POWER PLOW IN A MULTI-DIMENSIONAL VIBRATION ISOLATION SYSTEM

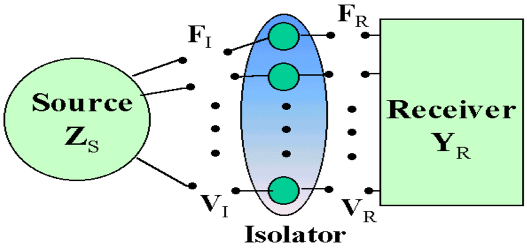

Fig. 1 Schematic representation of multi-dimensional vibration isolation system

We first consider a multi-dimensional and multi-point mounted isolation system that is in general represented by a source, isolators, and a receiver as shown schematically in Fig. 1. A time-averaged vibration power transmitted from the source onto the receiver is expressed by

$$P(\omega) = \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{F}_R^H(\omega)\mathbf{V}_R(\omega)\right\} \quad \text{(1-a)},$$
$$= \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{V}_R^H(\omega)\mathbf{F}_R(\omega)\right\} \quad \text{(1-b)}.$$

where $\mathbf{F}_R$ and $\mathbf{V}_R$ are force and velocity vectors at the connection points on the receiver respectively and the superscription, H, represents hermitian matrix.

Using a receiver mobility($\mathbf{Y}_R$) or impedance($\mathbf{Z}_R$), the vibration power can also be represented as follows[6]:

$$P(\omega) = \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{F}_R^H(\omega)\mathbf{Y}_R(\omega)\mathbf{F}_R(\omega)\right\} \quad \text{(2-a)},$$
$$= \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{F}_R^H(\omega)\mathbf{Y}_R^H(\omega)\mathbf{F}_R(\omega)\right\} \quad \text{(2-b)},$$
$$= \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{V}_R^H(\omega)\mathbf{Z}_R^H(\omega)\mathbf{V}_R(\omega)\right\} \quad \text{(2-c)},$$
$$= \tfrac{1}{2}\mathrm{Re}\left\{\mathbf{V}_R^H(\omega)\mathbf{Z}_R(\omega)\mathbf{V}_R(\omega)\right\} \quad \text{(2-d)}.$$

When six(three-translational and three-rotational)-DOF are considered at each of the *n* connection points, the force $\mathbf{F}_R$ and velocity $\mathbf{V}_R$ are 6*n* x 1 vectors and $\mathbf{Y}_R$ and $\mathbf{Z}_R$, 6*n* x 6*n* matrices. In addition, it can be said that the vibration power is transmitted to the receiver through 6*n* paths. The total vibration power transmission to the receiver can be represented by the sum of each of the power transmissions through the 6*n* paths as follows:

$$P(\omega) = \sum_{i=1}^{6n} P_i(\omega) = \sum_{i=1}^{6n} \frac{1}{2}\mathrm{Re}\{F_i^*(\omega)V_i(\omega)\}$$
$$= \sum_{i=1}^{6n} \frac{1}{2}\mathrm{Re}\{V_i^*(\omega)F_i(\omega)\} \quad (3),$$

where * denotes the complex conjugate. Using Eqs. (2-a, b, c, d), the power transmission respectively through the i-th path can be expressed by

$$P_i(\omega) = \frac{1}{2}\mathrm{Re}\{F_i^* \sum_{j=1}^{6n} Y_{ij}F_j\} \quad \text{(4-a)},$$
$$= \frac{1}{2}\mathrm{Re}\{(\sum_{j=1}^{6n} F_j^* Y_{ij}^*)F_i\} \quad \text{(4-b)},$$
$$= \frac{1}{2}\mathrm{Re}\{(\sum_{j=1}^{6n} V_j^* Z_{ij}^*)V_i\} \quad \text{(4-c)},$$
$$= \frac{1}{2}\mathrm{Re}\{V_i^* \sum_{j=1}^{6n} Z_{ij}V_j\} \quad \text{(4-d)}.$$

3. APPLICATION TO COMPRESSOR

We consider here the outdoor compressor mounting system of an air conditioner. Vibration generated in the compressor is transmitted to the case structure along several paths and then gives rise to sound radiation from the case. We first define the vibration transmission paths and investigate the contribution of each path to the total vibration power transmission by employing the vibration power method based on Eq. (2-c).

The major vibration transmission paths in the compressor system can be represented by compressor mounting points, suction pipe and discharge pipe as schematically drawn in Fig. 2.

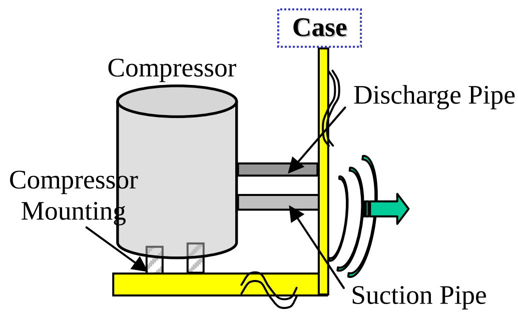

Fig. 2 Vibration power transmission paths in compressor system

3.1 Measurement points and directions

The compressor is mounted at 3 points on the base pan of the case. Therefore, it can be said that the power transmission through the mounting points is the sum of the power transmissions through each of the mounting points. Furthermore, only 3-DOF(one-translational and two-rotational) out of 6-DOF at each mounting point for the power approximations are considered as shown in Fig. 3

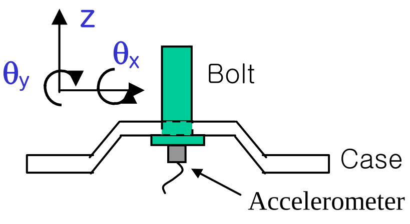

Fig. 3 Degree of freedoms in compressor mounting point for power approximation

Vibration transmission paths from the compressor to the case through pipes are shown schematically in Fig. 4.

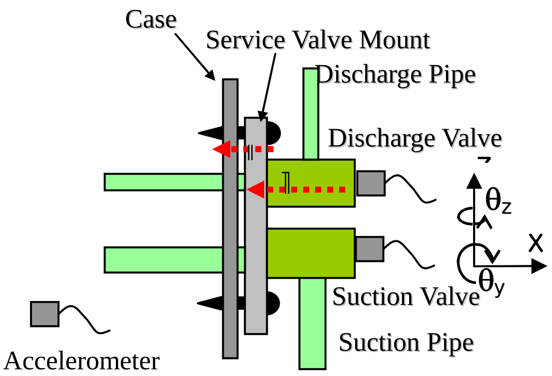

Fig. 4 Vibration power transmission paths through service valve mount

Vibration generated in the compressor is transmitted to the suction/discharge valves fixed to the service valve mount through the suction/discharge pipes. Then, the vibration induced in the suction/discharge valve is transmitted to the case through the service valve mount. Therefore, two vibration transmission paths near the service valve mount can be taken into consideration. One is the vibration power flow from the suction/discharge valves to the service valve mount indicated as Ⅰ, the other one is from the service valve mount to the case indicated as Ⅱ. The latter, however, makes it difficult to distinguish between the suction pipe path and the discharge pipe path. We hereby estimated the vibration power transmission from the suction/discharge valves to the service valve mount. In addition, we also considered 3-DOF (one-translational and two-rotational) at each of the suction/discharge valves for the power approximation as shown in Fig. 4.

3.2 Formulation for power approximations

As mentioned earlier, the power transmissions are approximated based on Eq. (2-c), which means that translational and rotational velocities at the connection points of the receiver under compressor's operating condition and the receiver impedance are required to estimate the power approximations. In this study, impedance matrix is obtained from taking inverse of the receiver mobility matrix.

Because 3-DOF at each of the 5 connection points(including 3 compressor mounting points) are taken into account, velocity $\mathbf{V}_R$ in Eq. (2-c) becomes 15 x 1 vector expressed as

$$V_R = [V_{1z} \quad V_{2z} \quad V_{3z} \quad V_{sx} \quad V_{dx} \quad \Omega_{1x} \quad \Omega_{2x} \quad \Omega_{3x} \quad \Omega_{sy} \quad \Omega_{dy} \quad \Omega_{1y} \quad \Omega_{2y} \quad \Omega_{3y} \quad \Omega_{sz} \quad \Omega_{dz}] \qquad (5),$$

where the subscript, i, denotes the i-th compressor mounting point, s, suction valve and d, discharge valve.

By using Eq. (5) and 15 x 15 impedance matrix of the receiver, Eq.(2-c) can be expressed in a simple notation as shown in Eq. (6),

$$P = \frac{1}{2}\mathrm{Re}\left\{ [V_1^*, \cdots, V_5^*, V_6^*(\Omega_1^*), \cdots, V_{15}^*(\Omega_{10}^*)] \begin{bmatrix} Z^*_{(1,1)} & \cdots & Z^*_{(1,15)} \\ \vdots & \ddots & \vdots \\ Z^*_{(15,1)} & \cdots & Z^*_{(15,15)} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_5 \\ V_6(\Omega_1) \\ \vdots \\ V_{15}(\Omega_{10}) \end{bmatrix} \right\} \qquad (6).$$

We got translational and rotational velocities at each connection point under actual operating condition with accelerometers of PCB 352C66 series and TAP sensor of Kistler 8832A series respectively. Mobility measurement was carried out after the compressor was detached from the outdoor unit, which means the source was removed. Connection points of each path were excited by mini shaker over the frequency range 1-1600Hz. In addition, I-type moment arm was used for moment excitation[7], and an impedance head (B&K8001), for the point mobility measurement.

3.3 Experimental results

First of all, we investigate the effects of the rotational power transmission which is often ignored mainly due to difficulties in instrumentation. When the rotational terms are not measured and/or ignored for simplicity, the velocity vector in Eq. (5) is represented by [6],

$$V_{RT} = [V_{1z} \quad V_{2z} \quad V_{3z} \quad V_{sx} \quad V_{dx} \quad 0 \quad \cdots \quad 0] (7).$$

Using Eqs. (2-c) and (7), the power transmission with the rotational terms excluded can be represented by

$$P_T(\omega) = \frac{1}{2}\mathrm{Re}\left\{\mathbf{V}_{RT}^*(\omega)\mathbf{Z}_R^*(\omega)\mathbf{V}_{RT}(\omega)\right\} \quad (8).$$

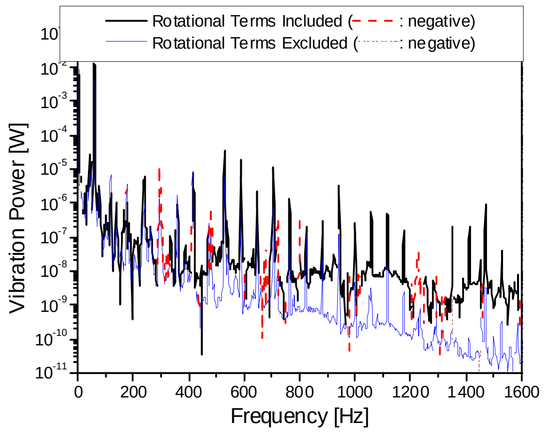

Fig. 5 Total vibration power with rotational terms included/excluded

Fig. 5 shows the total power transmission to the case with the rotational terms included and excluded. It can be seen that neglecting the rotational terms leads to underestimation over high frequency range. Investigation of assembly configuration of the pipes to the service valve mount might explain this phenomenon. That is, the suction/ discharge valves fixed on the service valve mount seems to plays a role of moment arm in vibration transmission. Therefore, it can be concluded that not only translation motions but also rotational motions should be taken into consideration for the vibration approximations through the pipes especially.

Fig. 5 shows negative value in power flow that is obviously erroneous result. There might be many reasons for this phenomenon. First among these is the fact that we didn't consider all the vibration transmission paths, which means that minor paths were ignored. However, it can be thought easily that it is a task of great difficulty to investigate the effects of minor paths on the power approximation in practice.
Another possible reason is the numerical errors in taking the inverse of the mobility matrix with the experimental errors and it can be examined by looking at the condition number of the mobility matrix. Fig. 6 shows the condition number of the mobility matrix. The higher value of condition number here indicates that the error associated with each matrix element can be distorted seriously after taking the inverse of the whole matrix. In order to investigate the effect of these singularity problems, the frequency ranges where negative power flow appears are indicated with arrows in Fig. 6. By comparison of the peaks in condition number and the arrows in Fig. 6, it can be said that negative power at 480 and 800Hz possibly resulted from the errors induced by taking the inverse of the matrix. There must be other physical explanations about the negative power over other frequency ranges.

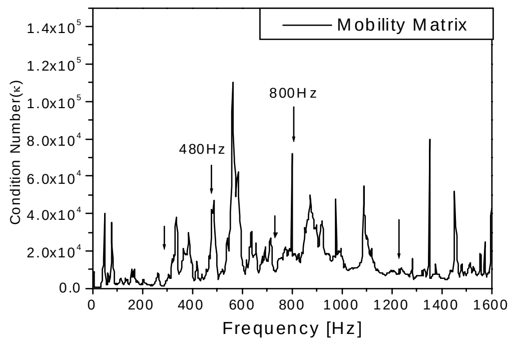

Fig.6 Condition number of receiver mobility matrix

The total power transmission to the case is the sum of the power transmissions through the 3-major paths as shown in Eq. (9).

$$P = P_M + P_S + P_D \qquad (9),$$

where P_M, P_S, and P_D means the power transmission through the compressor mounting, suction pipe, and discharge pipe respectively.

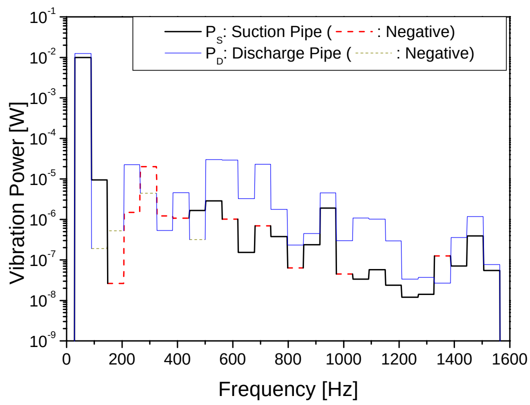

Fig. 7 Vibration power transmission to the case through path suction pipe and discharge pipe

Fig. 7 shows that the vibration power transmissions though the suction pipe and the discharge pipe. It can be seen that the vibration power transmission through the discharge pipe is mainly dominant over the one through the suction pipe over the frequency range of interest. It seems to be caused by the fact that the fluid is discharged much faster in the thinner discharge pipe with the higher pressure build-up arisen from the smaller cross-sectional area.

Negative power flow in Fig. 7 can be interpreted differently from the one in Fig. 5. For example, the negative power in the power transmission through the suction pipe near 550Hz, means that the power transmitted to the case through the discharge pipe is dominant over the other paths, and some power of it is transmitted back to the compressor through the suction pipe. That is, total power transmission to the receiver should be positive, however, the power transmission through a single path/point could be negative for the multi-dimensional vibration isolation system.

The power transmission through each of the paths can be easily represented as a ratio to the total power transmission whose experimental result is shown in Fig. 8.

By the fact that the vibration powers transmitted to the case through the discharge pipe and the mounting points are cancelled out considerably around 1300Hz, it can be said that power cancellations between different connection points or DOF motions should be considered for the effective isolation. That is, the mounting points isolation may give rise to increase of total power transmission around 1300Hz.

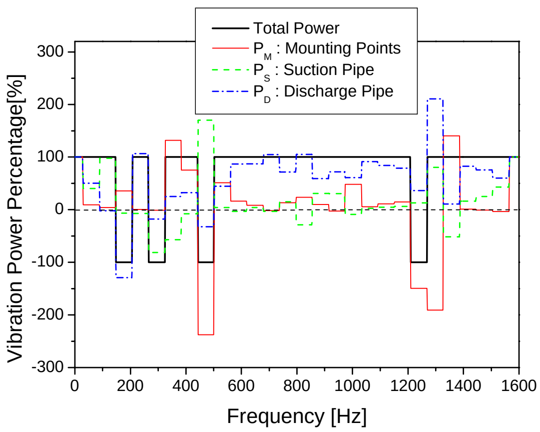

Fig. 8 Percentage of vibration power along each path to total vibration power

It can be seen in Figs. 7 and 8 that the vibration power method makes it possible to investigate the dominant path among the several vibration transmission paths and provides a guide for effective isolation.

4. CONCLUSION

The concept of vibration power has been employed for vibration transmission path analysis of a commercial air conditioner compressor system. We compared the power transmissions through the compressor mounting points, suction pipe and discharge pipe which are considered the major paths. Furthermore, 3(one-translational and two-rotational) DOF at each connection point are considered in order to investigate the effects of the rotational terms.

Experimental results for the compressor system showed that the power approximation could be underestimated especially over a high frequency range when the rotational terms are ignored. By the multi-dimensional vibration power path analysis, it was found that the power transmission through the discharge pipe is dominant over the other paths in general over the frequency range of interest. By analogy with the approach applied to the compressor system, it is feasible to employ the concept of vibration power effectively for the analysis of multi-dimensional vibration isolation systems.

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