

Capacity of Interference-Limited Ad Hoc Networks with Infrastructure Support

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Abstract—In this letter, we consider the capacity of ad hoc networks with infrastructure support. Although Grossglauser-Tse mobile network model enables $\Theta(1)$ per-node throughput scaling, the mobility assumption may be too unrealistic to be accepted in some practical situations. One of the key observations we acquired is that the infrastructure support plays the same role played by the mobility in the Grossglauser-Tse model. We show that nodes can utilize the randomly located infrastructure support instead of mobility when nodes are nearly static. In this case, we show that the per-node throughput of $\Theta(1)$ is still achievable when the number of access points grows linearly with respect to the number of nodes.

Index Terms—Ad hoc wireless network, throughput capacity, infrastructure support, mobility.

I. INTRODUCTION

A key concern in ad hoc networks is the throughput scaling law that may depend on many aspects of networks such as power control, scheduling strategies, routing schemes, network topology, and physical characteristics.

In their seminal paper [1], Gupta and Kumar showed that per-node throughput of $\Theta(1/\sqrt{n \log n})$ is attainable assuming that n number of mobile nodes are placed randomly. Roughly speaking[§], $f(n) = \Theta(g(n))$ means that $f(n)$ and $g(n)$ are of the same rate of growth. This result is rather pessimistic since per-node throughput should decrease very fast as the number of mobile nodes increases. In the subsequent paper [2], Grossglauser and Tse showed that per-node throughput of $\Theta(1)$ is achievable when mobility is fully exploited. Although this result seems to be optimistic at the first glance, in some realistic situations, nodes are nearly static. Consequently, delays experienced by source-destination pairs will be intolerable [3] and the throughput per node becomes very small. However, we still think that it is possible to achieve the capacity of $\Theta(1)$ by exploiting infrastructure support, even if the nodes are static. In this letter, the mobility is considered not as an essential component but as an additional component that can *potentially* increase the throughput of ad hoc networks.

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[§]Formally, $f(n) = \Theta(g(n))$ if there are constants $c_1 > 0$, $c_2 > 0$, n_0 such that $\forall n > n_0$ it is true that $c_1 g(n) \leq f(n) \leq c_2 g(n)$.

Regarding the throughput capacity of ad hoc networks with infrastructure support, there have been several works [4]–[6]. Note that these works adopted the protocol model [1] for their interference model rather than the physical model [2], which is our case. Analytical results based on the protocol model may overestimate the capacity of ad hoc networks since they do not take interference from other users into consideration. This letter reveals that the per-node throughput of $\Theta(1)$ is also attainable even though the physical model is employed.

II. NETWORK MODEL

We consider a random network with infrastructure support where mobility can be potentially exploited. There are two tiers where *infra tier* is an infrastructure network composed of access points and *ad hoc tier* is composed of nodes. All access points and nodes are randomly located in the disk of unit area (of radius $1/\sqrt{\pi}$). To make our model mathematically tractable, the bandwidth between any two access points is assumed to be relatively enough.

There are n nodes in the ad hoc tier. For each *slotted* time t , we randomly designate $n_S = \theta n$ of the nodes as source nodes and the remaining $n_R = (1 - \theta)n$ nodes as destination nodes. The *source density* parameter θ falls within the open interval $(0, 1)$. Similar to [2], a destination node can be also a relay node. Since the designation changes at every time slot, the role of each node is varying. The number of access points n_A is assumed to be proportional to n , i.e., $n_A = \psi n$ where ψ is the *infrastructure density* parameter falling within $[0, 1]$. The ad hoc tier model used in this letter is not very different from the one employed in [2].

In the ad hoc tier, a source node or relay node sends data to other nodes at R bits/s through a single common channel. Each access point has $K (> 0)$ uplink channels where each channel has the bandwidth of R bits/s. We assume the full orthogonality among K channels such that the transmissions occurring within one channel do not interfere with the other channels. We know that downlink traffic may not be evenly distributed to all access points but may be concentrated at a specified access point. For simplicity, we assume that downlink bandwidth is relatively abundant compared with uplink bandwidth.

A. Interference Model

For the interference model, we adopt a *physical* model where the main features of ad hoc networks are specified with the signal power of a node and the interference signal from other nodes. At time slot t , let $P_i(t)$ be the transmission power of node i and $\gamma_{ij}(t)$ be the channel gain from node i

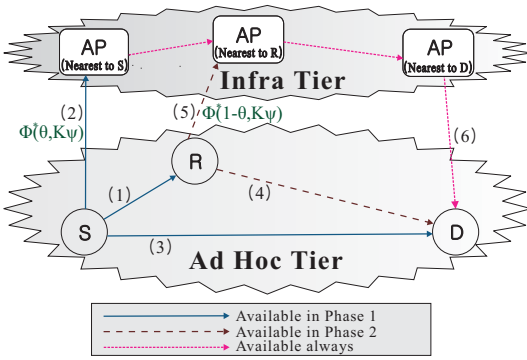


Fig. 1. The proposed scheduling policy.

to node (or access point) j , such that the received power at j is $P_i(t)\gamma_{ij}$. The transmission from i to j at rate R bits/s through channel c is successful if

$$\frac{P_i(t)\gamma_{ij}(t)}{N_0 + \frac{1}{L} \sum_{k \in S_c, k \neq i} P_k(t)\gamma_{kj}(t)} \geq \beta$$

where S_c is the set of senders transmitting through channel c , β is the signal-to-interference ratio (SIR) requirement for successful communication, N_0 is the background noise power, and L is the *processing gain* of the system. The channel gain is assumed to be

$$\gamma_{ij}(t) \stackrel{\text{def}}{=} \frac{1}{|X_i(t) - X_j(t)|^\alpha} \quad (1)$$

where α is the path loss exponent greater than 2 and $\{X_i(t)\}$ is the location of node i at time slot t .

B. Proposed Scheduling Policy

For the moment, we assume that the node location processes $\{X_i(t)\}$ are independent, stationary, and ergodic. The sets of source nodes, destination nodes, access points are denoted by \mathcal{S} , \mathcal{R} and \mathcal{A} , respectively. Destination nodes also play the role of potential receivers, i.e., relay nodes. Thus nodes in \mathcal{R} will be also called *relay nodes*. Let us focus on a certain time slot t and its subsequent time slot $t+1$. There are two phases in scheduling policy η as shown in Fig. 1.

Phase I (Source Nodes Transmit): Each source node S transmits packets to its nearest relay node R through the single common channel (arrow (1) in Fig. 1) and transmits packets to its nearest access point AP through one of K uplink channels (arrow (2) in Fig. 1) simultaneously. With very low probability of $\Theta(1/n)$, the nearest relay node R can be the *true* destination D of the source node (arrow (3) in Fig. 1).

Phase II (Relay Nodes Transmit): Each relay node R transmits packets to its nearest relay node only if the relay node R has packets whose true destination D is the nearest relay node (arrow (4) in Fig. 1). Concurrently, each relay node R transmits packets to its nearest access point AP through K uplink channels (arrow (5) in Fig. 1) in a random manner.

These two phases are interleaved: In the odd time-slots, Phase I is run. In the even time-slots, Phase II is run. Once an access point receives packets, the packets are assumed to be delivered to the access point which is nearest to the true destination within a brief instant. Then the access point transmits them to the true destination (arrow (6) in Fig. 1).

For simplicity, two kinds of mode are considered: In pure ad hoc mode, arrows (1), (3) and (4) are used. In pure infra mode, arrows (1), (2), (5), (6). It is already proven that $\Theta(1)$ per-node throughput is attainable in pure ad hoc mode *with* mobility support [2].

III. CAPACITY IMPROVEMENT WITH INFRASTRUCTURE SUPPORT

Theorem 1 shows that there are $\Theta(n)$ feasible S-D pairs in pure infra mode, that is to say, per-node throughput of $\Theta(1)$ is attainable *without* mobility support.

Theorem 1: For ad hoc networks with scheduling policy η and the identical transmission power of $p \neq O(n^{-\alpha/2})$ ^{†,††}, the expected number $E\{N_t\}$ of feasible source-access point pairs in Phase I is $\Theta(n)$, i.e.,

$$\lim_{n \rightarrow \infty} \frac{E\{N_t\}}{n} = \Phi^*(\theta, K\psi) > 0 \quad (2)$$

where the probability $\Phi^*(\theta, K\psi)$ is defined in Eq. (9). Furthermore, the expected number $E\{N_t\}$ of feasible relay-access point pairs in Phase II is $\Theta(n)$, i.e.,

$$\lim_{n \rightarrow \infty} \frac{E\{N_t\}}{n} = \Phi^*(1 - \theta, K\psi) > 0. \quad (3)$$

(Note that the probability (9) is *independent* of n .)

Proof: We consider a fixed time slot t . Let U_1, \dots, U_{n_S} be the random positions of the sources in \mathcal{S} . Let V_1, \dots, V_k be the positions of access points in \mathcal{A} where $k = \psi n$. These random variables are i.i.d. uniformly distributed on the open disk of unit area. For each node $s \in \mathcal{S}$, let its intended access points $a(s) \in \mathcal{A}$ be the access point that is nearest to s among all access points in \mathcal{A} . Since there are K uplink channels and source node s chooses one channel randomly, without loss of generality, we assume that channel 1 is chosen by s .

We now analyze the probability of successful transmission for each source-access point pair. By symmetry, we can just focus on one such pair, say $(1, a(1))$. The event of successful transmission depends on the positions U_1, \dots, U_{n_S} and V_1, \dots, V_k . Let Q_i be the received power from source node i at access point $a(1)$, and

$$Q_i = p|U_i - V_{a(1)}|^{-\alpha}.$$

The node $a(1)$ satisfies

$$a(1) = \arg \min_{j \in \mathcal{A}} p|U_1 - V_j|.$$

The total interference at access point $a(1)$ is given by $I = \sum_{i \neq 1} Q_i$. The SIR for the transmission from sender 1 at access point $a(1)$ is given by

$$\text{SIR} = \frac{Q_1}{N_0 + \frac{1}{L}I}.$$

Step 1. Finding the Asymptotic Distribution of Q_1

[†]Roughly, $() = (())$ means that $()$ does not grows faster than $()$. Thus, $() \neq (())$ means that $()$ grows faster than $()$.

^{††}For the case $= \Theta(-\alpha/2)$, Eqs. (2) and (3) are also satisfied. We do not deal with this case for brevity.

We now analyze the asymptotics of Q_1 and I as $n \rightarrow \infty$. Because p is independent of j , Q_1 becomes

$$Q_1 = \max_{j \in \mathcal{A}} Z_j$$

where $Z_j = p|U_1 - V_j|^{-\alpha}$. Following an analogous procedure used in [2], one can show that

$$\frac{1}{p(\pi\psi n)^{\alpha/2}} Q_1 \rightarrow Q_\alpha^* \quad (4)$$

where Q_α^* has a cdf

$$F_{Q_\alpha^*}(x) = \begin{cases} \exp(-x^{-2/\alpha}), & x \geq 0 \\ 0, & x < 0. \end{cases}$$

Step 2. Finding the Asymptotic Distribution of I

Since each source randomly chooses a channel among K uplink channels between the source and its nearest access point, the total interference in a channel becomes a random sum of random variables Q_i . Since the probability that a sender chooses channel 1 is $1/K$, the total interference experienced by s becomes $I = \sum_{i \in S_1, i \neq 1} Q_i$ where S_1 denotes the set of sources that transmit data through channel 1. Then the cardinality of S_1 has the following binomial distribution:

$$\Pr\{|S_1| = m\} = \binom{n}{m} \left(\frac{1}{K}\right)^m \left(1 - \frac{1}{K}\right)^{n-m}.$$

To rearrange I into a sum of a deterministic number of random variables, we now derive the generating function $\mathbf{E}\{s^I\}$ of I (See, e.g., [7]). Using the fact that the generating function of a random sum is a compound function [8, pp. 287], $\mathbf{E}\{s^I\}$ becomes

$$\mathbf{E}\{s^I\} = \left(\frac{1}{K} \mathbf{E}\{s^{Q_i}\} + 1 - \frac{1}{K}\right)^n.$$

Therefore, we can regard I as a sum of $n_S - 1$ number of Q_i^+ which has a cdf

$$F_{Q_i^+}(x) = \begin{cases} 1 - \frac{1}{K} + \frac{1}{K} F_{Q_i}(x), & x > 0 \\ 0, & x \leq 0. \end{cases} \quad (5)$$

Following an analogous procedure used in [2], one can show that

$$\frac{1}{p[\pi\Gamma(1-2/\alpha)(\theta n - 1)/K]^{\alpha/2}} I \rightarrow I_\alpha^* \quad (6)$$

where $\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$, and I_α^* is the *stable distribution* defined in [9].

Step 3. Asymptotic Independence of Q_1 and I

Finally, we can easily show that I and Q_1 are asymptotically independent following an analogous procedure used in [2]. Combining this last fact with Eqs. (4) and (6), we get the result on the probability of successful transmission from source node 1 to access point $a(1)$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr\{\text{SIR} \geq \beta\} &= \lim_{n \rightarrow \infty} \Pr\left\{\frac{Q_1}{N_0 + \frac{1}{L}I} \geq \beta\right\} \\ &= \lim_{n \rightarrow \infty} \Pr\left\{\frac{p(\pi\psi n)^{\alpha/2} Q_\alpha^*}{N_0 + \frac{1}{L} p[\pi\Gamma(1-2/\alpha)(\theta n - 1)/K]^{\alpha/2} I_\alpha^*} \geq \beta\right\}. \end{aligned} \quad (7)$$

Furthermore, If $p \neq O(n^{-\alpha/2})$, that is, p grows faster than $n^{-\alpha/2}$ (e.g., p is a constant or $p = \Theta((\log n/n)^{\alpha/2})$), then $n^{\alpha/2}$ terms are canceled out and the Eq. (7) becomes

$$\Pr\left\{\frac{Q_\alpha^*}{I_\alpha^*} \geq \frac{\beta}{L} \left[\frac{\theta}{K\psi} \Gamma\left(1 - \frac{2}{\alpha}\right)\right]^{\alpha/2}\right\} \quad (8)$$

which is independent of n . Thus Eq. (2) is satisfied with

$$\Phi^*(\theta, K\psi) \stackrel{\text{def}}{=} \theta \cdot \Pr\left\{\frac{Q_\alpha^*}{I_\alpha^*} \geq \frac{\beta}{L} \left[\frac{\theta}{K\psi} \Gamma\left(1 - \frac{2}{\alpha}\right)\right]^{\alpha/2}\right\} \quad (9)$$

and the expected number of feasible source-access point pairs is $\Theta(n)$. Similarly, we can verify Eq. (3) easily. ■

The probabilities of successful transmission $\Phi^*(\theta, K\psi)$ and $\Phi^*(\theta, K\psi)$ are independent of n and strictly positive. Thus, the above theorem says that the expected numbers of source-access point pairs and relay-access point pairs grow linearly with respect to the number of nodes n if the number of access points grows linearly with the number of nodes.

Access points in pure infra mode play the same role played by relay nodes in pure ad hoc mode. Thus, similar to [2], we have proved the following theorem.

Theorem 2: In pure infra mode, each node achieves throughput of $\Theta(1)$.

IV. CONCLUSION

We have proved that the per-node throughput of ad hoc networks with infrastructure support is still $\Theta(1)$ based on the physical model, which captures essential characteristics of interference-limited ad hoc networks.

The transmission power requirement that, p can be decreased on the order of $\frac{1}{n^{\alpha/2}}$ as n grows, is somewhat surprising at a first glance. However, note that it is unrealistic since the channel gain γ_{ij} in Eq. (1) is invalid when two nodes are very close [10]. Thus, we note that the adoption of more *accurate physical model* is crucial for establishing more *genuine* per-node throughput in ad hoc wireless networks.

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