Reproduction of a Plane-Wave Sound Field Based on Wave Number Domain Focusing :Comparison with Ambisonics and WFS

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Abstract

Recently a novel method of reproducing a plane wave sound field based on wave number domain focusing is proposed, which makes it possible to generate plane wave propagating to a desired direction in a region by concentrating sound energy on a desired area in wave number domain.
In this paper, we introduce the method briefly, and then compare it with existing methods; Ambisonics and Wave Field Synthesis. Wave number domain focusing has many advantages compared with Ambisonics and WFS on the aspects of reproduction accuracy and the number of required loudspeakers.

Introduction

In the research on control of sound field providing 3-D sound sensation by using loudspeaker array, reproduction of a plane-wave sound field is a fundamental problem because plane-wave gives a feeling that notional sound source exists in the direction plane-wave comes from. Several methods for it have been proposed, among them Ambisonics[1] and WFS(Wave Field Synthesis)[2] are representative.
Ambisonics reproduce the sound field, which is recorded by the sound field microphones, by using circular array of loudspeakers. It can reproduce plane-wave propagating to any direction at the position.
WFS is based on Rayleigh integral, and makes it possible for linear array of loudspeakers to generate wave front which is almost same as that generated by original notional source in a relatively large area.
Recently a novel method[3] based on wave number domain focusing is proposed. This method generates plane-wave propagating to a desired direction in a region by concentrating sound energy on a point in wave number domain.
In this paper, we introduce the method by wave number domain focusing briefly, and then compare it with Ambisonics and WFS respectively by numerical experiment.

Theory: wave number domain focusing

Problem definition

The plane wave is transformed to a delta function in wave number domain. It means that a plane wave can be generated by concentrating sound energy on a point in wave number domain. Therefore, this problem is analogous with focusing problem in spatial domain. For the focusing of sound energy, we apply acoustic brightness control and acoustic contrast control[4].

Focusing in wave number domain: acoustic brightness control & contrast control

Let us consider N loudspeakers fixed in space, and we denote their location as

$\vec{r}_s^{(n)}$ (n=1, ... , N), filter coefficients to them as $\hat{s}(\vec{r}_s^{(n)})$. Denoting transfer function input between the coefficients $\hat{\mathbf{s}} = \left[\hat{s}(\vec{r}_s^{(1)}) \ \cdots \ \hat{s}(\vec{r}_s^{(N)})\right]^T$ and sound pressure in wave number domain $\hat{p}(\vec{k})$ at the position $\vec{k}$ as $\hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_\mathbf{s})$, we can express the sound field in vector form:

$$\hat{p}(\vec{k}) = \hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_s)\hat{\mathbf{s}}\,. \tag{1}$$

Introducing a spatial correlation matrix $\mathbf{R}_{0,k} = \frac{1}{A_{0,k}}\int_{A_{0,k}} \hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_s)^H \hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_s) dA(\vec{k})$ and omitting constants, the spatial average of acoustic potential energy in a region $A_{0,k}$ can be written as a following simpler equation,

$$\begin{aligned} e_{0,k} &= \int_{A_{0,k}} \left|\hat{p}(\vec{k})\right|^2 dA(\vec{k}) \\ &= \hat{\mathbf{s}}^H \left[\int_{A_{0,k}} \hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_s)^H \hat{\mathbf{h}}(\vec{k}|\,\vec{\mathbf{r}}_s) dA(\vec{k})\right]\hat{\mathbf{s}} \\ &= \hat{\mathbf{s}}^H \mathbf{R}_{0,k} \hat{\mathbf{s}} \end{aligned} \tag{2}$$

The acoustic brightness is defined as the ratio of acoustic potential energy in desired bright zone to input power, can be written as follows,

$$\alpha_k = \frac{\hat{\mathbf{s}}^H \mathbf{R}_{b,k} \hat{\mathbf{s}}}{\left|\hat{H}_{0,k}\right|^2 \hat{\mathbf{s}}^H \hat{\mathbf{s}}} \tag{3}$$

where $\hat{H}_{0,k}$ is a normalization constant and $\mathbf{R}_{b,k}$ is a spatial correlation matrix in the bright zone, in this case, the point at which $\vec{k} = \vec{k}_0$ in wave number domain. The solution that maximizes acoustic brightness can be obtained by eigenvalue analysis. So that, equation (3) is rearranged as follows:

$$\mathbf{R}_{b,k}\hat{\mathbf{s}}_\alpha = \alpha_{k,\max}\left|\hat{H}_0\right|^2 \hat{\mathbf{s}}_\alpha\,. \tag{4}$$

Here $\alpha_{k,\max}$ is the maximum eigenvalue and $\hat{\mathbf{s}}_\alpha$ is eigenvector corresponds to $\alpha_{k,\max}$.

On the other hand, the acoustic contrast is defined as the ratio of acoustic potential energy in bright zone to that in dark zone. Therefore it can be written as follows,

$$\beta_k = \frac{\hat{\mathbf{s}}^H \mathbf{R}_{b,k} \hat{\mathbf{s}}}{\hat{\mathbf{s}}^H \mathbf{R}_{d,k} \hat{\mathbf{s}}} \tag{5}$$

where $\mathbf{R}_{d,k}$ is a spatial correlation matrix in the dark zone which is the total area except the bright zone. (In this case, bright zone is so small as a point that total zone can be used instead of dark zone.) This equation can be rearranged as a following eigenvalue problem:

$$\mathbf{R}_{d,k}^{-1}\mathbf{R}_{b,k}\hat{\mathbf{s}}_{\beta} = \beta_{k,\max}\hat{\mathbf{s}}_{\beta} \tag{6}$$

Here $\beta_{k,\max}$ is the maximum eigenvalue and $\hat{\mathbf{s}}_{\beta}$ is eigenvector corresponds to it.

Comparison of wave number domain focusing with ambisonics and WFS

In the reproduction problem, important issues are 'how precisely the method can reproduce original sound field' and 'how large area the method can reproduce' by using as small number of loudspeakers as possible. Thus we compare the error rate in the control zone when the same number of loudspeakers is used for each method. The error rate is defined from difference between the sound pressure values of the original field and of the reproduced field in control zone as follows,

$$\overline{e} = \frac{\sqrt{E\left(\left|\hat{p}_0(\vec{r}) - \hat{p}_r(\vec{r})\right|^2\right)}}{\sqrt{E\left(\left|\hat{p}_r(\vec{r})\right|^2\right)}} \tag{7}$$

where $\hat{p}_0(\vec{r})$ is sound pressure value at the position $\vec{r}$ in original field, $\hat{p}_r(\vec{r})$ in reproduced field, and $E(\ )$ indicates average value.
First we compare wave number domain focusing with Ambisonics and then with WFS.

Computational experiment setting

We consider 2-D plane as shown in Figure 1, where λ is wavelength. The control zone locates in the center of the plane and its size is $L_0\lambda \times L_0\lambda$.
2-D circular array of loudspeakers is used to compare wave number domain focusing with Ambisonics as shown in Figure 2 (left). The loudspeakers stand on the circle which is centered at origin and radius is $r_0\lambda$. While, we use linear array for comparing wave number domain focusing with WFS, the length of the array is $2r_0\lambda$ and it locates in left side of control zone as shown in Figure 2 (right). .

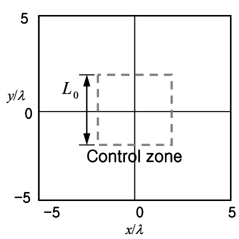


Figure 1. control zone

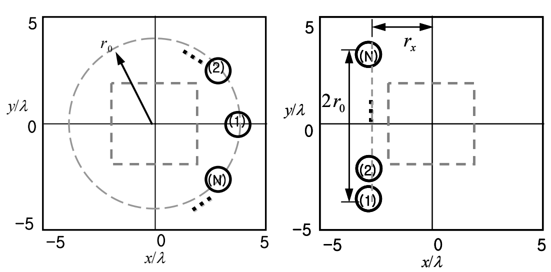


Figure 2. location of loudspeakers,
(left) circular array (right) linear array

The loudspeakers input signal

In Ambisonics, the input signal feeding the i-th loudspeaker can be expressed as[5]:

$$\hat{s}(\vec{r}_s^{\,(i)}) = \frac{1}{N} + \frac{2}{N}\sum_{n=1}^{M}\cos[n(\theta - \phi_i)], \quad \phi_i = (i-1)\frac{2\pi}{N}, \quad i = 1,2,\cdots. \tag{8}$$

Here θ is the direction of propagation of plane wave in original field, M is the sound pick-up order of the spatial encoding, and ϕ_i is the angle indicating location of the loudspeaker.
In WFS, the input signal can be expressed as[2],

$$\hat{s}(\vec{r}_s^{\,(i)}) = \sqrt{\frac{2\pi}{jk}}\sqrt{\frac{\sigma_0}{\rho_0 + \sigma_0}}\left|\vec{r}_s^{\,(i)} - \vec{r}_n\right|^{\frac{1}{2}}\hat{p}(\vec{r}_s^{\,(i)}) \tag{9}$$

where σ_0 is the distance between position of listener and the array line of loudspeakers, and ρ_0 is the distance between notional source and the array line of loudspeakers. The vector $\vec{r}_n$ indicates the position of a notional source.

Comparison wave number domain focusing with ambisonics

Let us assume the original field is a plane wave field. Loudspeakers are considered as plane wave sources and the shape of the array is circular. And we assume that the size of control zone(L_0) is 4λ, the number of loudspeakers(N) is 30, the angle of plane wave for reproducing is $1 \sim 360^\circ$.
The results of reproduction in case of $\theta = 10^\circ$ are in Figure 3~6. All pressure scale is normailized with respect to the value of center position. The error rates with respect to the angle of plane wave are in Figure 7 (left). We can see the error rates of brightness control and contrast control are smaller than those of Ambisonics for the most angles,

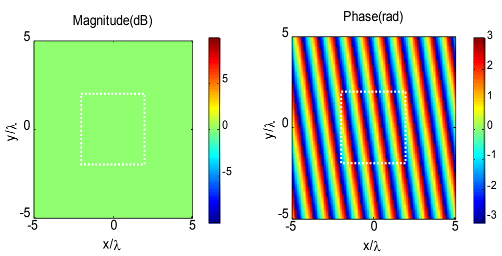


Figure 3. Original Sound Field

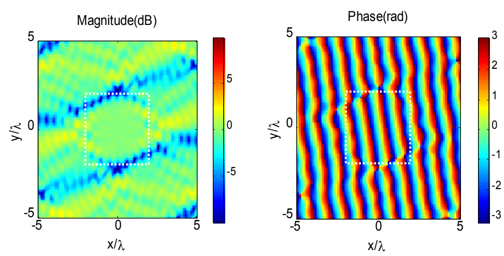


Figure 4. ambisonics (order M=10)

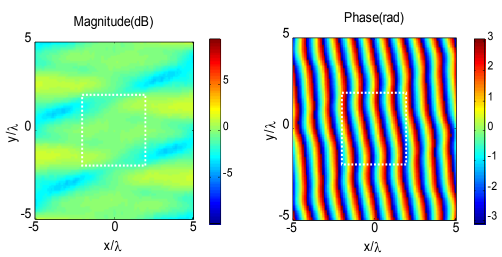


Figure 5. Brightness control

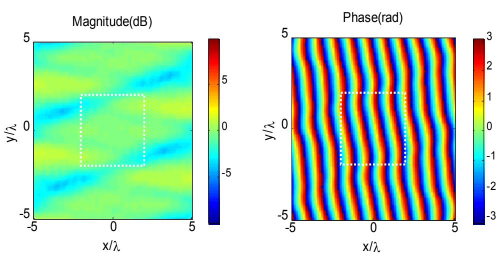
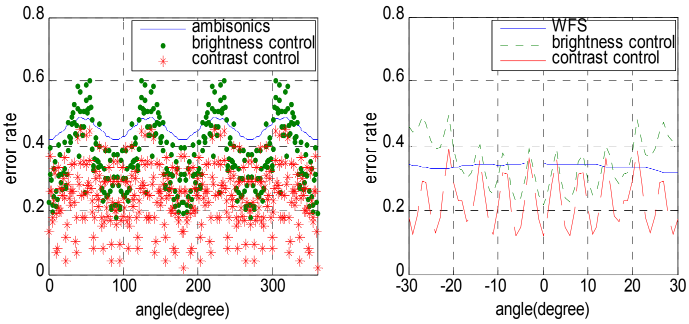


Figure 6. Contrast control



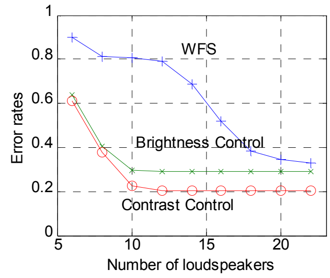


Figure 7. Error rates w.r.t. angle (left)with Ambisonics, (right)WFS

Figure 8. Error rates w.r.t. the number of loudspeakers

therefore wave number domain focusing is more precise.
Practically speaking, it is noteworthy that loudspeakers cannot be plane wave sources and an Ambisonic sound pick-up for order greater than 2 is unfeasible. Thus the error rates of Ambisonics in practical situation would be bigger and controllable zone too small. On the other hand, the error rates of wave number domain focusing wouldn't be much different because it has no assumption for loudspeakers. But a process of measuring transfer function between loudspeakers and control zone is needed.

Comparison wave number domain focusing with WFS

In WFS, the loudspeakers are considered as dipole sources and the array shape is linear. (Circular array in WFS isn't reasonable mathematically because Kirchhoff-Helmholtz integral cannot be derived into the integral composed of only monopoles or dipoles as in Rayleigh integral in circular array case.) We consider original field as a field generated by a notional source that locates at $(-10\cos\theta, -10\sin\theta)$. Let us assume a case that the size of control zone(L_0) is 4λ, the number of loudspeakers(N) is 20, the length of linear array($2r_0$) is 10λ, the distance from the

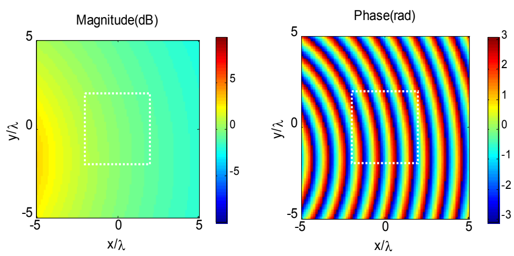


Figure 9. Original Field

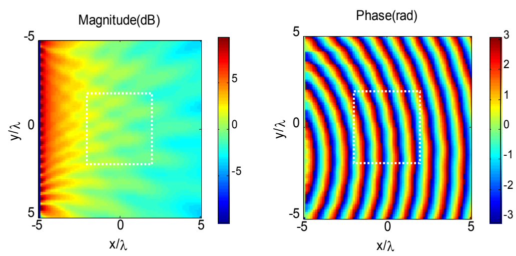


Figure 10. WFS

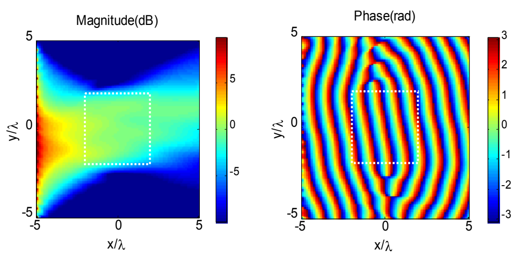


Figure 12: Brightness control

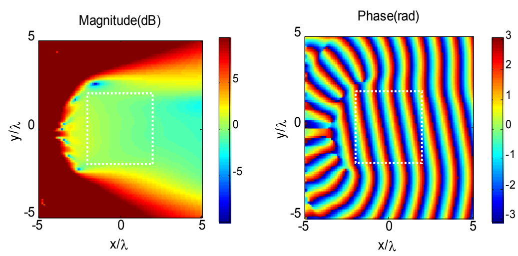


Figure 13: Contrast control

array to center of control zone(r_x) is 5λ , and the angle of plane wave is $-30^\circ \sim 30^\circ$. The results of reproduction in case of $\theta = 10^\circ$ are in Figure 9~12, all pressure scale is normailized with respect to the value of center position. And the error rates of WFS, brightness control and contrast control with respect to angle of propagating are in Figure 7 (right). For the most angles, the error rates of brightness and contrast control are smaller than that of WFS. But strictly speaking, this result doesn't show wave number domain focusing is more precise than WFS because the errors of WFS strongly depend on the position of notional source.

It is noteworthy that wave number focusing needs smaller number of loudspeakers for reproducing the control zone even if controllable zone of WFS is larger than those of wave number focusing. In WFS, as the smaller number of loudspeakers is used, the larger error rates are included because of spatial aliasing as shown in Figure 8. While in wave number focusing, error rates remain in relatively low level even if the number of loudspeakers decreases. In other words, wave number domain focusing has the advantage of reproducing control zone using small number of loudspeakers.

Conclusions

Wave number domain focusing is an effective method to generate plane waves propagating to any direction by focusing sound energy on a point in wave number domain.

And it has been shown that reproduction by wave number domain focusing is more precise than that by Ambisonics for the control zone when circular array of loudspeakers are used.

In addition, it has been shown that the area of reproduction is smaller than that in WFS but wave number domain focusing has the advantage that the number of loudspeakers required to reproduce the control zone is fewer.

Acknowledgements

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