

# **An Active Geophone Sensor with Optimized State Variable Filter for Measuring Low‑Band Frequencies**

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#### **Abstract**

An active vibration-isolation system (AVIS) utilizes a geophone sensor, a type of velocity sensor, to control microvibration. The structure of the sensor is modeled by mass, damper, and spring. The mathematical model of the geophone sensor is a second-order model with a resonant frequency. However, at low-band frequencies, the response characteristic is nonlinear and phase delay occurs. Compared with the ideal velocity signals of the system, the velocity signals measured from the geophone sensor were distorted in low-band frequencies. Consequently, this measurement issue in feedback control loops can afect the stability and performance of the AVIS. This paper proposes design rules for a state-variable flter (SVF) that can compensate for the nonlinearity of the geophone sensors in low-band frequencies and evaluates vibration attenuation performance of the AVIS by applying the proposed SVF. To evaluate the efectiveness of the flter in compensating for the nonlinear response of the geophone sensor, we compared Bode plots generated through simulation and experimental results obtained using a dynamic signal analyzer. The experimental results demonstrated that the proposed SVF efectively reduces the resonance peak of the geophone sensor and expands the frequency bands that maintain a constant magnitude in range of 0.8–10 Hz. By applying the geophone sensor with SVF to AVIS, the microvibration attenuation improved to −18.4 dB near 4.5 Hz.

Keywords State variable filter · Active vibration-isolation system · Geophone sensor · Low-frequency compensation · Vibration measurement

### **1 Introduction**

Active vibration-isolation systems (AVISs) are commonly used to attenuate microvibrations in various felds, including the semiconductor industry, microscopy, and high-precision measurement systems [\[1](#page-10-0)[–4\]](#page-10-1). Although passive vibrationisolation systems (PVIS) do not require sensors or actuators, they cannot guarantee vibration-isolation performance near the resonant frequency. Therefore, AVISs are required to improve the microvibration attenuation performance at the resonant frequency [\[5](#page-10-2)–[9\]](#page-10-3). In particular, the geophone sensor, which have features such as cost efectiveness, light

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weight, simple structure, and high tensile strength has been commonly used as a velocity sensor in AVIS. Ding et al. [\[10\]](#page-10-4) analyzed the signal to noise ratio of geophone sensors used in AVIS and improved performance by dividing and control-ling the frequency bands. Tonoli et al. [\[11](#page-10-5)] specifically analyzed the performance of AVIS which has voice coil motor and geophone sensor based on simulation and experiment. Laro et al. [\[12\]](#page-10-6) analyzed the tilt-horizontal coupling problem of geophone sensors used in AVIS and proposed a solution.

However, the geophone sensor faces problems with lowfrequency responses owing to its mechanical–electrical structure [\[13\]](#page-10-7). The mathematical model of the geophone sensor is a second-order model with a mass, damper, and spring. The zeros are close to origin on the complex s-plane than the poles. Therefore, it has a slope in the low-band frequencies, resulting in distorted velocity signals from the geophone sensor because of nonlinearity. Therefore, the response of the geophone sensor cannot maintain a constant magnitude, resulting in a nonlinear section in the low-frequency band. Therefore, the signal-to-noise ratio deteriorates in a microvibration, making improvement of control

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performance difficult. The resonant frequency of AVIS is designed to be in the low-band frequencies. The control performance degrades in the bandwidth of the feedback control loop because the response at the resonant frequency (lowfrequency section) of AVIS is output smaller than the actual response, thereby afecting the stability and control performance of AVISs with respect to the microvibration attenuation [[14–](#page-10-8)[19\]](#page-11-0). Consequently, the controller operates based on a distorted velocity signal and margin of the designed controller may not be ideal [[20–](#page-11-1)[22](#page-11-2)]. Although these problems are resolved using a low gain, the performance may degrade in AVISs. To improve the signal measured by the sensors, various approaches such as multi-sensor [[23](#page-11-3)[–26](#page-11-4)], sensor-less control [[27](#page-11-5)], signal conditioning circuit [[28\]](#page-11-6), and modifed electromechanical system [[29](#page-11-7)[–31](#page-11-8)], specially designed sensor [[32](#page-11-9), [33\]](#page-11-10) have been proposed. In addition, research on combining a sensor and a flter for maintaining constant-magnitude response has been proposed; however, the method of designing a circuit difers depending on the model of the sensor [\[34](#page-11-11), [35](#page-11-12)].

This study proposes design rules for a state-variable flter (SVF) to enhance the low-band frequencies of geophone sensors in AVISs for high-performance vibration control. The SVF, which is an active flter, comprises a capacitor, resistor and operational amplifer; it employs a state feedback structure that analyzes the response characteristics through state-space modeling for current operational amplifers or enhanced piezoelectric properties, such as signal-tonoise ratio, sensitivity, and maintaining a constant-magnitude response [\[36](#page-11-13), [37\]](#page-11-14). Previous studies have attempted to improve frequency characteristics; however, the AVIS system presented in this study requires a further improved lowfrequency band. An SVF can be represented as a linear combination of a high-pass flter (HPF), bandpass flter (BPF), and low-pass flter (LPF). The signal output of each flter becomes a state variable, resulting in a second-order system. The SVF can adjust the cutoff frequency and damping ratio independently; these can be determined by assuming that the specifc element values used in SVF have the same value and must meet a certain ratio [[38\]](#page-11-15). Additionally, the damping ratio can be adjusted using specifc resistor values. Analog flters typically take into account the coupling issues of circuit component values owing to their response characteristics. Consequently, an SVF offers a more flexible circuit design by adjusting the cutoff frequency and damping ratio independently. By applying SVF to a geophone sensor for resonant frequency reduction and low-frequency amplifcation, new design rules suitable for enhancing the vibrationisolation performance of AVISs are obtained. To evaluate the compensation for nonlinear response of the geophone sensor using SVFs, we compared Bode plots obtained from experimental results with dynamic signal analyzer (DSA). The experimental results demonstrated that the proposed SVF effectively reduces the resonance peak of geophone sensors and expands the frequency bands that maintain a constant magnitude ranging from 0.8 to 10 Hz. By applying the geophone sensor with SVF to an AVIS, the microvibration attenuation improved to  $-18.4$  dB near 4.5 Hz.

The remainder of this paper is organized as follows. In Sect. [2,](#page-1-0) the effect of the compensation range on the plant is evaluated through simulations combining the plant and geophone sensor models. Section [3](#page-6-0) describes the fabrication of an analog circuit design of the SVF based on a mathematical model and evaluates the resonance peak attenuation and extension of the range that maintaining constant-magnitude response. Section [4](#page-7-0) presents the experimental results of microvibration attenuation of the geophone sensor with SVF to validate the improved control performance of the AVIS. Finally, Sect. [5](#page-9-0) concludes the study.

## <span id="page-1-0"></span>**2 Modeling and Simulation of Nonlinear Response Compensation**

## **2.1 Electromechanical Modeling of Geophone Sensor**

The proposed velocity sensor is a geophone sensor GS-11D, which converts velocity into electrical signals. When floor vibrations are transmitted to the geophone sensor surface, an electromotive force is generated inside the coil owing to the interaction between the permanent magnet and coil according to the Faraday's law. The magnitude of the electromotive force represents the linear characteristics of the time derivatives of the magnetic fux density. Subsequently, the magnitude of the velocity was predicted by measuring the voltage using the geophone sensor [\[39](#page-11-16)[–41\]](#page-11-17).

<span id="page-1-2"></span><span id="page-1-1"></span>
$$
m_s \ddot{x}_m + c_s (\dot{x}_m - \dot{x}_g) + k_s (x_m - x_g) + G_S \dot{q} = 0
$$
 (1)

$$
m_s \ddot{x}_s + c_s \dot{x}_s + k_s x_s + G_s \dot{q} = -m_s \ddot{x}_g \tag{2}
$$

Equations [\(1](#page-1-1)) and [\(2](#page-1-2)) represent the mathematical models of the mechanical parts, and Fig. [1](#page-2-0) shows that electromotive force  $G_{\rm s}\dot{q}$  is generated according to the Faraday's law owing to the relative displacement  $x_s = x_m - x_g$  of the floor vibration  $x_g$  and displacement  $x_m$  of the inertia mass  $m_s$ . The relationship between the magnet and the inertia mass can be expressed in terms of the damping coefficient  $c_s$  and spring constant  $k_s$ .  $V_0$  is the magnitude of the electrical output generated by the geophone sensor, and  $q$  is the charge.  $G<sub>s</sub>$  is a constant proportional to the magnetic fux density and has units  $[V/(m/s)]$ . Equation  $(3)$ , which indicates the electrical component, can be obtained using the following secondorder diferential equation:





<span id="page-2-0"></span>**Fig. 1 a** Electromechanical schematic and **b** structure of geophone sensor

$$
L_s \ddot{q} + R_s \dot{q} - G_s \dot{x}_s = 0. \tag{3}
$$

Equations ([2\)](#page-1-2) and ([3\)](#page-2-1) can be expressed using the Laplace transform **L** (assumption that all of the initial condition is zero and variables  $q$ ,  $x_s$ ,  $x_g$ , and  $x_m$  related to time derivatives are changed to  $Q$ ,  $X_s$ ,  $X_g$ , and  $X_m$ ), respectively, as follows:

$$
m_s s^2 X_s + c_s s X_s + k_s X_s + G_s s Q = -m_s s^2 X_g \tag{4}
$$

$$
L_s s^2 Q + R_s s Q - G_s s X_s = 0 \tag{5}
$$

By combining Eqs. ([4](#page-2-2)) and ([5\)](#page-2-3), the output  $X<sub>s</sub>$  can be expressed as Eq. ([6\)](#page-2-4).

$$
m_s s^2 X_s + c_s s X_s + k_s X_s + \frac{G_s^2 s X_s}{L_s s + R_s} = -m_s s^2 X_g \tag{6}
$$

Here, the interaction between the motion of the coil and permanent magnet can be expressed using the Faraday's Law of electromagnetic induction. The geophone has a similar structure to the voice coil transducer, the induced electromotive force of the coil can be expressed as the relative velocity between the coil and the magnet. A voice coil transducer converts electrical and mechanical energy into each other. Let the relative velocity of the coil and the permanent magnet be  $v$ , the external force that maintains equilibrium with the electromagnetic force is  $f$ , the number of turns of coil is *n*, the potential diference between both ends of the coil is  $\varepsilon$ , and the electric current in the coil is *i*. This transducer follows Faraday's law  $\epsilon = 2\pi n r B v = Tv$ . *T* is transducer constant.  $x<sub>s</sub>$  is the relative displacement of the coil and permanent magnet, and  $G<sub>s</sub>$  is the sensitivity of the sensor. Therefore, the induced electromotive force of the coil can be expressed as follow:

$$
V_0 = -G_s s X_s \tag{7}
$$

Equation [\(8](#page-2-5)) is obtained by combining Eqs. [\(6](#page-2-4)) and ([7](#page-2-6)) for the voltage output  $V_0$  with the applied displacement  $X_g$ .  $sX_g$  <span id="page-2-1"></span>is the speed of foor vibration and is the Laplace transform expression of the differential of displacement  $X<sub>g</sub>$ .

$$
\frac{V_0}{sX_g} = \frac{m_s G_s s^2}{m_s s^2 + c_s s + k_s + \frac{G_s^2 s}{L_s s + R_s}}
$$
(8)

<span id="page-2-7"></span><span id="page-2-5"></span>If  $R_s$  is large, Eq. ([8\)](#page-2-5) is reduced to:

<span id="page-2-2"></span>
$$
\frac{V_0}{sX_g} \approx \frac{G_s s^2}{s^2 + \frac{c_s}{m_s} s + \frac{k_s}{m_s}}
$$
(9)

<span id="page-2-3"></span>The transfer function in Eq.  $(9)$  $(9)$  $(9)$  can be expressed as Eq. [\(10](#page-2-8)) by substituting the parameters presented in Table [1.](#page-2-9) The table was flled based on the datasheet provided by the geophone sensor manufacturer and includes the parameters and sensitivities. Additionally, the specifcations of elements were measured at a vibration input of 100 Hz or less.

<span id="page-2-8"></span><span id="page-2-4"></span>
$$
\frac{V_0}{sX_g} = \frac{G_s s^2}{s^2 + \frac{c_s}{m_s} s + \frac{k_s}{m_s}} = \frac{32s^2}{s^2 + 18s + 760}
$$
(10)

The frequency response obtained from the geophone sensor model shown in Fig. [2](#page-3-0) indicates that a resonant frequency is 4.5 Hz and nonlinearity is occurred at lowband frequencies (below 10 Hz). This nonlinear response

<span id="page-2-9"></span>**Table 1** Specifcations of the geophone sensor (GS-11D)

<span id="page-2-6"></span>

Properties	Symbols	Values
Resonant frequency	$rac{1}{2\pi}\sqrt{\frac{k_s}{m_s}}$	$4.5 \pm 0.75$ (Hz)
Damping coefficient	$c_{s}$	$0.34 \pm 20\%$ (Ns/m)
Inertia mass	$m_{\rm s}$	$23.6 \pm 5\%$ (g)
Sensitivity gain	$G_{s}$	$32 \pm 10\%$ (V/(m/s))
DC Coil resistance $@25$ °C	$R_{\rm c}$	$380 \pm 5\%$ ( $\Omega$ )
Maximum coil excursion p-p	max(x <sub>s</sub> )	$2.5 \, \text{(mm)}$
Coil inductance	L,	$50 \pm 5\%$ (mH)



<span id="page-3-0"></span>**Fig. 2** Frequency response of the geophone sensor (GS-11D)

causes errors between the velocity signal measured by the geophone sensor and system's actual behavior, leading to performance degradation and changes in the design margins for the AVIS controller. To mitigate this performance degradation, we propose an SVF to improve the responsiveness in low-band frequencies and verify the designed SVF by applying it to the AVIS. Section [2.2](#page-3-1) describes the plantmodeling process.

#### <span id="page-3-1"></span>**2.2 Modeling of AVIS**

Equations  $(11)$  $(11)$  $(11)$ – $(16)$  $(16)$  $(16)$  present the mathematical model of the AVIS. This plant includes the mass *m* (upper plate and stage), damping coefficient  $c_i(i = x, y, z)$ , spring constant  $k_i$ ( $i = x, y, z$ ), moment of inertia  $i_i$ ( $i = x, y, z$ ), axial displacement of the upper plate  $x_p, y_p, z_p, \theta_{xp}, \theta_{yp}, \theta_{zp}$ , axial displacement of the ground  $x_b$ ,  $y_b$ ,  $z_b$ ,  $\theta_{xb}$ ,  $\theta_{yb}$ ,  $\theta_{zb}$ , input force/moment  $F_i/M_{\theta_i}$  (*i* = *x*, *y*, *z*) and distance between the center of gravity and isolator  $p_i(i = x, y, z)$  can be modeled as a second-order system in Fig. [3](#page-3-4) [\[40](#page-11-18)[–42\]](#page-11-19). The six degrees-of-freedom system was analyzed on the z-axis only to study the performance of the SVF. Table [2](#page-4-0) lists the specifcations of the experimental parts.

$$
F_x(t) = m\ddot{x}_p(t) + c_x(\dot{x}_p(t) - \dot{x}_b(t)) + k_x(x_p(t) - x_b(t)) - c_x p_z \dot{\theta}_{yp}(t) - k_x p_z \theta_{yp}(t)
$$
(11)

$$
F_{y}(t) = m\ddot{y}_{p}(t) + c_{y}(\dot{y}_{p}(t) - \dot{y}_{b}(t)) + k_{y}(y_{p}(t) - y_{b}(t)) - c_{y}p_{z}\dot{\theta}_{xp}(t) - k_{y}p_{z}\theta_{xp}(t)
$$
\n(12)



<span id="page-3-4"></span>**Fig. 3** Modeling for passive vibration-isolation system: **a** X–Z Plane, **b** Y–Z Plane

$$
F_z(t) = m\ddot{z}_p(t) + c_z(\dot{z}_p(t) - \dot{z}_b(t)) + k_z(z_p(t) - z_b(t))
$$
 (13)

$$
M_{\theta_x}(t) = i_{xx} \ddot{\theta}_{xp}(t) + \left(c_z p_y^2 + c_y p_z^2\right) \left(\dot{\theta}_{xp}(t) - \dot{\theta}_{xb}(t)\right) + \left(k_y p_z^2 + k_z p_y^2\right) \left(\theta_{xp}(t) - \theta_{xb}(t)\right) + c_y p_z \dot{y}_p(t) + k_y p_z y_p(t)
$$
(14)

$$
M_{\theta_{y}}(t) = i_{yy}\ddot{\theta}_{yp}(t) + (c_{x}p_{z}^{2} + c_{z}p_{x}^{2})(\dot{\theta}_{yp}(t) - \dot{\theta}_{yb}(t)) + (k_{x}p_{z}^{2} + k_{z}p_{x}^{2})(\theta_{yp}(t) - \theta_{yb}(t)) - c_{x}p_{z}\dot{x}_{p}(t) - k_{x}p_{z}x_{p}(t)
$$
\n(15)

$$
M_{\theta_z}(t) = i_{zz}\ddot{\theta}_{zp}(t) + \left(c_x p_y^2 + c_y p_x^2\right) \left(\dot{\theta}_{zp}(t) - \dot{\theta}_{zb}(t)\right) + \left(k_x p_y^2 + k_y p_x^2\right) \left(\theta_{zp}(t) - \theta_{zb}(t)\right)
$$
(16)

## <span id="page-3-2"></span>**2.3 Simulation Results for Low‑Band Frequencies with Respect to Compensation Range of SVF**

<span id="page-3-3"></span>An inverse transfer function of the geophone sensor for maintaining a constant-magnitude response causes instability, as **Table 2** Specifcations of

<span id="page-4-0"></span>

shown in Eq.  $(10)$ , because zeros exist at the origin of the s-plane. Therefore, a compensation range is required to improve the nonlinearity of the sensor response. However, the compensation range is signifcantly afected by the values of the circuit components. Generally, the capacitances are of the order of a few microfarads; thus, high capacitance and resistance values are impractical. Therefore, limiting the compensation range for low-band frequencies is necessary for using practical capacitance and resistance values. In this study, capacitance and resistance of a compensation range for

<span id="page-4-1"></span>



maintaining a constant-magnitude response was proposed to be less than 1  $\mu$ F and 1 M $\Omega$ . The output error according to the compensation range was calculated using Eq. [\(17\)](#page-5-0). The timedomain responses for each compensation range based on the three cases are shown in Fig. [4](#page-4-1). The plant model represents the ideal output of the plant, whereas the uncompensated model represents a signal that is distorted by including the sensor model in the plant model. The numbers (No.) represent the output of the plant model with the sensor model and SVF with respect to the compensation range listed in Table [3.](#page-5-1) When the compensation range increases, the compensated signal of the plant approximates the ideal signal. The error between the ideal signal and compensated signal of the plant is expressed as follows:

$$
error = \left( \left| \frac{\text{plant model area} - \text{compensated model area}}{\text{plant model area}} \right| \right) \tag{17}
$$

<span id="page-5-0"></span>Here, the transfer function  $(TF_{comp})$  of SVF model should satisfy the following three conditions:  $TF_{comp \ s\rightarrow\infty} \approx 0 \text{ dB}$ , locations of zero at 4.5 Hz, and a second-order system. In this study, compensation range No. 3 was selected, which matched about 92% of the ideal signal of the plant, and a second-order compensation flter that satisfed the slope and magnitude conditions was chosen. The Bode plot of the proposed SVF that satisfes the previously mentioned conditions is the same as that of No. 3, and the transfer function is given by Eq.  $(18)$  $(18)$  $(18)$ .

<span id="page-5-1"></span>



<span id="page-5-2"></span>**Fig. 5** Block diagram of the proposed SVF for expanding the range of constant-magnitude response of geophone sensor

<span id="page-6-2"></span>



<span id="page-6-3"></span>**Fig. 6** Bode plots of state variable of the proposed SVF

$$
TF_{comp} = -\frac{0.329s^2 + 8.072s + 280.5}{0.329s^2 + 1.997s + 6.325}
$$
(18)

## <span id="page-6-0"></span>**3 Analog Circuit Design for the SVF**

The proposed SVF satisfes a linear combination of HPF, BPF, and LPF. Therefore, the overall transfer function can be expressed as a linear combination of the partial transfer functions, and each output can be expressed as a state variable. The total output was determined using a linear combination. Based on the state space model, a block diagram of the SVF is shown in Fig. [5](#page-5-2). The overall transfer function satisfes Eq. ([18](#page-6-1)), with the specifcations listed in Table [4](#page-6-2). Furthermore, the frequency response of each state variable can be represented as shown in Fig. [6](#page-6-3) and defned by Eqs. ([19](#page-7-1))–([21\)](#page-7-2).

<span id="page-6-1"></span>The proposed analog circuit is illustrated in Fig. [7](#page-6-4). The transfer functions of the circuit's state variables, which are expressed in Eqs.  $(19)$ – $(21)$  $(21)$  $(21)$ , are functions of resistor and capacitor. The parameters listed in Table [4](#page-6-2) were used to determine the transfer functions.



<span id="page-6-4"></span>**Fig. 7** Active analog circuit corresponding to Eq. ([12](#page-3-3))



<span id="page-7-3"></span>**Fig. 8** Photographs of the manufactured SVF

> **Experimental result of SVF with DSA** Simulation model of SVF **4 Experimental Result of Microvibration**  Compensated sensor mo **Control Based on Compensated Geophone**   $-Ilnc$

> > <span id="page-7-1"></span><span id="page-7-0"></span>**Sensor by SVF**

<span id="page-7-4"></span>**Fig. 9** Bode plots of the geophone sensor with SVF compensating for the low-band frequencies

<span id="page-7-2"></span>The experimental setup for the AVIS performance evaluation is shown in Figs. [10](#page-7-5) and [11](#page-8-0) [[43](#page-11-20)]. The control algorithm applied to the control block diagram in Fig. [12](#page-8-1) and

An analog circuit based on Fig. [7](#page-6-4) was manufactured, as shown in Fig. [8](#page-7-3). The frequency response of the SVF was measured using an Agilent DSA 35670A, as shown in Fig. [9.](#page-7-4) As a result, the proposed flter design efectively reduced the resonance peak of the geophone sensors and expanded constant-magnitude frequency bands from 0.8 to 10 Hz.

$$
HPF = -R_1 R_3 R_5 \frac{\left(R_6 C_2 s + 1\right) \left(R_4 C_1 s + 1\right)}{R_1 R_3 R_5 \left(R_4 R_6 C_1 C_2 s^2 + \left(R_6 C_2 + R_4 C_1\right) s + 1\right) + R_2 R_4 R_6} = -\frac{0.329 s^2 + 1.997 s + 3.025}{0.329 s^2 + 1.997 s + 6.325}
$$
(19)

$$
BPF = -R_1R_4R_5 \frac{R_6C_2s + 1}{R_1R_3R_5(R_4R_6C_1C_2s^2 + (R_6C_2 + R_4C_1)s + 1) + R_2R_4R_6} = -\frac{6.075s + 27.5}{0.329s^2 + 1.997s + 6.325}
$$
(20)

$$
LPF = -R_1 R_4 R_6 \frac{1}{R_1 R_3 R_5 (R_4 R_6 C_1 C_2 s^2 + (R_6 C_2 + R_4 C_1) s + 1) + R_2 R_4 R_6} = -\frac{250}{0.329 s^2 + 1.997 s + 6.325}
$$
(21)

<span id="page-7-5"></span>





Resonant frequency of Sensor (4.5Hz)





<span id="page-8-0"></span>**Fig. 11** Experimental setup for active vibration isolation

<span id="page-8-1"></span>

LQR

y: Plant output

**LQG** 

her Kant

<span id="page-9-1"></span>

Table [5](#page-9-1) can be represented as a linear-quadratic Gaussian (LQG) function that consists of a linear-quadratic estimator (LQE) for estimating state variables and linearquadratic regulator (LQR) for regulating control inputs [[44](#page-11-21)–[49](#page-12-0)]. Model updates are required depending on the inertia change due to moving stage. The LQG is the control algorithm based on the state space. It is easier to update the model than the traditional error-based PID controller. In this paper, the adaptive LQG controller is applied for model update. The second-order diferential equation in Eqs.  $(11)$ – $(16)$  is the system matrix *A*; *B* represents the input matrix; *C* represents the output matrix as shown in Fig. [12](#page-8-1). Therefore, matrices *A*, *B*, and *C* can be represented in a state space. The state variables were updated using the *L* gain, and the estimated state variables were fed back to control the system through the *K* gain. Characteristics of the observer and regulator for the *L*, *K*-matrix are obtained by solving the Riccati equation. Evidently, the resonance frequency  $(w_n)$  increased by three to four times and the damping ratio  $(\zeta)$  also increased compared with the characteristics of the existing plant. Resonance frequency and damping ratio are related to the control bandwidth and attenuation of peak resonance. As a result, the proposed AVIS means 3–4 times faster control bandwidth and peak reduction compared with the existing PIVS. The objective of this study was to enhance the control performance by expanding the low-band frequencies and reducing the peak resonant frequency of the geophone sensor. Because the sensor output is calculated to update the state variables in the LQE, using the uncompensated sensor output as the feedback signal for the controller causes errors in the estimator. The attenuation performance can be improved using a compensated sensor output to control the system. In this section, we compare the attenuation performance of the geophone sensors with and without SVF. By applying the geophone sensor with SVF to the AVIS, the microvibration attenuation was improved to 18.4 dB near 4.5 Hz as shown in Fig. [13](#page-9-2).

## <span id="page-9-0"></span>**5 Conclusion**

In general, the resonant frequency of an AVIS is designed to attenuate foor vibrations at low frequencies, and a geophone sensor is used to measure the velocity. Geophone sensors have the advantages of being lightweight, small, low-cost, and easy to maintain owing to their simple structure. However, when used in the AVIS, the attenuation performance degrades at the resonant frequency of the geophone sensor. These problems are caused by nonlinearity in the range of low-band frequencies.

This study designed a simulation model of an SVF to improve the performance of the low-band frequencies of geophone sensors in an AVIS. The experimental results

<span id="page-9-2"></span>**Fig. 13** Performance-result comparison of various vibration-isolation systems: passive, active, and compensated active system with the proposed SVF



demonstrated that the proposed SVF efectively reduced the resonance peak of the geophone sensors and expanded the range of constant-magnitude response from 0.8 to 10 Hz. Subsequently, the linear response range increased, and the proposed SVF was able to reduce the peak near 4.5 Hz by 18.4 dB in the AVIS. The proposed design rules enabled independent designs of the HPF, BPF, LPF, phase inversion, and linear combination. Therefore, it can also be applied to increase the linear response range of a pressure or force sensor that uses the piezoelectric efect with a resonant frequency in the high-frequency range. Consequently, the usefulness and practicality of this study was obtained in the form of flter design rules optimized for the response characteristics of various types of sensors.

Future research will be to optimize the values of the circuit elements of the SVF. In this paper, the transfer function and circuit elements were matched as Eqs. [\(19\)](#page-7-1)–([21](#page-7-2)). Therefore, it is not difficult to obtain the transfer function and circuit elements of SVF for application to various sensors. However, there are realistic limitations to the range of values of circuit elements, and determining appropriate values of circuit elements is time-consuming. It is necessary to develop an algorithm to fnd the optimal values of SVF's circuit elements. If we will propose the optimization algorithm, the optimized values which have maximized fat frequency range can be proposed when limited range of values of circuit elements. As a result, the design of optimized SVFs will be automated and less time consuming.

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