Extended Layout of Stiffeners to Increase Fundamental Frequency of Shell Structures

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Abstract

Structural Dynamics Modification (SDM) is to improve dynamic characteristics of a structure, more specifically of a base structure, by adding or deleting auxiliary (modifying) structures. In this paper, the focus will be concentrated on the optimal layout of the stiffeners which are attached to the plate to maximize 1st natural frequency. Recently, a new topology method was proposed by yamazaki. He uses growing and branching tree model. In this paper, to overcome limitations of the method, modified tree model will be suggested. To expand the layout of stiffeners, non-matching problem will be considered. The problem is solved by using local lagrange multiplier without the mesh regeneration. Moreover CMS(Component mode synthesis) method is employed to reduce the computing time of eigen reanalysis using reduced component models.

1. Introduction

Structural dynamics modification(SDM) is a method to improve dynamic characteristics of a base structure. There are mainly three ways of adding or deleting auxiliary structures, changing material property of some parts of whole structure and transforming the shape of the structure without variation of weight. Above all, adding stiffeners such as beams to shell structures is widely used because of convenience and comfort. In this research, the target system is confined to the shell plate and SDM will be performed through adding stiffeners.

Many people have studied on topology optimization of stiffeners to improve dynamic characteristics of shell structures. Existing methods perform SDM using stiffeners having fixed shapes such as straight line and ‘L’. This restriction makes a limited layout of stiffeners. Therefore research on a method to obtain extended layout of stiffeners is needed.

Recently, Yamazaki proposed growing and branching tree model and applied it to the optimization of extended layout of stiffeners[1]. However the method deals with problems that nodes of plate and stiffeners match each other. Thus, strictly speaking, it has also a limitation that the topology of stiffeners is restricted. To obtain extended stiffeners layout in a broad sense, this paper suggests modified tree model considering non-matching problem. In previous researches, mesh regeneration has been accomplished every time nodes of base structure and stiffeners are combined inconsistently. The iterative mesh regeneration increases the computing time. To solve the problem, connection method using local lagrange multiplier proposed by Park[2] is used. That method has an advantage of combining structures without mesh regeneration.

2. Topology optimization

2.1 modified tree model

Yamazaki suggested growing and branching tree model to increase fundamental frequency of a plate as adding stiffeners. Although the tree model is efficient to perform SDM, it has several limitations as mentioned in introduction. In this chapter, modified tree model is proposed to obtain extended stiffener layout through complementing those restrictions. The modified tree model is composed of main three steps below.

▪ Step 1 : initialization
  - positions and the number of seeds (the starting point which stiffeners are placed)
▪ Step 2 : optimization
  - eigen sensitivity is performed
  - determine where the next stiffeners are created
▪ Step 3 : growth
  - new stiffeners are created based on step 2

In step 3, the next stiffeners can grow at the end points of stiffeners created at the previous iteration. Step 2 and 3 are repeated until the total volume of stiffeners reaches the given amount.

2.2 eigen sensitivity

To determine the growing direction of stiffeners, eigen sensitivity is used. It is obtained by equation (1). The design parameter θ is an angle between a plate and each stiffener.

$$\frac{\partial(1/\omega_1^2)}{\partial\theta_j} = u_1^T\left(\frac{1}{\omega_1^2}\frac{\partial M}{\partial\theta_j} - \frac{1}{\omega_1^4}\frac{\partial K}{\partial\theta_j}\right)u_1 \qquad (1)$$

Where ω_1 is 1st natural frequency of base structure. M and K is mass and stiffness matrix. u_1 is a first mode shape.

3. Synthesis of structures having non-matching nodes

3.1 Need for interface frame

In past time, when there are non-matching nodes between base structure and stiffeners, regeneration of mesh of the structures was performed to agree the nodes each other. However, it took much computing time. To cope with the problem, synthesizing non-matched structures using lagrange multiplier will be employed. The details are discussed in [2]. To apply the method, interface frame have to be determined firstly. If the interface frame is determined, the synthesis is performed through combining not directly substructures each other but the nodes of substructures and interface frame at the connection area as you see in figure 1.

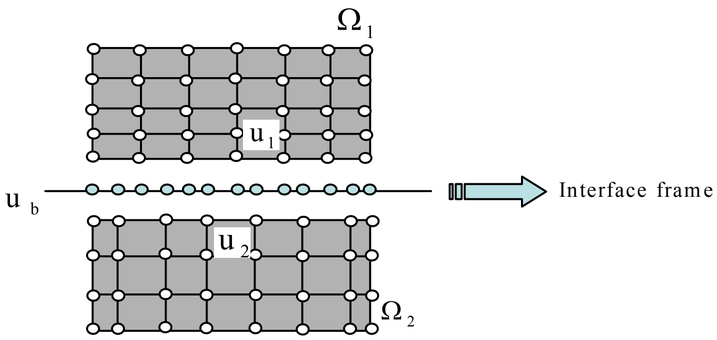

Figure 1. Connection of three substructures

3.2 eigen analysis of non-matching structure

Using substructures, we can set up Hamiltonian equation as equation (2).

$$\delta\int(T-V+\Pi_\lambda)dt=0,\ T=\sum_{s=1}^{n_s}T_s,\ V=\sum_{s=1}^{n_s}V_s \qquad (2)$$

Where $\Pi_\lambda=\sum_{s=1}^{n_s}\int_{\Gamma_s}\lambda_s(u_s-u_b)d\Gamma_s$

Here, T is kinetic energy, V is potential energy, u_b is displacement of interface frame, λ local lagrange multiplier and Π_λ constraint that displacement of interface frame and substructures. Through above Hamiltonian equation, we can obtain equation of motion of synthesized structure. The equation of motion is represented such as (3).

$$\begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{0} & -\mathbf{C}_1^T & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 & \mathbf{0} & \mathbf{0} & -\mathbf{C}_2^T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (\mathbf{L}_{b1})^T & (\mathbf{L}_{b2})^T \\ -\mathbf{C}_1 & \mathbf{0} & \mathbf{L}_{b1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{C}_2 & \mathbf{L}_{b2} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_b \\ \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{Bmatrix} = \omega^2 \begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_b \\ \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{Bmatrix} \qquad (3)$$

Where, M and K means mass and stiffness matrix, **C** is a matrix extracting DOFs corresponding to a structure connected with interface frame. L_b is a matrix connecting interface frame with substructures.

In addition, if we define[4]

$$\mathbf{B}=\begin{bmatrix} -\mathbf{C}_1 & \mathbf{0} & \mathbf{L}_{b1} \\ \mathbf{0} & -\mathbf{C}_2 & \mathbf{L}_{b2} \end{bmatrix},\ \mathbf{Q}=\mathbf{I}-\mathbf{B}^T(\mathbf{B}\mathbf{B}^T)^{-1}\mathbf{B}$$

$$\mathbf{M}=\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},\ \mathbf{K}=\begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},\ \mathbf{u}=\begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_b \end{Bmatrix},\ \boldsymbol{\lambda}=\begin{Bmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{Bmatrix}$$

The equation (3) is defined as a reduced eigen problem.

$$(\mathbf{B}^T\mathbf{B}+\mathbf{Q}^T\mathbf{K}\mathbf{Q})\mathbf{u}=\boldsymbol{\omega}^2\mathbf{Q}^T\mathbf{M}\mathbf{Q}\mathbf{u} \qquad (4)$$

4. component mode synthesis

4.1 reduced model

The method proposed in this paper has a drawback that it takes a little bit computing time due to optimization process. The optimization process is needed to determine where the next stiffeners come up. However, because the system matrix is large, it takes much computing time to take an eigen analysis. To reduce the time, component mode synthesis method is used[3]. Because it needs only the DOFs corresponding to connecting area of substructures and modal coordinates with same numbers corresponding to available mode shapes, it can perform eigen analysis faster.

Substructures can be expressed as a partitioned form.

$$\begin{bmatrix} M_{ii} & M_{ic} \\ M_{ci} & M_{cc} \end{bmatrix} \begin{Bmatrix} \ddot{u}_i \\ \ddot{u}_c \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ic} \\ K_{ci} & K_{cc} \end{bmatrix} \begin{Bmatrix} u_i \\ u_c \end{Bmatrix} = \begin{Bmatrix} f_i \\ f_c \end{Bmatrix} \quad (5)$$

Where i indicates interior coordinates, and c indicates connection coordinates.

Assuming now that the connection coordinates are fixed, $\{u_c\} = \{0\}$, and that no external forces are acting at the interior DOFs, $\{f_i\} = \{0\}$, the corresponding equation of motion becomes

$$[M_{ii}]\{\ddot{u}_i\} + [K_{ii}]\{u_i\} = \{0\} \quad (6)$$

From (6), each interior DOF displacement can now be approximated by a linear combination of mass-normalized eigen vectors and modal coordinates.

$$\{u_i\} = [\phi_{im}]\{p_m\} \quad (7)$$

In addition, assume that displacements at the interior coordinates are related to the connection ones as follows

$$\{u_i\} = [\phi_{ic}^*]\{u_c\} \quad (8)$$

where

$$[\phi_{ic}^*] = -[K_{ii}]^{-1}[K_{ic}]$$

4.2 comparison of computing time during eigen analysis

The table 1 shows how much the computing time of eigen analysis can be reduced in applying CMS. The table show that when eigen analysis is performed for combined structure using reduced model, the first natural frequency can be approximated accurately, and the computing time also can be reduced.

	w/ CMS	w/o CMS
Natural frequency	5.175Hz	5.175Hz
Computing time	1.64(s)	2.27(s)

Table 1. Plate with 10×10 nodes

5. Simulation result

To verify performance of the modified tree model considering non-matching nodes, it is applied to a square plate with asymmetric boundary condition and L-shaped plate. The geometry of plates and stiffeners is shown in figure 2.

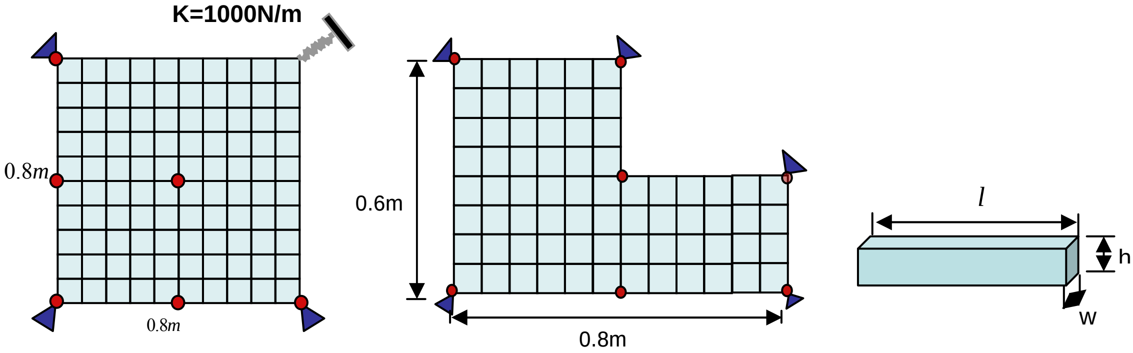

Figure 2. Geometry of plates and stiffeners

The thickness of plate is 0.003m and density is 7860 kg/m^2. The size of a beam is 0.005m, 0.01m and 0.08m ($w \times h \times l$). The total volume of stiffeners is restricted to 10% of volume of each plate. The simulation result is like figure 3.

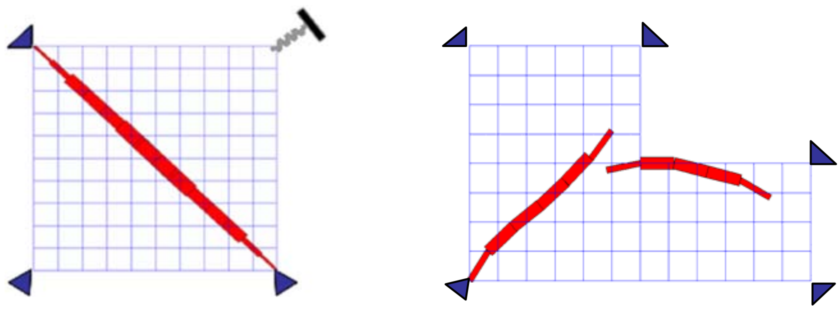

Figure 3. Simulation results

In the case of square plate, the stiffeners are placed diagonally on the plate. In other papers show, similar result is obtained. Particularly, while stiffeners are thin at the both corners of plate, they are thick at the center. As stiffeners grow, it happens that the stiffeners are overlapped in same places. In this case, the overlapped stiffeners are considered as the width of stiffeners being increase. The natural frequency is increased 4.9Hz to 5.75Hz. In L-shape plate, we can obtain a little curved layout. The result indicates that we can obtain extended layout of stiffeners. The natural frequency is increased 23.6Hz to 29.14Hz.

6. Conclusions

In this paper, modified tree model is proposed to increase a fundamental frequency of base structure. As considering non-matching problem, modified tree model can make more extended layout stiffeners. However, according to the positions and the number of initial seeds, the simulation results can be different. Generally, it is known that to add stiffeners to the places with high strain energy is efficient to increase natural frequencies. Therefore we can locate the seed to the place with high strain energy. Still, to decide the position and the number systematically is required.

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