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A simple idea to obtain acoustic scattering matrix of human head

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Abstract [575] Virtual 3D sound technology has directly relation with human head scattering problem. We commonly use HRTF (head related transfer function) that sees head scattering in terms of the transfer function.

In this study, we propose an experimental method, which allows us to have acoustic scattering holography that can predict the far-field scattered field. This utilizes the measured nearfield pressures to obtain scattered field around human head. Because of complicated geometrical shape of human head, we use BEM based NAH (nearfield acoustic holography) methods. By using this method, we first determine local impedance of head surface and then, we obtain scattered field by arbitrary source positions using this information.

1 INTRODUCTION

There are several methods to study acoustic scattering problems, using theoretical solution, numerical method such as boundary element methods and experimental methods. But for using theoretical solution or numerical method, we have to know the geometrical shape and acoustic properties of surface. It is also not worthy that scattered field is the function of the incident field, therefore it depends on source positions. In other words, we have to measure scattered field for every source positions which we want to know. Because of these reasons, we are going to use BEM-based acoustic holography to get the scattered field of arbitrary shaped scatterer and arbitrary incident waves. If we use BEM-based acoustic holography, we can obtain acoustic properties of surface, such as pressure, particle velocity, and local impedance. So we can predict scattered pressure of any points by arbitrary source positions.

Generally, BEM-based acoustic holography has been widely used and the method can deal with the arbitrary source shapes by measuring the near-field pressures.[1-4] In this method, the distribution of surface acoustic parameters of the source can be reconstructed by multiplying the inverse of transfer matrix and the measured field pressure vector.

In this study, we use these chacracteristics of BEM-based acoustic holography to predict scattered field. This method allows us to reconstruct distribution of surface acoustic parameters of scatterer, especially surface impedance. By using BEM modeling data which represent scatterer's geometrical shape, and measurement data around scatterer obtained by holography, we can obtain acoustic parameters of node points and also scattered field generated by arbitrary source positions. In this paper, theoretical formulation is discussed by using matrix-vector form of K-H (Kirchhoff-Helmholtz) integral equation.

2 THEORETICAL FOMULATION

2.1 Brief overview of BEM based NAH

First of all, matrix-vector form of K-H integral equation can be written as

$$\mathbf{p_f} = \mathbf{D_f p_s} + \mathbf{M_f v_s} \text{ in the domain,} \quad (1)$$

$$\mathbf{D_s p_s} = \mathbf{M_s v_s} \text{ on the boundary.} \quad (2)$$

Here, $\mathbf{p_s}$, $\mathbf{v_s}$ are the pressure and velocity vector on the surface, respectively, $\mathbf{p_f}$ denotes the field pressure vector in the domain, $\mathbf{D_s}$, $\mathbf{M_s}$ indicate the dipole and monopole matrices on the surface, and $\mathbf{D_f}$, $\mathbf{M_f}$ are those corresponding to field pressures, respectively. From Eq. (1), the following field pressure can be described only by the surface velocity provided $\mathbf{D_s}^{-1}$ exists:

$$\mathbf{p_f} = (\mathbf{M_f} + \mathbf{D_f D_s}^{-1}\mathbf{M_s})\mathbf{v_s} = \mathbf{G v_s} \quad (3)$$

Here, $\mathbf{G}$ is the vibro-acoustic transfer matrix correlating the surface normal velocity and the field pressure that contains the geometric information of the system as well. If the field pressure at *m* points is known, the surface velocity at *n*(<*m*) nodes can be uniquely determined. This can be accomplished by utilizing the overdetermined least-squared solutions approach and the singular value decomposition (SVD) technique. The SVD of the transfer matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \mathbf{U ? W^H} \quad (4)$$

Where

$$\mathbf{?} = \mathbf{diag(?_1, ?_2, \cdots, ?_n), ?_1 \geq ?_2 \geq \cdots \geq ?_n \geq 0,}$$

$$\mathbf{u_i}^{\mathbf{H}}\mathbf{u_j} = \mathbf{d_{ij}}, \mathbf{w_i}^{\mathbf{H}}\mathbf{w_j} = \mathbf{d_{ij}} \quad (5)$$

Here, $\mathbf{d_{ij}}$ is the Kronecker delta, the elements of diagonal matrix $\mathbf{?}$ are singular values $\mathbf{?_i}$, and $\mathbf{U}$, $\mathbf{W}$ mean the vectors, each of which has orthonormal columns. Physically, $\mathbf{u_i}$ and $\mathbf{w_i}$ are the wave vectors that decompose the distribution of field pressure on the hologram plane and that of surface velocity on the source plane at a selected frequency. By using Eq.(4), one can derive the following inverse equation:

$$\mathbf{v_s} = \mathbf{G^+ p_f} = \mathbf{(G^H G)^{-1} G^H p_f}$$

$$= \mathbf{W ?^{-1} U^H p_f} \quad (6)$$

Here, the superscripts ''+'' and '' $\mathbf{H}$ '' signify the pseudoinverse and the Hermitian operator, respectively. If the transfer matrix $\mathbf{G}$ is generated by the nonsingular BEM and the field pressure $\mathbf{p_f}$ is measured, the surface velocity $\mathbf{v_s}$ can be determined from Eq.(6). Also we can determine surface pressure $\mathbf{p_s}$ from Eq.(2).

2.2 Calculation of Scattering matrix using BEM techinque

If we measure total pressure vector $\mathbf{p}_{\mathbf{f\_T}}$ and incident pressure vector $\mathbf{p}_{\mathbf{f\_I}}$ in the domain, we readily know scattered pressure vector $\mathbf{p}_{\mathbf{f\_SC}}$ of field points around the scatterer as following;

$$\mathbf{p}_{\mathbf{f\_SC}} = \mathbf{p}_{\mathbf{f\_T}} - \mathbf{p}_{\mathbf{f\_I}} \tag{7}$$

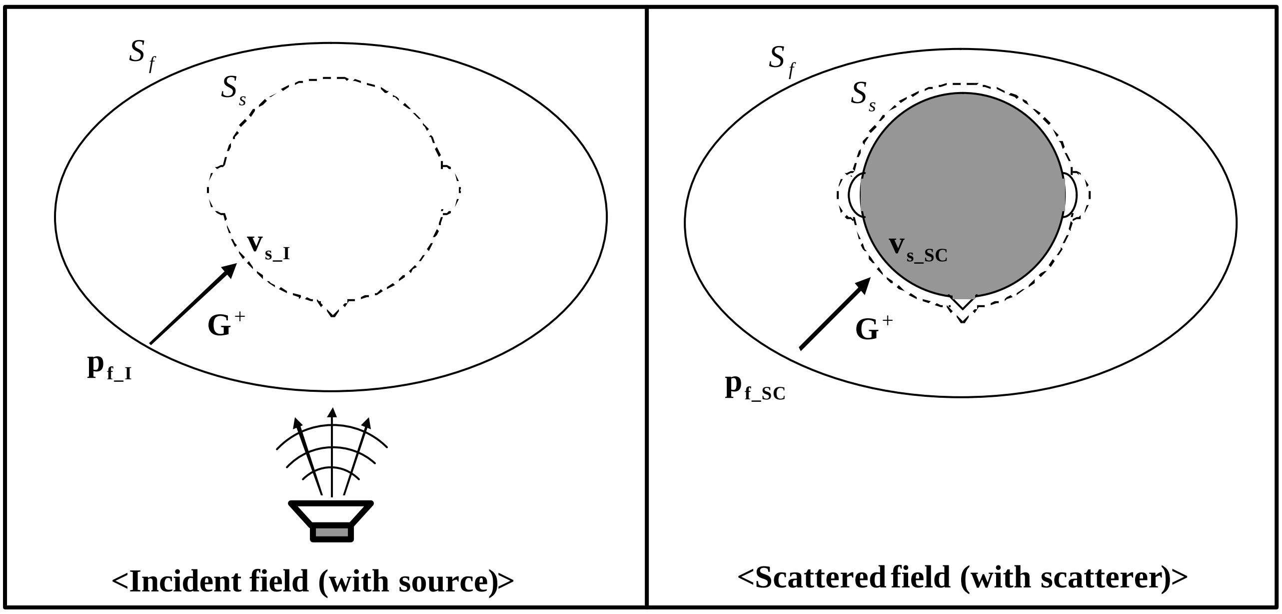

Figure 1: *Define regions and variables of incident field and scattered field*

First, we define incident field and scattered field using given boundaries and variables illustrated in Figure.1. Incident field means that there is a source of certain position and no scatterer inside of region S_s. And scattered field means that there is a scatterer and no source outside of region S_f.
BEM based NAH technique mentioned in 2.1 is appliable both incident field and scattered field. So surface velocity of incident field and scattered field can be determined by Eq.(6);

$$\begin{aligned}\mathbf{v}_{\mathbf{s\_I}} = \mathbf{G}^{+}\mathbf{p}_{\mathbf{f\_I}} &= (\mathbf{G}^{\mathbf{H}}\mathbf{G})^{-1}\mathbf{G}^{\mathbf{H}}\mathbf{p}_{\mathbf{f\_I}} \\ &= \mathbf{W?}^{-1}\mathbf{U}^{\mathbf{H}}\mathbf{p}_{\mathbf{f\_I}}\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{v}_{\mathbf{s\_SC}} = \mathbf{G}^{+}\mathbf{p}_{\mathbf{f\_SC}} &= (\mathbf{G}^{\mathbf{H}}\mathbf{G})^{-1}\mathbf{G}^{\mathbf{H}}\mathbf{p}_{\mathbf{f\_SC}} \\ &= \mathbf{W?}^{-1}\mathbf{U}^{\mathbf{H}}\mathbf{p}_{\mathbf{f\_SC}}\end{aligned} \tag{9}$$

Also we can determine surface pressure of incident field and scattered field from Eq.(2).
If we use BEM modeling of human head, we can determine local impedance of node points because surface impedance is related to surface pressure and velocity of total field. Local impedance of node point can be determined by Eq.(10) as following;

$$\mathbf{z_i} = \frac{(\mathbf{p}_{s\_T})_i}{(\mathbf{v}_{s\_T})_i} = \frac{(\mathbf{p}_{s\_I})_i + (\mathbf{p}_{s\_SC})_i}{(\mathbf{v}_{s\_I})_i + (\mathbf{v}_{s\_SC})_i} = \frac{\mathbf{?c}}{\mathbf{ß_i}} \tag{10}$$

Here, **?** , **c** are the density of air and sound speed, respectively, $\mathbf{ß_i}$ denotes local admittance.
If we determine local impedance of head surface, We can predict scattered field by arbitrary source position using local impedance of node points.

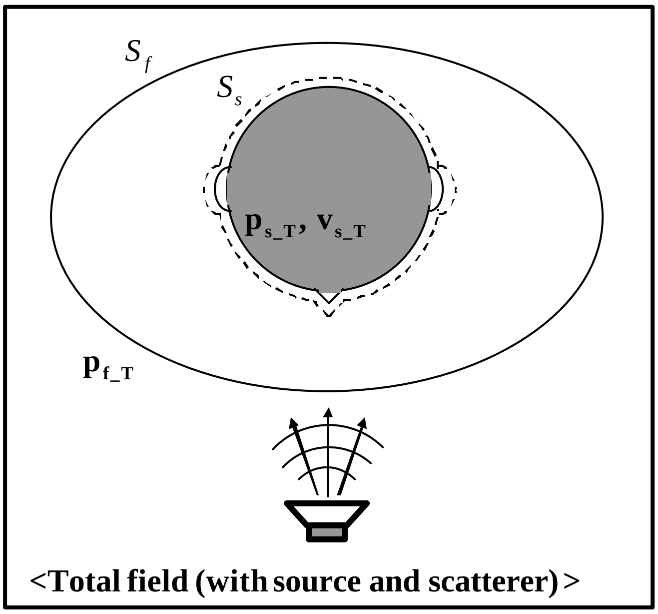

Figure 2: *Define regions and variables of total field*

We can define total field using regions and variables defined by Figure.2. Matrix-vector form of K-H integral equation for total field can be written by Eq.(11) same as Eq.(2).

$$\mathbf{D}_s\mathbf{p}_{s\_T} = \mathbf{M}_s\mathbf{v}_{s\_T} - \mathbf{p}_{s\_I} \text{ on the boundary} \tag{11}$$

Here, $\mathbf{p}_{s\_T}$, $\mathbf{v}_{s\_T}$ are the pressure and velocity vector of total field on the surface, respectively, $\mathbf{p}_{s\_T}$ is the pressure of incident field on the surface, $\mathbf{D}_s$, $\mathbf{M}_s$ mean the dipole and monopole matrices on the surface.
Also, from Eq. (10), the following pressure vector of total field, $\mathbf{p}_{s\_T}$ can be described by the local impedance of surface and surface velocity vector of incident field as following

$$\mathbf{p}_{s\_T} = \mathbf{Z}\mathbf{v}_{s\_T} \tag{12}$$

Here, $\mathbf{Z}$ is diagonal matrix which denotes local impedance of surface. Consequently, velocity vector of total field can be represented by surface pressure vector of incident field from Eq. (11) & Eq. (12)

$$\mathbf{v}_{s\_T} = (\mathbf{M}_s - \mathbf{D}_s\mathbf{Z})^{-1}\mathbf{p}_{s\_I} \tag{13}$$

Also, surface pressure vector of total field can be represented by surface pressure vector of incident field.

$$\mathbf{p}_{s\_T} = \mathbf{p}_{s\_I} + \mathbf{p}_{s\_SC} = \mathbf{D}_s^{-1}\{(\mathbf{M}_s - \mathbf{D}_s\mathbf{Z})^{-1} - \mathbf{I}\}\mathbf{p}_{s\_I} \tag{14}$$

$$\Rightarrow \ \mathbf{p}_{s\_SC} = [\mathbf{D}_s^{-1}\{(\mathbf{M}_s - \mathbf{D}_s\mathbf{Z})^{-1} - \mathbf{I}\} - \mathbf{I}]\mathbf{p}_{s\_I} \tag{15}$$

So we can obtain scattering matrix $\mathbf{S}$ which means transition matrix between suface pressure vector of scattered field and incident field as following.

$$\mathbf{S} = [\mathbf{D}_s^{-1}\{(\mathbf{M}_s - \mathbf{D}_s\mathbf{Z})^{-1} - \mathbf{I}\} - \mathbf{I}] \tag{16}$$

For example, if we want to obtain scattered field by point source which is located a certain position, we can determine surface pressure $\mathbf{p}_{s\_I}$ and velocity $\mathbf{v}_{s\_I}$ of incident field by definition. So we can also deterimine surface prssure of scattered field scattering matrix from Eq.(16).

3 CONCLUSION

In this study, we propose BEM based NAH technique to predict scattered field by human head model. It is very useful to predition of scattered field by arbitrary source positions. By using this method, we can determine local impedance of human head surface, so we can obatain scattered field by arbitrary source positions using measurement data of only one source position.

But there are limitations because measurement points have to be larger than the node points of BEM modeling. So more studies about the errors due to the numbers of measurement points and experimental verification are needed.

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