Thermal Noise Model for Short-Channel MOSFETs

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1. Introduction

Due to continuous reduction of channel length in CMOS technologies, CMOS has become a viable candidate for RF applications. Accurate modeling of RF CMOS is very important to optimize circuit performance and yield as well as to decrease develop time. In spite of lots of progress in RF CMOS I-V and Q-V modeling, its thermal noise behavior in short channel MOSFET is not well understood yet and even very controversial. Therefore noise modeling as well as low noise circuit design have been done quite empirically.

It has been often reported that the channel thermal noise generated in short channel MOSFETs is higher than predicted by the long channel noise model. Although there are many literatures to try to explain such the increase of channel thermal noise for short channel MOSFETs by introducing hot electron effect in velocity saturation region, an exact thermal noise model for short channel MOSFETs have not been derived. And recently many experimental and simulation works have addressed that the noise generated due to hot electron in velocity saturation region is negligible [1,2]. Also the previous models did not correctly take into account velocity saturation effect, which is important to IV characteristics in short channel MOSFETs, since the accurate impedance field [3] in short channel devices was not used. The previous models considered the infinitesimal segment of MOSFET channel region as linear resistor and formulated the drain thermal noise by using noise source based on the Nyquist theory. However, such view can not be applied for short channel MOSFETs, because velocity degradation due to lateral field makes the channel of MOSFET nonlinear.

2. Thermal Noise Model

The channel of a MOSFET is divided into two regions. One is the so called gradual channel region with length of $L = L_{eff} - \Delta L$ and the other is velocity saturation region with length of ΔL. Since the contribution of carriers in the velocity saturation region to the drain thermal noise is negligible, we only consider the contribution of the carriers in the gradual channel region to formulate the drain thermal noise. Since mobility of carriers in short channel MOSFETs is degraded due to lateral field, infinitesimal channel segment cannot be thought as a linear resistor. Therefore we cannot follow Tsividis' approach in short channel MOSFETs, which is based on thermal noise formulation of infinitesimal linear ohmic resistor [4]. Therefore, we follow the Van der Ziel's approach, who formulate thermal noise based on diffusion noise source. Diffusion noise source can be applied at even nonlinear resistor [5].

To calculate the impedance field to drain terminal at channel region x, a MOSFET is divided into two MOSFETs as shown in Fig.1. The noisy segment at channel region x can be modeled as a noiseless segment with a Norton generator representing the local current fluctuation and the current noise source can be divided into two current noise sources, which have same amplitude but are oppositely directed at x and $x+\Delta x$ as shown in the Fig.1. δI is short-circuited drain noise currents induced by two local noise sources at x and $x+\Delta x$. The local noise source is given by diffusion noise source [5]:

$$\overline{\delta i_n{}^2} = 4q^2 B D_n n_s \frac{W}{\Delta x} \tag{1}$$

,where q is the electronic charge, B is the bandwidth, D_n is the diffusion coefficient, n_s is the areal carrier number density, W is the device width, direction of x is along the channel.

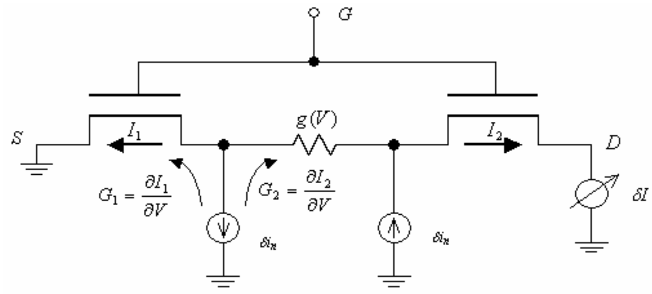

Fig.1. Impedance Field Method

The drain drift current with velocity saturation effect can be written as

$$I = g_o(V)\frac{dV/dx}{1+\dfrac{dV/dx}{E_C}} \tag{2}$$

,where $g_o(V) = WC_{ox}\mu_o(V_{GT}-V)$, C_{ox} is the gate oxide capacitance per unit area, μ_o is the effective mobility, V_{GT} is the effective gate overdrive voltage $(= V_{GS} - V_{TH})$, E_C is the critical field.

Drain currents of both MOSFETs with length x and $L-x$ is given by,

$$I_1 = \frac{\int_0^V g_o(V)dV}{x + V/E_C}, \quad I_2 = -\frac{\int_V^{V_{DS}} g_o(V)dV}{L - x + (V_{DS} - V)/E_C}$$

$$I = I_1 = -I_2 = \frac{\int_0^{V_{DS}} g_o(V)dV}{L + V_{DS}/E_C} \tag{3}$$

respectively ,where the direction for each current is indicated in Fig.1.

From eq.(3), the left side and right side small signal conductance $(= \partial I/\partial V)$ is obtained as follows:

$$G_1 \cdot (x + V/E_C) + I_1/E_C = g_o(V) \tag{4a}$$

$$G_2 \cdot (L - x + (V_D - V)/E_C) - I_2/E_C = g_o(V) \tag{4b}$$

Subtracting eq.(4.b) from eq.(4.a), we obtain

$$\frac{G_2}{G_1+G_2} = \frac{x+V/E_C}{L+V_D/E_C} \tag{5}$$

Then, the drain terminal noise current due to two local noise sources at x and $x+\Delta x$ is given by

$$\Delta I = \delta i\left(\left.\frac{G_2}{G_1+G_2}\right|_{x+\Delta x} - \left.\frac{G_2}{G_1+G_2}\right|_{x}\right) = \delta i_n \frac{\Delta x + \Delta V/E_C}{L+V_D/E_C} \tag{6}$$

Since

$$I(\Delta x+\Delta V/E_C)=g_o(V)\Delta V \tag{7}$$

Combining eq. (6) and eq.(7) leads to the impedance field,

$$\Delta I=\delta i_n \frac{g_o(V)\Delta V}{I(L+V_D/E_C)} \tag{8}$$

This gives the contribution of the noisy element at x to the drain current noise. The contributions of all similar elements in the channel are assumed uncorrelated, and one can thus find the mean square value of their combined effects by adding the individual mean square value. Considering ΔV as a differential variable and integrating (8) over the entire channel, the spectral density of the total drain current noise is obtained,

$$\begin{aligned}\overline{i_{dn}^2}&=\overline{(\sum\Delta I)^2}=\overline{\sum(\Delta I)^2}\\&=\sum 4q^2BD_nn_s\frac{W}{\Delta x}\cdot\frac{{g_o}^2(V)\Delta V^2}{I^2(L+V_D/E_C)^2}\end{aligned} \tag{9}$$

We substitute $\Delta x=\left(\frac{g_o(V)}{I}-\frac{1}{E_C}\right)\Delta V$ using eq.(7), then eq.(9) leads to

$$\overline{i_{dn}^2}=\sum 4q^2BD_nn_sW\cdot\left(\frac{g_o(V)}{I}-\frac{1}{E_C}\right)^{-1}\cdot\frac{{g_o}^2(V)\Delta V}{I^2(L+V_D/E_C)^2} \tag{10}$$

Let's define the channel conductance including the velocity degradation due to lateral field as

$$g(V)\equiv\frac{g_o(V)}{1+E/E_C}=g_o(V)-I/E_C \tag{11}$$

,where E is electric field at the channel region x.

In order to obtain the numerical solution of drain thermal noise given by eq.(10), the electric field dependences of the diffusivity must be given. We assume that the diffusion coefficient is constant over the channel [6,7],

$$D_n=D_o=\frac{kT_o}{q}\mu_o \tag{12}$$

,where D_o is the diffusion constant at low field, k is the Boltzmann's constant and T_o is the ambient temperature.

Substituting eq.(11) and eq.(12) into (10) gives,

$$\overline{i_{dn}^2}=\frac{4kT_oB}{IL^2(1+V_D/LE_C)^2}\int_0^{V_{DS}}g^2(V)(1+E/E_C)^3dV \tag{13}$$

In the case of $E_C\to\infty$ (or long channel limit), eq.(13) reduces to $\overline{i_{dn}^2}=\frac{4kT_oB}{IL^2}\int_0^{V_{DS}}g^2(V)dV$ [5].

3. Relation of Thermal Noise with Inversion Charge

Although eq.(13) is the final result, it is not an useful form. Therefore, we should approximate eq.(13) to obtain useful analytic equation. From eq. (13),

$$\overline{i_{dn}^2}=\frac{4kT_oB}{IL^2(1+V_D/LE_C)^2}\int_0^{V_{DS}}g^2(V)\left(1+3E/E_C+3(E/E_C)^2+(E/E_C)^3\right)dV \tag{14}$$

Eq.(14) consists of four integrals. It is straightforward to evaluate the first, the second, and the third integral with the applied bias. However, electric field E should be obtained as a function of x to evaluate the fourth integral. After that, the result of the fourth integral was approximated with talyor series. Adding each evaluated integral leads to,

$$\begin{aligned}<i_{dn}^2>&\approx 4kT_oBg_{do}\left[\frac{1-u+u^2/3}{1-u/2}-\frac{1}{12}\frac{V_{DS}/LE_C}{(1+V_{DS}/LE_C)(1-u/2)}u^2\right]\\&\approx 4kT_oBg_{do}\frac{1-u+u^2/3}{1-u/2}\end{aligned} \tag{15}$$

,where $g_{do}=\mu_o\frac{W}{L}C_{ox}V_{GT}$ is the drain conductance at $V_{DS}=0V$ and $u=V_{DS}/V_{GT}$. Now, let's evaluate the total inversion charge Q_N as a function of applied biases,

$$\begin{aligned}Q_N&=\frac{1}{\mu_o}\int_0^L g_o(x)dx=\left(\frac{L^2}{\mu_o}\right)g_{do}\left[\frac{1-u+u^2/3}{1-u/2}+\frac{1}{12}\frac{V_{DS}/LE_C}{1-u/2}u^2\right]\\&\approx\left(\frac{L^2}{\mu_o}\right)g_{do}\left[\frac{1-u+u^2/3}{1-u/2}\right]\end{aligned} \tag{16}$$

From eq.(15) and eq.(16), we can express the drain thermal noise with total inversion charge as follows:

$$<i_{dn}^2>=4kT_oB\frac{\mu_o}{L^2}Q_N \tag{17}$$

To check the validity of the approximation used to derive eq.(17), we evaluate the drain thermal noise given by eq.(13) and the thermal noise given by eq.(17) numerically. Although maximum difference between the value of the thermal noise given by eq.(13) and the value of the thermal noise given by eq.(17) occurs at saturation condition, the maximum difference is below 5% for all channel lengths, gate biases, and drain biases. Note that L is the length of the gradual channel region, not the length of total channel region and μ_o is only gate bias dependent effective mobility, not the mobility including the degradation due to lateral field.

4. Conclusion

In this paper, a physics-based thermal noise model for short channel MOSFET was derived by using impedance field method. The model includes two important physical effects for short channel MOSFETs. One is the velocity saturation effect due to lateral field and the other is the carrier heating effect in gradual channel region. Conclusively, eq.(17) was obtained for short channel MOSFETs, which turned out to be the same equation from used for long channel MOSFET[4].

With the eq.(17), the increase of drain thermal noise in short channel MOSFET biased at saturation region seems to be originated from the increase of Q_N and the decrease of L, which is called the static feedback effect and the channel length modulation effect, respectively.

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