C-31

A Systematic Extraction Method for Noise Sources and Correlation Coefficient in MESFET

Jong-Hee Han, *Student Member, IEEE*, and Kwyro Lee, *Senior Member, IEEE*

Department of Electrical Engineering, KAIST,
373-1, Kusong-dong, Yusong-gu, Taejon, Korea 305-701.

Abstract - A systematic extraction method for noise sources and correlation coefficient in the noise equivalent circuit of GaAs MESFET is proposed. It is based on the linear regression, which allows us to extract physically meaningful parameters from the measurement. The confidence level of the measured data can be easily examined from the linearity, y-intercept of the linear regression, and the scattering from the regression line. Furthermore, it is found that the time delay of correlation coefficient whose value is almost the same as that of the transconductance should be considered to model noise parameters accurately.

I. Introduction

The noise characteristics of a linear two-port system can be completely described by 2-port parameters such as Y, Z, S-parameters and the additional four noise parameters. There are several equivalent sets of these four noise parameters, depending on how to represent the two-port circuit[1]. In H-representation, the noise current source at the input in Y-representation is replaced by the noise voltage source. Generally speaking, the correlation between the noise sources has to be taken into account in any representation and the values of the 4 noise parameters should be determined from noise measurement at every bias point for each device.

Pospieszalski[2] assumed that the gate noise voltage and drain noise current sources are independent of each other. But, that assumption is not proved yet. Granted it is true in the intrinsic device, the parasitic gate-source capacitance, C_{gsp}, invokes the correlation between gate noise voltage and drain noise current sources[3]. Since it is very difficult to discriminate C_{gsp} from the intrinsic capacitance, C_{gsi}, by S-parameter measurement, the correlation coefficient has to be considered in the noise equivalent circuit.

Usually a numerical optimization technique is used for extracting the noise sources from the measured noise characteristics by minimizing the error between the measurement and model. Though conceptually accurate, numerical optimization often gives wrong answer due to the local extrema. This is especially true for the noise data since the data itself is very noisy. Furthermore, it is hard to figure out the confidence level of the extraction result. In this paper, we propose a new systematic method to determine the noise sources from the measurement by linear regression.

II. Noise modeling of MESFET

Fig.1 shows MESFET small signal equivalent circuit with H-representation noise sources whose correlation coefficient is C_{II}. Here, C_{II} is defined as $\langle v_{gn}\, i_{dn} * \rangle$ over $\sqrt{\langle v_{gn}{}^2\rangle\langle i_{dn}{}^2\rangle}$, which is a complex number in general. Note that C_{gs} in Fig.1 is the sum of C_{gsp} and C_{gsi}. The input equivalent noise voltage, v_{in}, and the noise current sources, i_{in}, can be derived in the intrinsic equivalent circuit(inside the dotted box in Fig.1) as follows. v_{in} is the input equivalent noise voltage source which generates the same output noise when the gate is short to ground in AC, given by

$$v_{in} \;=\; v_{gn} + \frac{1 + j\omega C_{gs} R_i}{g_m e^{-j\omega\tau}} i_{dn} \,. \qquad (1)$$

i_{in} is the input equivalent noise current generating the same output noise when the gate is open in AC, which can be written as

$$i_{in} \;=\; -\frac{j\omega C_{gs}}{g_m e^{-j\omega\tau}} i_{dn} \,. \qquad (2)$$

Noise conductance, G_n can be obtained by normalizing $\langle i_{in}^2\rangle$ by $4kT\Delta f$ as following

$$G_n = \frac{1}{4kT\Delta f}\frac{\omega^2 C_{gs}^2}{g_m^2}\langle i_{dn}^2\rangle . \tag{3}$$

The correlation impedance between v_{in} and i_{in}, Z_c, defined by $\langle v_{in}\, i_{in}*\rangle$ over $\langle i_{in}^2\rangle$ can be derived using Eqs.(1)-(2) as follows.

$$Z_c = -R_i + \frac{j}{\omega C_{gs}} + \frac{j g_m e^{-j\omega\tau} C_H}{\omega C_{gs}}\sqrt{\frac{\langle v_{gn}^2\rangle}{\langle i_{dn}^2\rangle}} . \tag{4}$$

C_H can be assumed to have only the real part in MESFET at low frequency, but the imaginary part of C_H increases at higher frequency. Ignoring the imaginary part of C_H gives rise to the large discrepancy in noise characteristics between measurement and model especially at high frequency as in Sec.IV. We found from the measurement that the angle of C_H is proportional to the frequency, and that its proportional constant can be approximated by g_m time delay, τ. Then C_H is can be written as $C_H e^{j\omega\tau}$. Then, the imaginary part of the optimum impedance, X_{opt}, can be written as

$$X_{opt} = \frac{1}{\omega C_{gs}}\left(1 + g_m C_H\sqrt{\frac{\langle v_{gn}^2\rangle}{\langle i_{dn}^2\rangle}}\right). \tag{5}$$

Note that X_{opt} is inversely proportional to ωC_{gs}, and that its proportional constant is dependent on C_H. On the other hand, the real part of the optimum impedance, R_{opt}, can be easily derived as

$$R_{opt}^2 = R_i^2 + \left(1 - C_H^2\right)\frac{\langle v_{gn}^2\rangle}{\langle i_{dn}^2\rangle}\left(\frac{\omega_T}{\omega}\right)^2 , \tag{6}$$

where ω_T is the unit current gain frequency given by g_m/C_{gs}. It is very interesting to notice that $R_{opt}^2 - R_i^2$ is inversely proportional to the square of frequency.

III. Extraction method of noise sources from measurement for MESFET

Three noise parameters, $\langle v_{gn}^2\rangle$, $\langle i_{dn}^2\rangle$, and C_H are extracted from the frequency dependent noise measurement. The combinations of these three parameters have relations with G_n(Eq.(3)), X_{opt} (Eq.(5)), and R_{opt}^2(Eq.(6)). Therefore, we can extract them from three plots summarized in Table below. Therefore, we can extract three unknown noise parameters from the slopes of plots using linear regression. The inherent advantages of the linear regression are :

i) there is no danger to go into local extrema, and

ii) the confidence level of the extraction results can be checked easily from the linearity of the plots, i.e., slope, y-intercept, and the scattering from the regression line.

The examples of the extraction plots are shown in Figs.2~4. Note that the slope of X_{opt} vs. $1/\omega C_{gs}$ is not 1. In the Pospieszalski's model where C_H is zero, X_{opt} should be exactly the same as $1/\omega C_{gs}$, which means the slope should be 1. But, as shown in Fig.3, the slope is 1.2, and the slope goes upto ~1.4 at some bias conditions.

Graph	G_n vs. $(\omega/\omega_T)^2$	X_{opt} vs. $1/\omega C_{gs}$	$\left(R_{opt}^2 - R_i^2\right)$ vs. $(\omega_T/\omega)^2$
Slope	$\frac{\langle i_{dn}^2\rangle}{4kT\Delta f}$	$1 + g_m C_H\sqrt{\frac{\langle v_{gn}^2\rangle}{\langle i_{dn}^2\rangle}}$	$\left(1 - C_H^2\right)\frac{\langle v_{gn}^2\rangle}{\langle i_{dn}^2\rangle}$

Table1. Three plots for the extraction of the noise parameters, and their idealized slopes.

IV. Comparison of model with measurement

The measured device is an ion-implated GaAs MESFET, whose geometry is $0.5 \times 300\mu m^2$. The minimum noise figure, the optimum impedance, and the noise resistance of our proposed model are compared with those of measurement. In addition to our model, we compare two more cases. One is when C_H is assumed to have only real part in noise parameter calculation, and the other is when C_H=0, i.e., Pospieszalski's model. The noise sources in Pospieszalski's model are extracted from the slopes in the plot of G_n vs. $(\omega / \omega_T)^2$ and $\left(R_{opt}{}^2 - R_i{}^2\right)$ vs. $(\omega_T / \omega)^2$. Comparison is performed at two bias conditions in Figs.5~6, both at 45 mA(V_{GS}=0.0V) and at 15 mA(V_{GS}=-0.8V) with V_{DS}=3.0V, respectively. Figs.5~6 show that our model has much better accuracy than the other two models in explaining frequency dependence of all four noise parameters.

V. Conclusion

A new extraction method for noise sources and correlation coefficient in the noise equivalent circuit of GaAs MESFET has been proposed. Since the extraction of noise sources and correlation coefficient is based on the linear regression, it allows us not only to extract physically meaningful parameters from the measurement in a systematic and straightforward way, but also to examine easily the confidence level of the measured data from the linearity of three plots and the scattering from the regression lines. The comparison of noise figure, optimum impedance, and noise resistance between our model and measurement shows excellent agreements both at high and at low drain current bias for a typical MESFET device studied in this paper.

Acknowledgment

The authors thank Dr. Chul-Soon Park at ETRI and Dr. Woong-Sik Cho at LG Electronics Inc. for their help in noise measurement.

References

[1] K. Hartmann, "Noise Characterization of Linear Circuits," *IEEE Trans. Circuit Syst.*, vol.CAS-23, no.10, pp.581~590, Oct. 1976.

[2] M. W. Pospieszalski, "Modeling of Noise Parameters of MESFET's and MODFET's and Their Frequency and Temperature Dependence," *IEEE Trans. Microwave Theory Tech.*, vol.MTT-37, no.9, pp.1340~1350, Sep. 1989.

[3] F. Danneville, *et al.* "Microscopic Noise Modeling and Macroscopic Noise Models : How Good a Connection?," *IEEE Trans. Electron Devices*, vol.ED-41, no.5, pp.779~786 , May 1994.

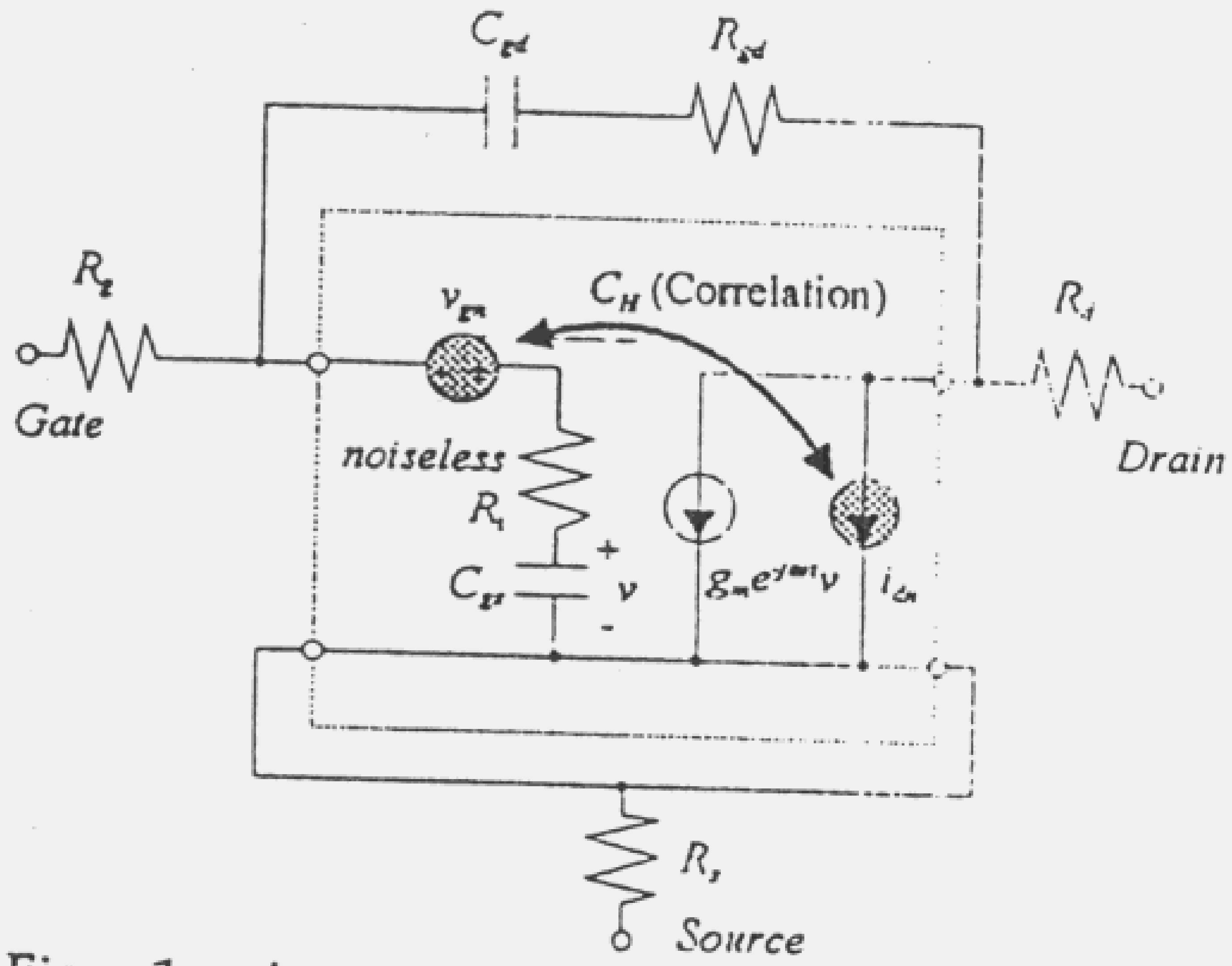

Fig. 1. A noise equivalent circuit in H-representation for MESFET.

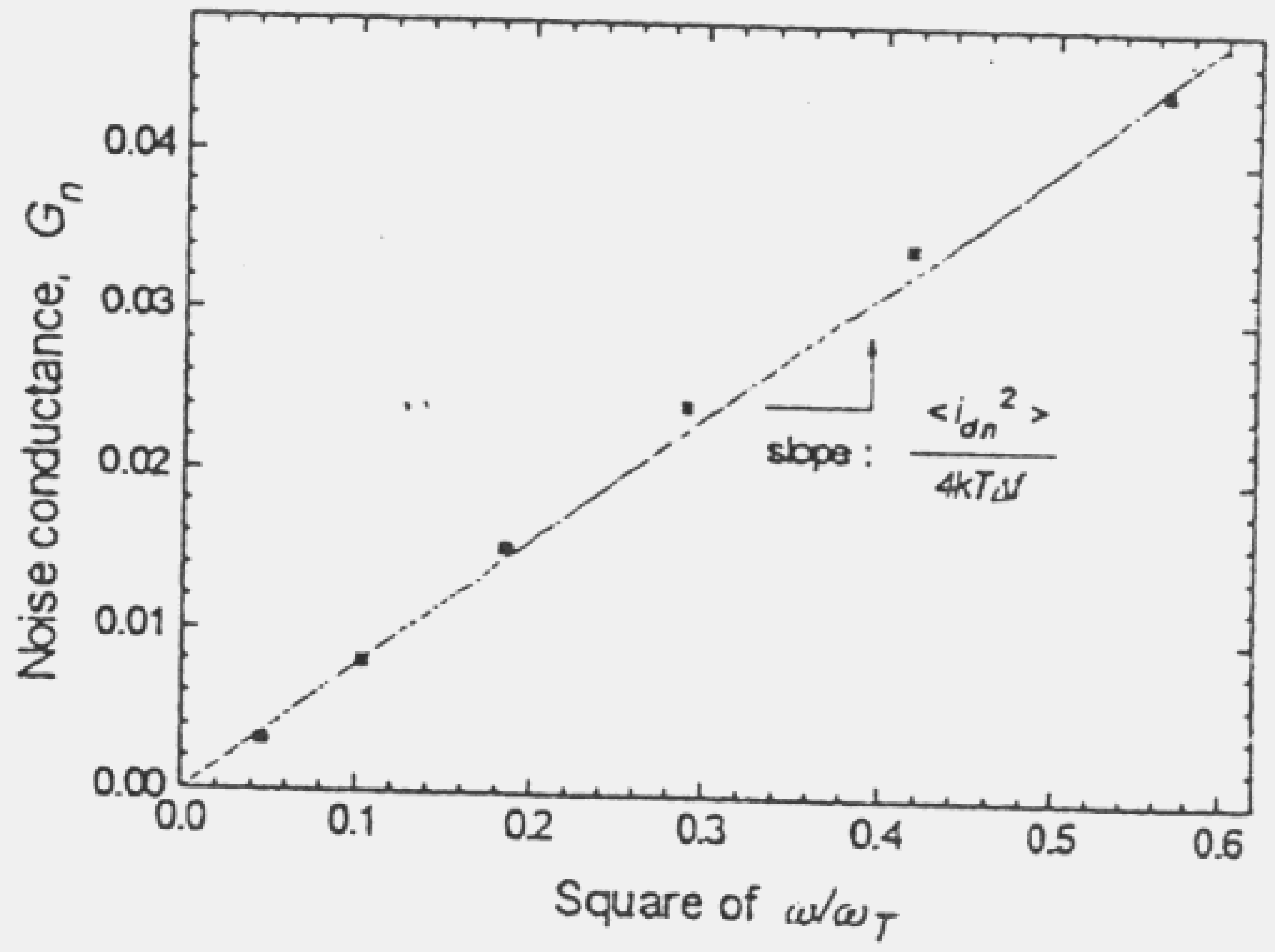

Fig.2. G_n vs. $(\omega / \omega_T)^2$ plot at V_{GS},V_{DS}=0.0, 3.0V.

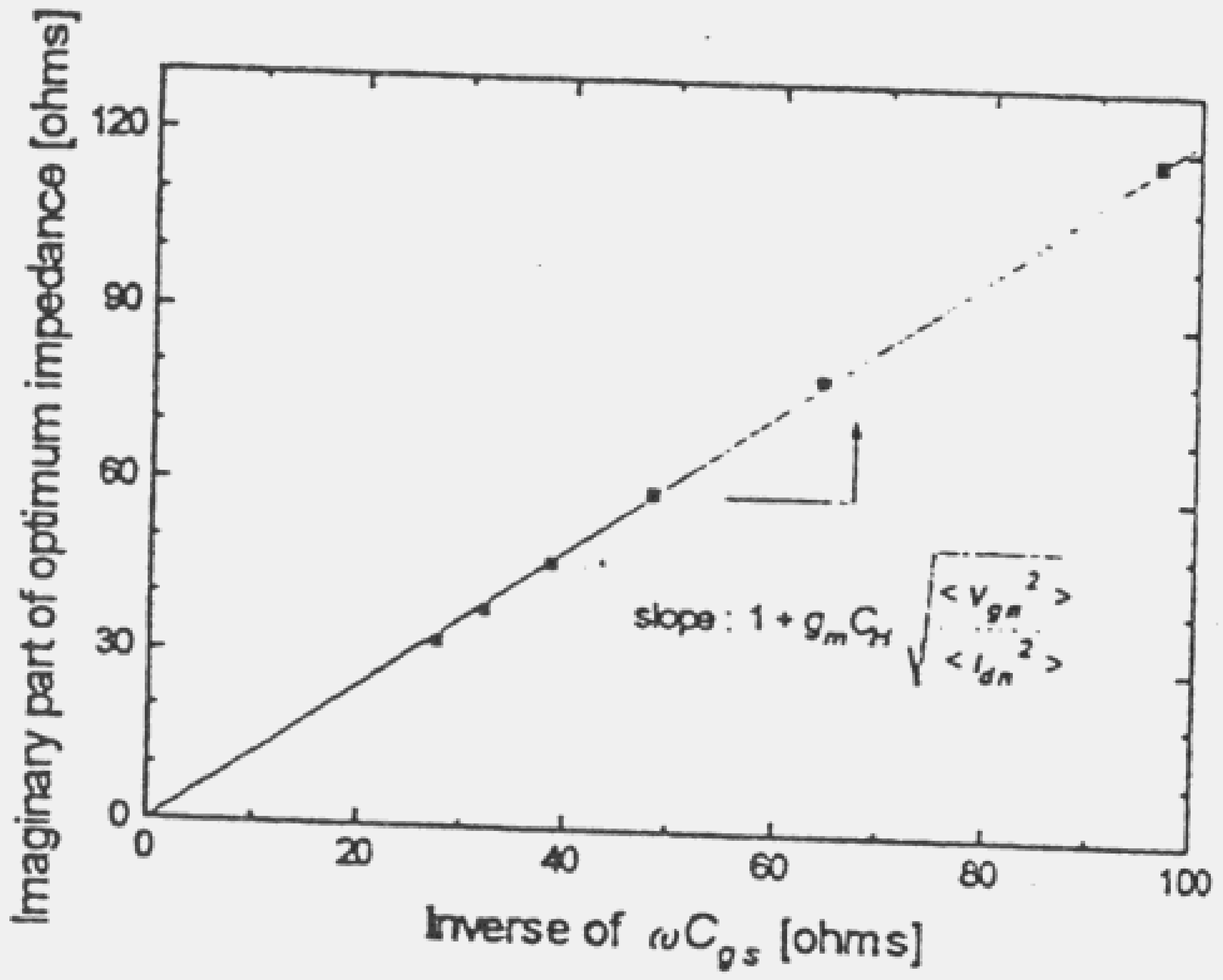

Fig. 3. X_{opt} vs. $(\omega_T / \omega)^2$

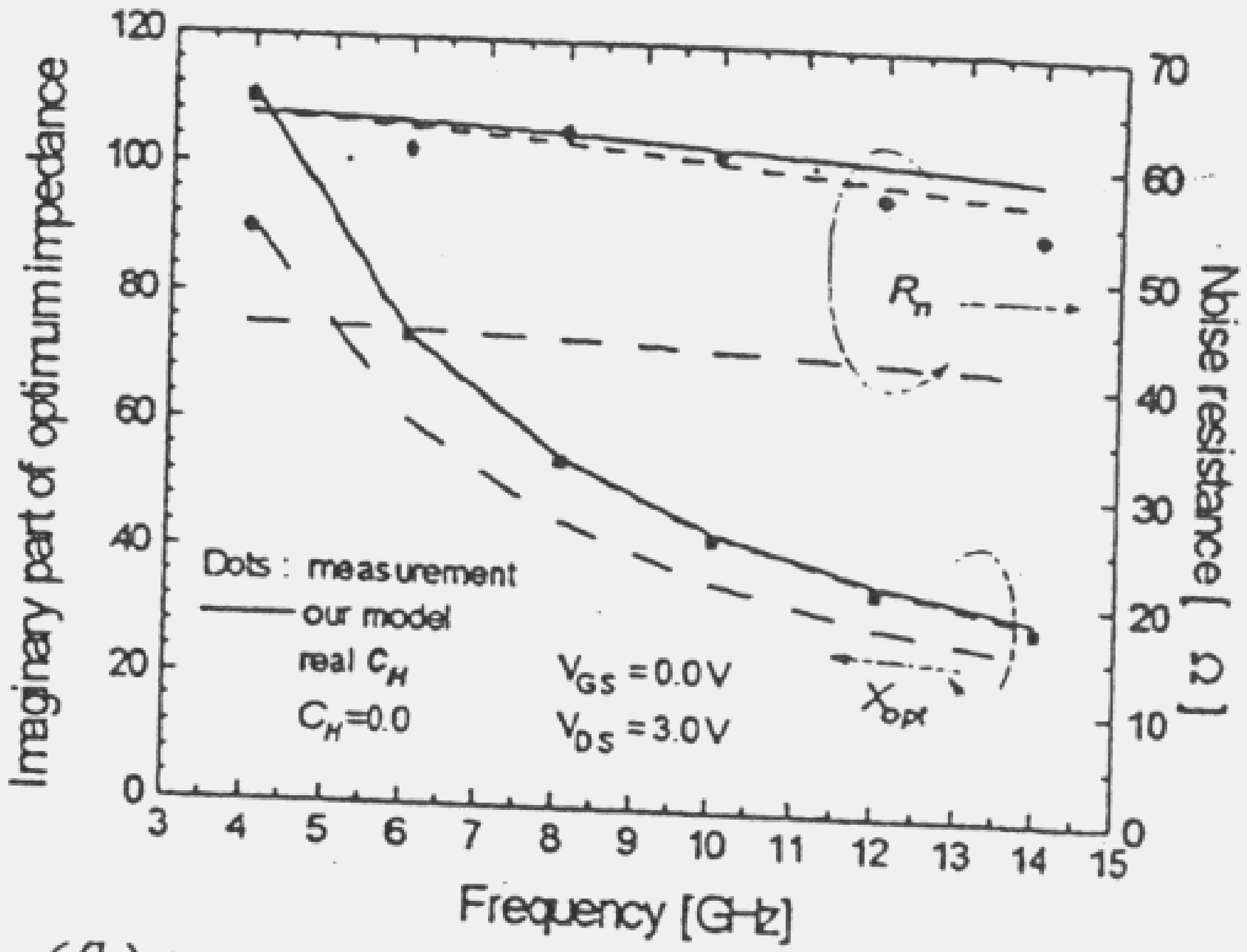

Fig.6(b) X_{opt} and R_n at V_{GS}, V_{DS}=0.0, 3.0V

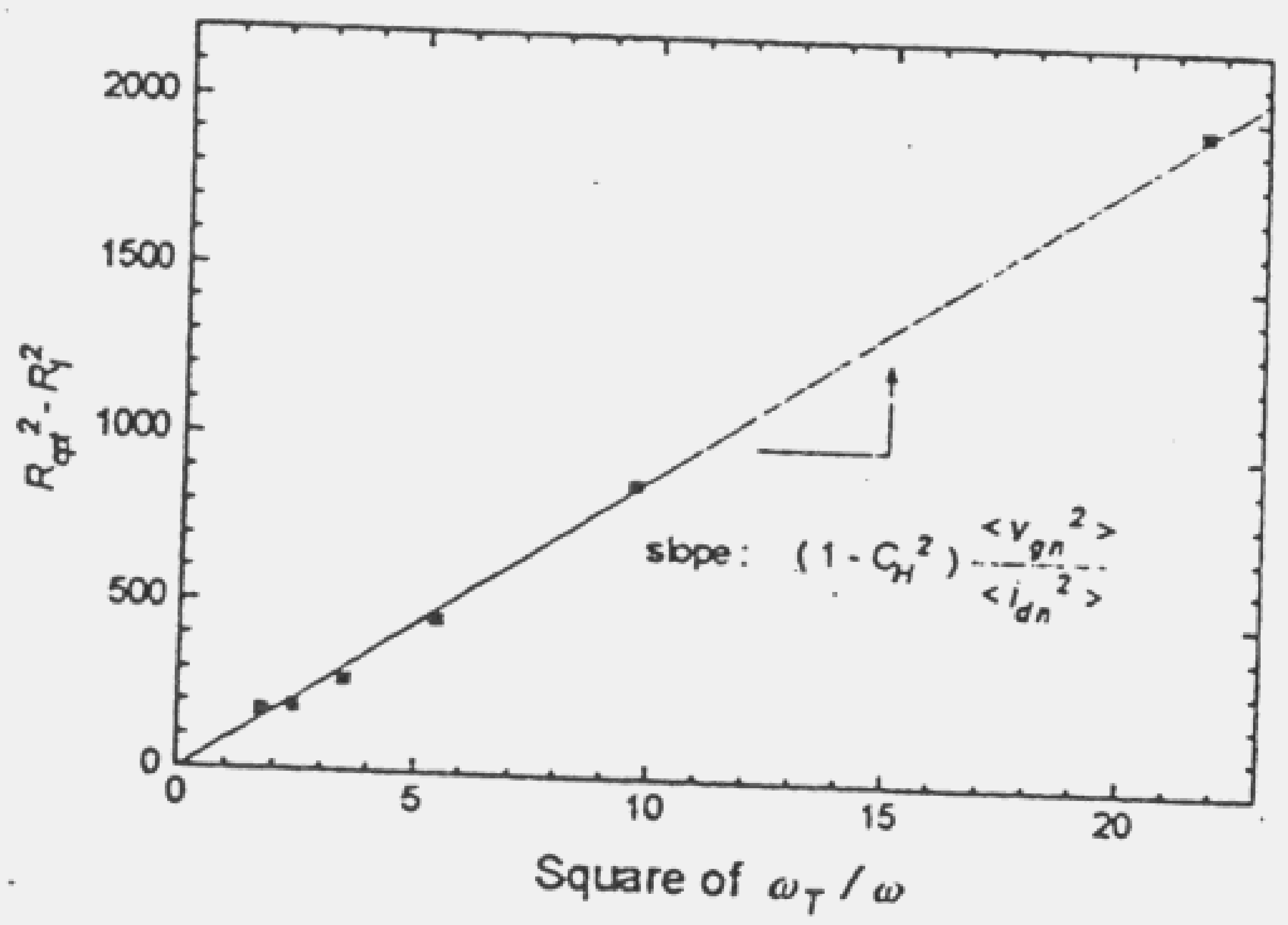

Fig. 4. $\left(R_{opt}^2 - R_i^2\right)$ vs. $(\omega_T / \omega)^2$

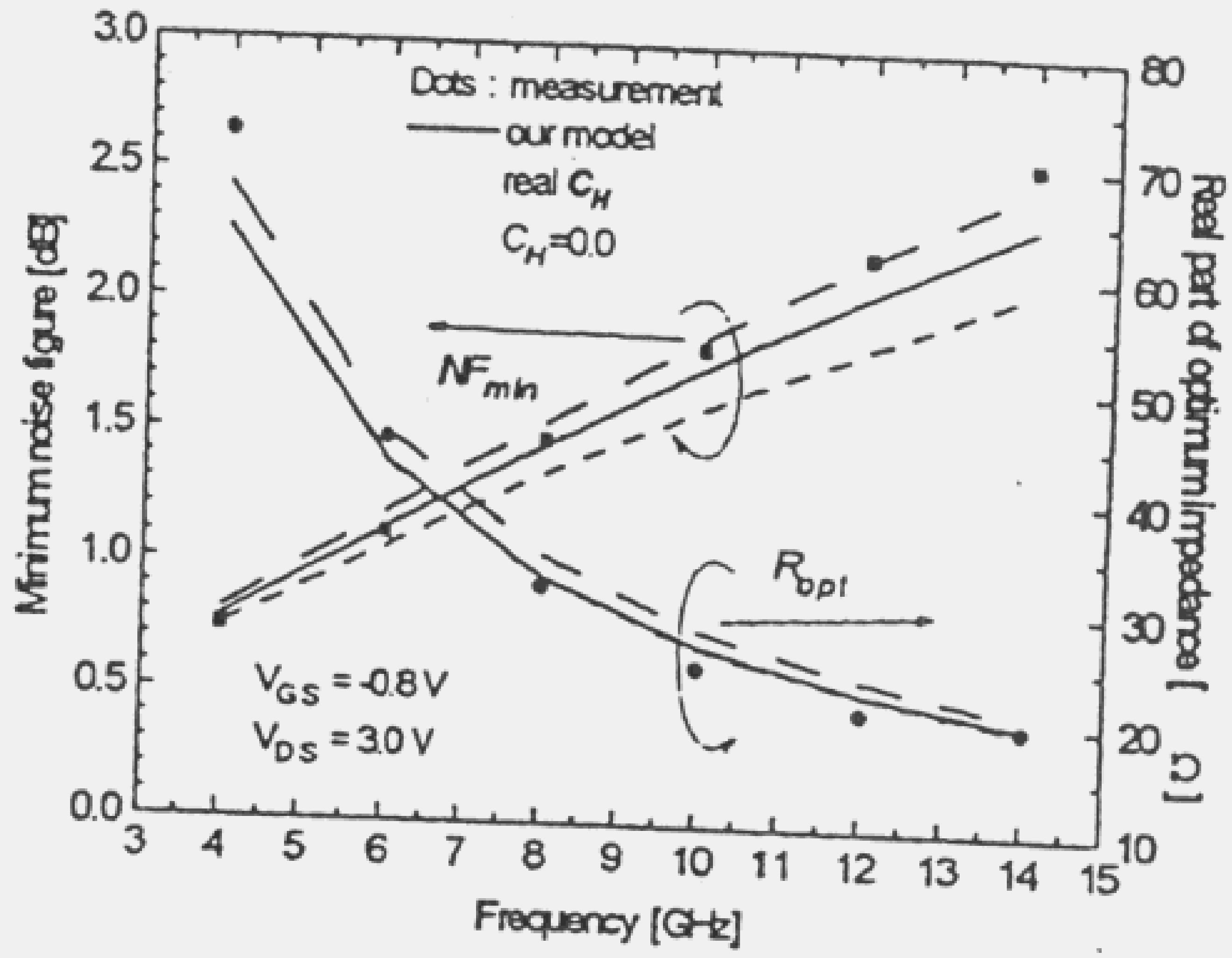

Fig.7 (a) NF_{min} and R_{opt} at V_{GS}, V_{DS}=0.0, 3.0V

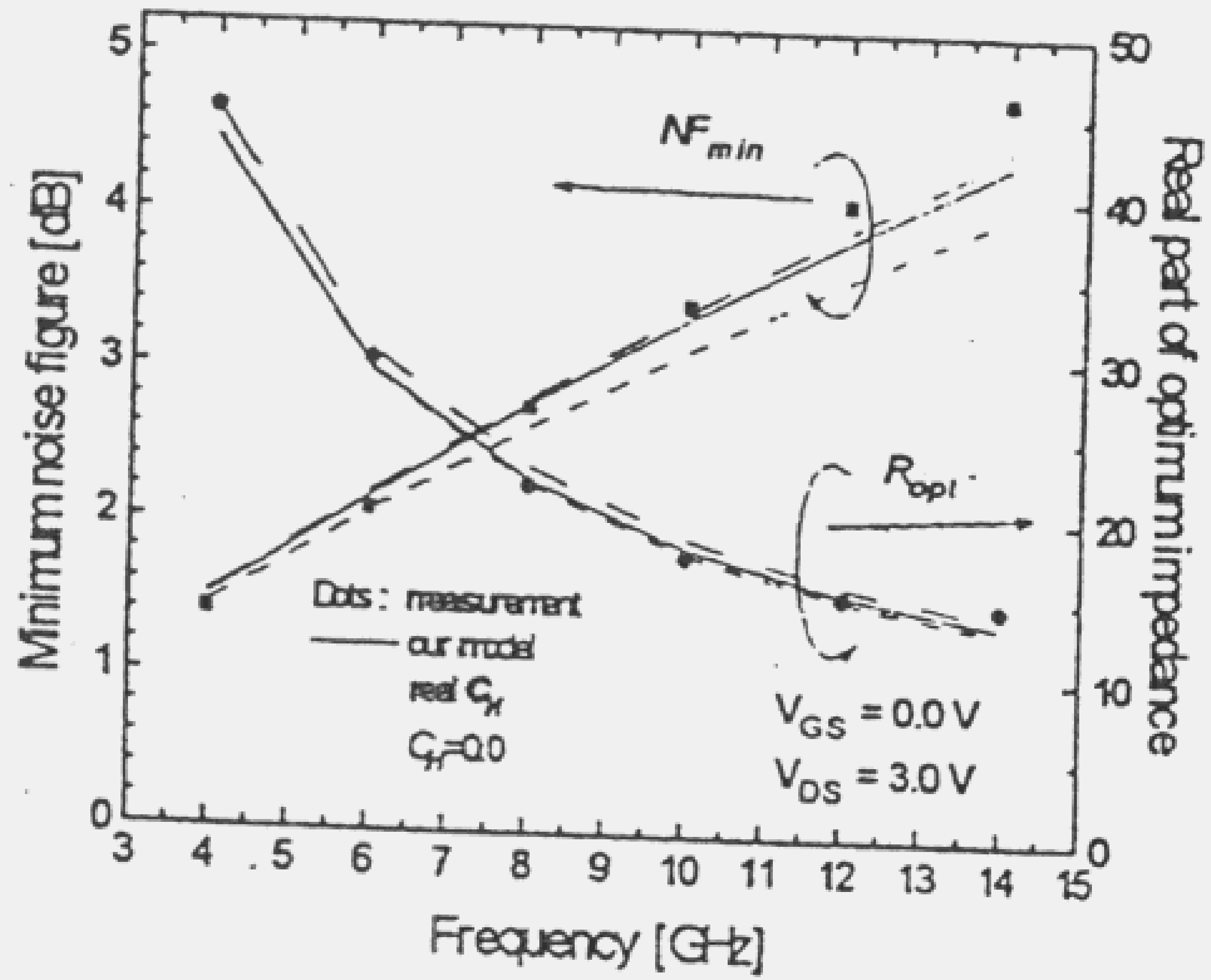

Fig.6 (a) NF_{min} and R_{opt} at V_{GS}, V_{DS}=0.0, 3.0V

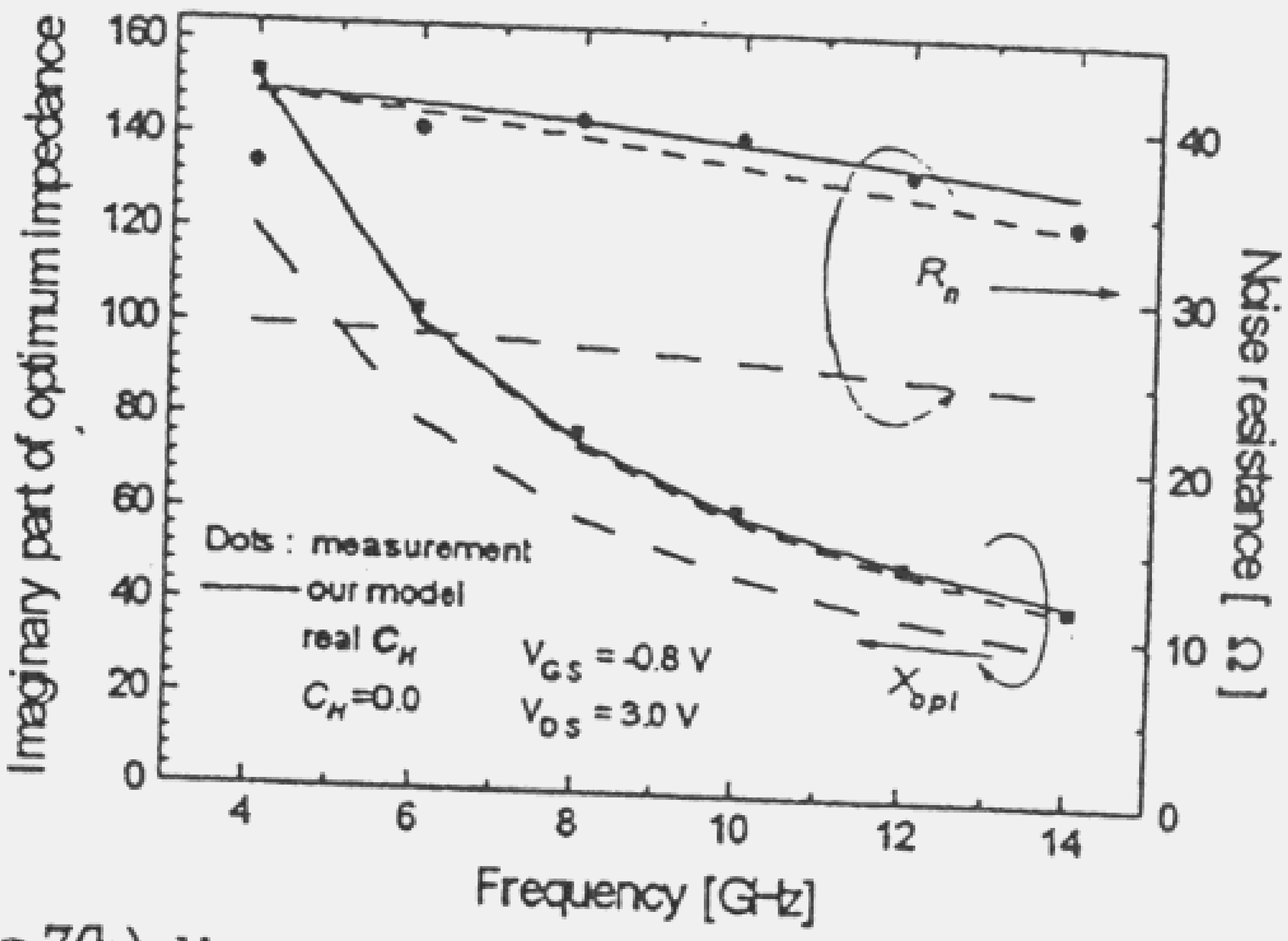

Fig.7(b) X_{opt} and R_n at V_{GS}, V_{DS} = 0.0, 3.0V. Dots are measured data. Solid lines are calculated one using our model, the short and long dashed lines are those with real C_H and zero C_H.