

# 3D Structure Recovery and On-line Calibration using Known Angles

Jong-Eun Ha

Department of Mechanical Engineering, KAIST  
373-1, Kusung-Dong, Yusong-Gu, Taejon, Korea  
jeha@cais.kaist.ac.kr

In-So Kweon

Department of Electrical Engineering, KAIST  
373-1, Kusung-Dong, Yusong-Gu, Taejon, Korea  
iskweon@cais.kaist.ac.kr

## ABSTRACT

A new calibration algorithm using only angles is presented. The proposed algorithm is based on the simple idea of the invariance of angles under the similarity transformation. Stratified calibration approaches using various scene constraints including angles have been proposed. However, the proposed algorithm uses only one type of scene constraint of angle, while directly recovering Euclidean structure from projective structure. We demonstrate the feasibility of the proposed algorithm through the experiment using synthetic and real images.

**Keywords:** Calibration, angle, 3D reconstruction

## 1. INTRODUCTION

Recent interesting applications such as virtual reality and augmented reality need metric information from image streams. The metric recovery of 3D scene structure from image sequences requires the calibration of cameras.

Classical calibration algorithms [1, 2] compute the relative positions of the cameras and their intrinsic parameters using a priori known 3D coordinates of points on a calibration target. After the calibration is done, we can compute Euclidean coordinates of scene points through the triangulation of the corresponding points between images. The need of calibration target limits its application in various areas such as varying cameras, on-line calibration, and 3D recovery from old video sequences. Therefore, the need of more relaxing calibration algorithm appears. The research toward this goal has been done in two directions: one is the calibration algorithm through the scene constraints other than the absolute 3D coordinates, and the other is the self-calibration that only requires correspondences among images.

In this paper, we focus on the calibration algorithm using scene constraints other than the absolute 3D coordinates. Faugeras [3] presented various scene constraints for the metric recovery of a scene, and he proposed a stratified framework: from projective to affine, and finally to Euclidean. He shows the relation between the image of the

absolute conic and known angle between two optical rays, which is a second order polynomial equation on the unknown coefficients of the image of the absolute conic. This relation is applicable to the known angle of lines between optical rays. To apply this as the constraint for the calibration using known angles between lines in 3D, the algorithm requires the information about the plane at infinity. Therefore, the algorithm presented by Faugeras [3] takes the form of stratified fashion.

In [4], experimental results by the algorithm described in [3] using real images are presented. They recovered affine geometry of a scene by computing a plane at infinity, which is determined from the vanishing points of the images of three sets of non-coplanar parallel lines. They recovered Euclidean structure of a scene up to a global scale using a priori information of some pairs of orthogonal lines. Boufama et al. [5] presented a calibration algorithm using scene constraints such as fixating a point, laying on the horizontal plane, vertical alignment and the distant between points in 3D, and they directly estimate the transformation matrix between the projective and the Euclidean using those scene constraints.

We present a calibration algorithm that only requires one type of scene constraint of known angles. The proposed algorithm directly recovers Euclidean structure of a scene up to the similarity transformation from projective structure using angles.

## 2. RELATED CALIBRATION METHODS USING ANGLES

In this section, we review the calibration algorithm of Faugeras [3] and Faugeras et al. [4] and compare them with the proposed method.

In [3], he first shows the relation between the image of the absolute conic and known angle between two optical rays. A conic is a curve defined by the locus of points on the projective plane and it satisfies the equation

$$S(\mathbf{x}) = \sum_{i,j=1}^3 a_{ij}x_i x_j = 0 \quad (1)$$

where  $a_{ij}$  is symmetric with respect to  $i$  and  $j$ . The matrix form of Eq.(1) is

$$S(\mathbf{x}) = \mathbf{x}^T \mathbf{W} \mathbf{x} = 0 \quad (2)$$

where  $\mathbf{W}$  is 3X3 symmetric matrix and defined up to a scale factor.

The angle between two optical rays in projective space can be defined using the Laguerre formula [6]. Let  $\mathbf{m}$  and  $\mathbf{n}$  be two image points and  $\langle \mathbf{C}, \mathbf{m} \rangle$  and  $\langle \mathbf{C}, \mathbf{n} \rangle$  be the two optical rays as shown in Fig. 1. Let  $\mathbf{a}$  be the angle two optical rays forms.  $\mathbf{M}_\infty$  and  $\mathbf{N}_\infty$  are the points of intersection with the plane at infinity by the two optical rays.  $\mathbf{U}$  and  $\mathbf{V}$  are the two points of intersections of the line  $\langle \mathbf{M}_\infty, \mathbf{N}_\infty \rangle$  with the absolute conic  $\Omega$ . The cross ratio  $\{\mathbf{M}_\infty, \mathbf{N}_\infty : \mathbf{U}, \mathbf{V}\}$  is preserved under the projection to the retinal plane. The angle between  $\langle \mathbf{C}, \mathbf{m} \rangle$  and  $\langle \mathbf{C}, \mathbf{n} \rangle$  is given by  $1/2i \log(\{\mathbf{m}, \mathbf{n}; \mathbf{u}, \mathbf{v}\})$  by the Laguerre formula.  $\mathbf{u}$  and  $\mathbf{v}$  are the images of points  $\mathbf{U}$  and  $\mathbf{V}$ . To obtain an angle we need the coordinates of  $\mathbf{u}$  and  $\mathbf{v}$  in the image plane.  $\mathbf{u}$  and  $\mathbf{v}$  are two intersection points of the line  $\langle \mathbf{m}, \mathbf{n} \rangle$  with the image of the absolute conic. Line  $\langle \mathbf{m}, \mathbf{n} \rangle$  can be represented by  $\mathbf{m} + q\mathbf{n}$ . The variable  $q$  is the projective parameter of line  $\langle \mathbf{m}, \mathbf{n} \rangle$ . The points  $\mathbf{u}$  and  $\mathbf{v}$  should lie on the image of the absolute conic. From this we obtain

$$S(\mathbf{m}) + 2qS(\mathbf{m}, \mathbf{n}) + S(\mathbf{n})q^2 = 0 \quad (3)$$

Let  $q_0$  and  $\bar{q}_0$  be the two roots that are complex conjugate and they are the projective parameters of  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\{\mathbf{m}, \mathbf{n}; \mathbf{u}, \mathbf{v}\} = \frac{0 - q_0}{0 - \bar{q}_0} : \frac{\infty - q_0}{\infty - \bar{q}_0} = \frac{q_0}{\bar{q}_0} = \exp[2i\mathbf{a}] \quad (4)$$

From the above equation, we can represent an angle  $\mathbf{a}$  as

$$\cos \mathbf{a} = -\frac{S(\mathbf{m}, \mathbf{n})}{\sqrt{S(\mathbf{m})S(\mathbf{n})}} \quad (5)$$

When we know the angle  $\mathbf{a}$  between two optical rays we can use it as estimating the image of the absolute conic. According to Eq. (5) this gives the following constraint on the coefficient of the equation of

$$S(\mathbf{m}, \mathbf{n})^2 = S(\mathbf{m})S(\mathbf{n})\cos^2 \mathbf{a} \quad (6)$$

is consisted of five independent parameters so that we need at least five angles between optical rays.

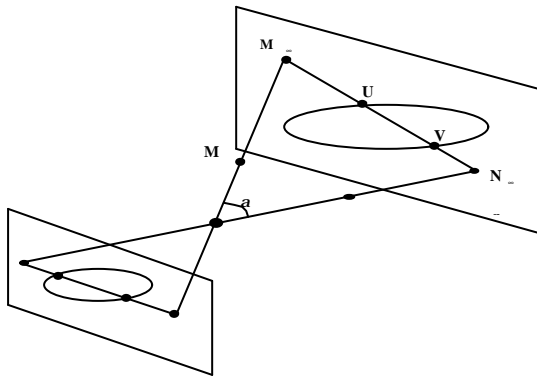


Fig. 1 The angle between two optical rays  $\langle \mathbf{C}, \mathbf{m} \rangle$  and  $\langle \mathbf{C}, \mathbf{n} \rangle$  by using the image of the absolute conic

Eq. (6) is a second order polynomial equation for the unknown  $q$ , thus we should solve the system of second order polynomial equation to obtain  $q$ . To apply this constraint for the calibration using angles between lines in 3D, Faugeras [3] suggests a stratified approach.

In the stratified approach of Faugeras et al. [4], they first recover affine geometry using a priori information such as 3D parallel lines, and then they recover Euclidean geometry of the scene up to a global scale factor using a priori information such as 3D angles between two lines in 3D. They recover sequentially projective structure, affine structure and Euclidean structure up to a global scale using a priori information according to the each step.

These approaches [3, 4] recover a metric structure in a stratified fashion and need various scene constraints according to each step. On the other hand, the proposed method recovers the homography matrix that directly relates projective structure and Euclidean structure without intermediate processes.

### 3. CAMERA CALIBRATION USING KNOWN ANGLES

The projection of a 3D scene point onto the image plane can be thought as a sequential step. First, there is a rigid body transformation between the world coordinates  $\mathbf{X}_w$  and the camera-centered coordinates  $\mathbf{X}_c$ . The next stage is perspective projection of  $\mathbf{X}_c$  onto  $\mathbf{x}$  in the image plane. Finally, the image coordinates  $\mathbf{x}$  are converted to the pixel coordinates  $\mathbf{m} = (u, v, 1)^T$ . These processes can be represented as:

$$\mathbf{m} = \mathbf{P}_{\text{euc}} \mathbf{X}_w \quad (7)$$

where

$$\mathbf{P}_{\text{euc}} = \mathbf{A} \mathbf{P}_0 \mathbf{T} = \begin{bmatrix} \mathbf{a}_u & \mathbf{g} & u_0 \\ 0 & \mathbf{a}_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}_3^T & 1 \end{bmatrix}$$

$\mathbf{P}_{\text{euc}}$  is the camera projection matrix for a perspective camera.  $\mathbf{a}_u$  and  $\mathbf{a}_v$  are the scale aspect of the  $x$  and  $y$  axis in the image,  $\mathbf{g}$  is a skew factor, and  $u_0$  and  $v_0$  are the image coordinate of the principal point.  $\mathbf{P}_0$  is the perspective projection matrix. In this projection under the pin-hole camera model, lens distortion is not considered. We assume that the skew in the camera is negligible:

$$\mathbf{g} = 0 \quad (8)$$

We can reconstruct a given scene up to the projective transformation only using corresponding points between images [8, 9]. In this case the projection equation is given by

$$\mathbf{m}_j^i \cong \mathbf{P}_{\text{proj}}^i \mathbf{Q} \mathbf{Q}^{-1} \mathbf{X}_{\text{proj}}^j \quad (9)$$

where  $\mathbf{m}_j^i$  is a 3x1 vector representing the  $j$ -th point in the  $i$ -th image,  $\mathbf{P}_{\text{proj}}^i$  is a 3x4 projective projection matrix of  $i$ -th camera,  $\mathbf{Q}$  is a 4x4 nonsingular homography matrix in  $P^3$  and  $\mathbf{X}_{\text{proj}}^j$  is a 4x1 projective structure in  $P^3$ .

Any nonsingular 4x4 matrix satisfies Eq. (9), therefore there could be infinite projective reconstructions that satisfy the correspondences. Among those  $\mathbf{Q}$  matrices, there exist a unique  $\mathbf{Q}$  matrix that transforms the projective structure to the Euclidean structure. Through this unique  $\mathbf{Q}$  matrix we can calibrate each camera and reconstruct a 3D scene up to similarity transformation. Such a  $\mathbf{Q}$  matrix satisfies the following relations:

$$\begin{aligned} \mathbf{P}_{euc}^i &\equiv \mathbf{P}_{proj}^i \mathbf{Q} \\ \mathbf{X}_{euc}^j &\equiv \mathbf{Q}^{-1} \mathbf{X}_{proj}^j \end{aligned} \quad (10)$$

where  $\mathbf{X}_{euc}^j$  is a metric structure of a 3D point.

In general, the matrix  $\mathbf{Q}$  consists of 15 parameters considering scale. We can further limit its element by selecting the world coordinate at the first camera's optical center. If we set the world coordinate system at the optical center of the first camera, we can obtain the projective projection matrix and the Euclidean projection matrix as  $\mathbf{P}_{proj}^0 = [\mathbf{I}_{3 \times 3} \ \mathbf{0}_3]$  and  $\mathbf{P}_{euc}^0 = [\mathbf{A}_0 \ \mathbf{0}_3]$ . If we substitute these relations into Eq. (10), we can obtain the element of  $\mathbf{Q}$  matrix:

$$\begin{aligned} \mathbf{P}_{euc}^0 &\equiv \mathbf{P}_{proj}^0 \mathbf{Q} \Leftrightarrow [\mathbf{A}_0 \ \mathbf{0}_3] \equiv [\mathbf{I}_{3 \times 3} \ \mathbf{0}_3] \mathbf{Q} \\ &\Leftrightarrow \exists (q_1 \ q_2 \ q_3 \ q_4) \left| \mathbf{Q} \equiv \begin{bmatrix} \mathbf{A}_0 & \mathbf{0}_3 \\ q_1 & q_2 & q_3 & q_4 \end{bmatrix} \right. \end{aligned} \quad (11)$$

Since  $\mathbf{Q}$  is defined up to a scale, we can represent it as:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{a}_u & 0 & u_0 & 0 \\ 0 & \mathbf{a}_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \\ q_1 & q_2 & q_3 & 1 \end{bmatrix} \quad (12)$$

Finally the matrix  $\mathbf{Q}$  is consisted of the intrinsic parameters of the first camera and the coefficient of plane at infinity,  $(q_1, q_2, q_3)$ .

Next, we describe an algorithm for computing unknown matrix  $\mathbf{Q}$  using known angles. Let us assume three points  $\mathbf{B}$ ,  $\mathbf{C}$  and  $\mathbf{D}$  are specified with their world coordinates  $\mathbf{X}_w^B$ ,  $\mathbf{X}_w^C$  and  $\mathbf{X}_w^D$  as shown in Fig. 2. If we set the world coordinate at the optical center of the first camera and if we know  $\mathbf{Q}$  matrix in Eq. (10), we can have Euclidean structure,  $s\mathbf{X}_w^B$ ,  $s\mathbf{X}_w^C$  and  $s\mathbf{X}_w^D$ , up to a global scale.

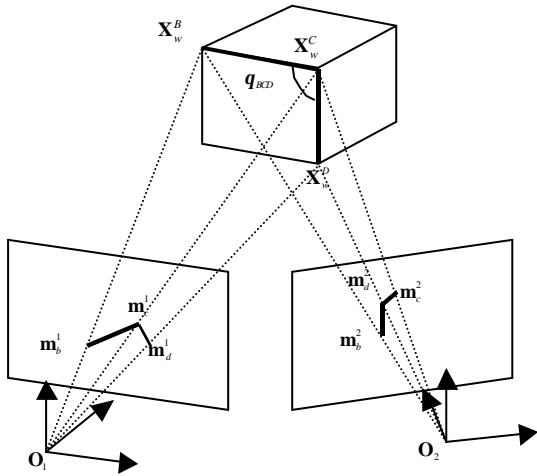


Fig. 2 The configuration of the imaging system

Since the Euclidean structure is recovered up to a scale, angle formed by the three points is invariant under the similarity transformation:

$$\cos q_{BCD} = \frac{\vec{\mathbf{CB}} \cdot \vec{\mathbf{CD}}}{\|\vec{\mathbf{CB}}\| \|\vec{\mathbf{CD}}\|} = \frac{\vec{\mathbf{C'B'}} \cdot \vec{\mathbf{C'D'}}}{\|\vec{\mathbf{C'B'}}\| \|\vec{\mathbf{C'D'}}\|} = \cos q_{B'C'D'} \quad (13)$$

where  $\vec{\mathbf{CB}} = \mathbf{X}_w^B - \mathbf{X}_w^C$ ,  $\vec{\mathbf{CD}} = \mathbf{X}_w^D - \mathbf{X}_w^C$ ,  $\vec{\mathbf{C'B'}} = s\mathbf{X}_w^B - s\mathbf{X}_w^C$  and  $\vec{\mathbf{C'D'}} = s\mathbf{X}_w^D - s\mathbf{X}_w^C$ .

Set  $\mathbf{Q}^{-1}$  and the projective structure of  $j$ -th point as:

$$\mathbf{Q}^{-1} = \begin{pmatrix} \mathbf{a}_u & 0 & u_0 & 0 \\ 0 & \mathbf{a}_v & v_0 & 0 \\ 0 & 0 & 0 & 0 \\ q_1 & q_2 & q_3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{Q}_1^{-T} \\ \mathbf{Q}_2^{-T} \\ \mathbf{Q}_3^{-T} \\ \mathbf{Q}_4^{-T} \end{pmatrix} \quad \mathbf{X}_{proj}^j = \begin{pmatrix} p_j \\ q_j \\ r_j \\ s_j \end{pmatrix} \quad (14)$$

where  $\mathbf{Q}_i^{-T}$  is the  $i$ -th row of  $\mathbf{Q}^{-1}$ .

By substituting Eq. (14) into Eq. (10), the Euclidean structure of  $j$ -th point can be represented as the combination of unknown matrix  $\mathbf{Q}$  and known projective structure.

$$\begin{aligned} \mathbf{X}_{euc}^j &= (X_{euc}^j \ Y_{euc}^j \ Z_{euc}^j)^T \\ &= \left( \frac{\mathbf{Q}_1^{-T} \cdot \mathbf{X}_{proj}^j}{\mathbf{Q}_4^{-T} \cdot \mathbf{X}_{proj}^j} \ \frac{\mathbf{Q}_2^{-T} \cdot \mathbf{X}_{proj}^j}{\mathbf{Q}_4^{-T} \cdot \mathbf{X}_{proj}^j} \ \frac{\mathbf{Q}_3^{-T} \cdot \mathbf{X}_{proj}^j}{\mathbf{Q}_4^{-T} \cdot \mathbf{X}_{proj}^j} \right)^T \end{aligned} \quad (15)$$

Using Eq. (15), we can derive a following constraint to compute unknown matrix  $\mathbf{Q}$  using the known angle  $q_{BCD}$ .

$$q_{BCD} = \cos^{-1} \left( \frac{\vec{\mathbf{CB}} \cdot \vec{\mathbf{CD}}}{\|\vec{\mathbf{CB}}\| \|\vec{\mathbf{CD}}\|} \right) = f(\mathbf{a}_u, \mathbf{a}_v, u_0, v_0, q_1, q_2, q_3) \quad (16)$$

Therefore, scene constraints - the angle invariance under similarity transformation - can be translated to a constraint on the intrinsic parameters. Eq. (16) is used to obtain the Euclidean projection matrix from the projective one. A nonlinear least squares method provides an approximate solution by minimizing the following criterion:

$$\begin{aligned} &\min \sum_{i=1}^N \left( q_{B_i C_i D_i} - \cos^{-1} \left( \frac{\vec{\mathbf{C}_i \mathbf{B}_i} \cdot \vec{\mathbf{C}_i \mathbf{D}_i}}{\|\vec{\mathbf{C}_i \mathbf{B}_i}\| \|\vec{\mathbf{C}_i \mathbf{D}_i}\|} \right) \right)^2 \\ &= \min \sum_{i=1}^N (q_{B_i C_i D_i} - f_i(\mathbf{a}_u, \mathbf{a}_v, u_0, v_0, q_1, q_2, q_3))^2 \end{aligned} \quad (17)$$

Each known angle from three points gives one constraint for the calibration. We use the Levenberg-Marquardt method [10] to solve the nonlinear minimization problem. An algorithm for the computation of the initial values for  $(q_1, q_2, q_3)$  is presented in the next section.

Projective reconstruction is done as follows. First, we compute the Fundamental matrix from correspondence between images using the method in [12]. We can compute the projective projection matrix from the Fundamental matrix, and finally we obtain the projective structure through the triangulation [7]. Projective reconstructions between two views have different projective basis, therefore it is necessary to equalize the projective reconstruction in a common basis. For this, we use the

method in [13]. Intrinsic parameters and extrinsic parameters are computed through the decomposition of the estimated Euclidean projection matrix [14].

#### 4. INITIAL VALUES FOR THE NONLINEAR MINIMIZATION

We need initial values to run the nonlinear minimization. The cost function of Eq. (17) has many local minima. Thus it is important to have good initial values close to the true ones to guarantee the convergence. The initial values by [11] often do not guarantee the convergence. This is due to the fact that a least-squares solution using the assumption that intrinsic parameters are constant under the varying cameras makes the initial values even worse. We propose a new initialization method for  $(q_1, q_2, q_3)^T$  using only two views.

Euclidean projection matrix can be represented as:

$$\mathbf{P}_{\text{euc}}^i = \mathbf{A}_i [\mathbf{R}_i \mathbf{t}_i]$$

where  $\mathbf{A}_i$  is the matrix consisted of the intrinsic parameters,  $\mathbf{R}_i$  is rotation matrix and  $\mathbf{t}_i$  is translation vector. Now, if we denote  $\mathbf{Q}^*$  as the first three columns of the matrix  $\mathbf{Q}$ , we can derive following relation from Eq. (10).

$$\mathbf{A}_i \mathbf{R}_i \cong \mathbf{P}_{\text{proj}}^i \mathbf{Q}^* \quad (18)$$

From Eq. (18), it follows

$$\begin{aligned} (\mathbf{A}_i \mathbf{R}_i)(\mathbf{A}_i \mathbf{R}_i)^T &\cong (\mathbf{P}_{\text{proj}}^i \mathbf{Q}^*) (\mathbf{P}_{\text{proj}}^i \mathbf{Q}^*)^T \\ &\cong \mathbf{P}_{\text{proj}}^i \mathbf{P}_{\text{proj}}^{iT} \quad \left( \begin{array}{l} \mathbf{A}_i \mathbf{R}_i \mathbf{A}_i^T = \mathbf{Q}^* \mathbf{Q}^{*T} \end{array} \right) \end{aligned} \quad (19)$$

$w_i$  is the dual image absolute conic and  $\Omega$  is the absolute dual quadric.

From Eq. (19), we obtain

$$\begin{aligned} \mathbf{I}_i &\begin{bmatrix} f_i^2 + u_0^2 & u_0 v_0 & u_0 \\ u_0 v_0 & f_i^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix} = \\ \mathbf{P}_{\text{proj}}^i &\begin{bmatrix} f_i^2 + u_0^2 & u_0 v_0 & u_0 & f_1 q_1 + u_0 q_3 \\ u_0 v_0 & f_i^2 + v_0^2 & v_0 & f_1 q_2 + v_0 q_3 \\ u_0 & v_0 & 1 & q_3 \\ f_1 q_1 + u_0 q_3 & f_1 q_2 + v_0 q_3 & q_3 & \|\mathbf{q}\|^2 \end{bmatrix} \mathbf{P}_{\text{proj}}^{iT} \quad (20) \\ &(f = \mathbf{a}_u = \mathbf{a}_v) \end{aligned}$$

We assume that the intrinsic parameters are constant through the first and the second camera. We use  $f$  computed from the algorithm proposed in [11], and take the initial values of the principal point as the center of the first image. Then, we are left with four unknowns  $\mathbf{I}_i$ ,  $(q_1, q_2, q_3)^T$  and Eq. (20) provides 6 equations. We compute the unknowns using four equations. Thus, we avoid the false initial values by least-squares using overconstrained equations. Experimental results support this fact.

#### 5. EXPERIMENTAL RESULTS

##### Comparison with Tsai calibration algorithm

First we compare the proposed algorithm with Tsai's [1] calibration algorithm. Fig. 3 shows an input image sequence,

which are captured while changing the position and the focus of camera. For the Tsai calibration, we used the 75 points on the calibration box, and 15 control angles for the proposed calibration algorithm.

Table 1 shows the initial values of  $(q_1, q_2, q_3)$  by the method in [11] and by the proposed algorithm. The true values of  $(q_1, q_2, q_3)$  are (-50.5, 12.33, -36.3). The method in [11] employs least-squares and presents a linear algorithm. We used 2~6 images for the least-squares solutions of the algorithm in [11]. The proposed method for the initial values of  $(q_1, q_2, q_3)$  provides two solutions at a fixed focal length. The proposed algorithm gives more accurate initial values, which guarantee the convergence of gradient-based search. The initial values computed by the method in [11] are often far from the ground truth values and result in a local minimum.

Table 2 shows the intrinsic parameters computed by the proposed and Tsai's calibration algorithm, respectively. We can observe that both results are comparable each other.

Tables 3 and 4 show the rotation and the translation parameters computed by the proposed and Tsai's algorithm. Specifically, the proposed algorithm computes the rotation and the translation by decomposing the estimated Euclidean projection matrix. In Table 3, the relative error of the rotation parameters with respect to the Tsai algorithm is presented. Table 4 shows the angular directional error of the estimated unit translation vector to the Tsai algorithm. Our angle-based algorithm computes accurate motion parameters, which are comparable with those by the Tsai algorithm, only using known angles between lines.

We reconstruct the given scene up to the similarity transformation. Fig. 4 shows the texture-mapped 3D structure with a proper scale, which is selected arbitrarily for display purpose. We can observe that the estimated 3-D structure preserves the good orthogonality and planarity of the original structure.

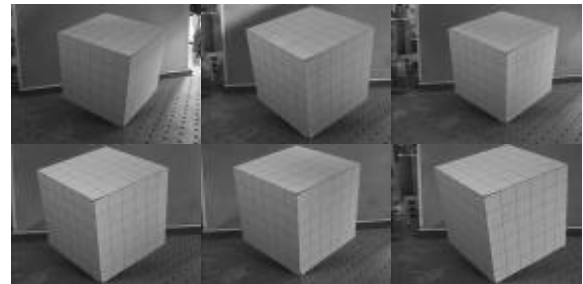


Fig. 3 The calibration box image sequence

Table 1 Comparison of the initial values of  $(q_1, q_2, q_3)$

Initialization method by [11]		Proposed initialization method	
$f$	$(q_1, q_2, q_3)$	$f$	$(q_1, q_2, q_3)$
760	(0.0, -6.84, -5.13)	650	(-39.4, 10.1, -37.3),
760	(-81.8, 1.78, -8.44)		(42.7, -1.34, -35.1)
760	(-82.6, 1.66, -7.85)	760	(-46.1, 11.6, -41.6),
760	(-94.1, -0.30, 0.54)		(50.1, -1.74, -38.6)
760	(-86.3, 0.92, -5.00)	960	(-57.2, 14.0, -50.9),
			(62.9, -2.35, -46.0)

Table 2 The computed intrinsic parameters.

	Proposed algorithm	Tsai [1] algorithm
	$\mathbf{a}_u, \mathbf{a}_v, u_0, v_0$	$\mathbf{a}_u, \mathbf{a}_v, u_0, v_0$
C1	759.3,759.3,340.6,228.8	758.9,760.0,340.4,230.3
C2	880.8,880.2,352.3,221.5	880.2,881.0,353.2,223.2
C3	860.3,860.3,366.9,219.1	859.9,861.2,366.6,220.8
C4	1092.2,1093.1,332.8,230.1	1091.6,1094.1,332.6,232.1
C5	1007.6,1006.6,335.6,232.8	1007.1,1007.6,335.4,234.7
C6	1032.7,1033.5,326.6,229.7	1032.0,1034.5,326.5,231.5

Table 3 The computed rotation parameters.

	Rotation [degree] $(q_x, q_y, q_z)$	
	Tsai [1]	Proposed (relative error [%])
C1-C2	(-1.028,-15.384,-4.071)	0.25
C1-C3	(1.512,-9.616,-2.655)	0.23
C1-C4	(1.039,-15.754,-3.248)	0.30
C1-C5	(1.472,-11.108,-1.945)	0.36
C1-C6	(0.895,-23.558,-4.386)	0.33

Table 4 The computed translation vector.

	Unit translation $(T_x, T_y, T_z)$	
	Tsai [1]	Proposed (angular directional error)
C1-C2	(-0.9697,0.0697,0.2343)	0.030 $^\circ$
C1-C3	(-0.6883,-0.1493,0.7099)	0.063 $^\circ$
C1-C4	(-0.7549,-0.0335,0.6550)	0.076 $^\circ$
C1-C5	(-0.7404,-0.1194,0.6615)	0.063 $^\circ$
C1-C6	(-0.8644,-0.0775,0.4968)	0.053 $^\circ$

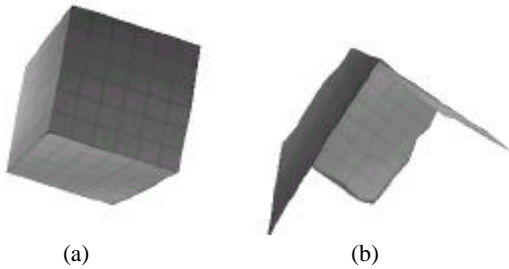


Fig. 4 The estimated 3-D structure by the proposed algorithm: (a) A texture mapped display; (b) a different perspective to check the orthogonality.

### Experimental results using real images

Fig. 5 shows a sequence of images of an outdoor building scene captured by a hand-held camcorder. Fig. 5(a) shows the control angles used for the calibration, and we assume the true value of the angle is  $90^\circ$ . The mean and standard deviation of angles formed by the control points from the estimated 3D structure is  $92.2^\circ/3.27^\circ$ . It is a good estimate since the window frames are manufactured to have the right angle. Fig. 6 shows the estimated 3D structure with a texture map. We can observe that the coplanarity of the original structure is well preserved in the recovered structure.



Fig. 5 Outdoor building images captured by a hand-held camcorder (a) the first image and the control angles (b) the second image.

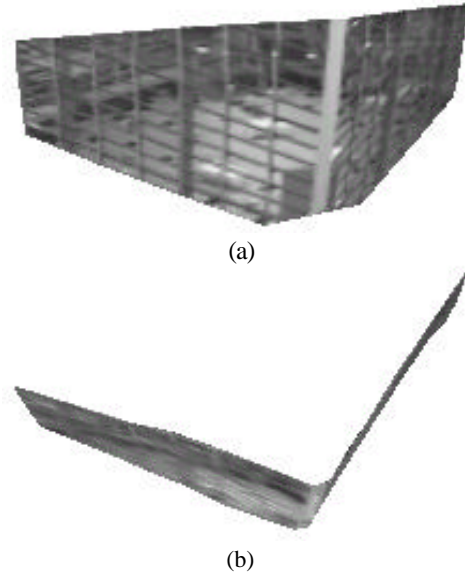


Fig. 6 (a) Texture mapped display of the estimated 3D structure by the proposed algorithm (b) a different view.

## 6. CONCLUSIONS

We have presented a calibration algorithm using a specific scene constraint of angles. The proposed algorithm directly recovers Euclidean structure up to the similarity transformation from projective structure as well as the camera parameters using one type scene constraint of angle. A similar angle-based stratified algorithm requires various scene constraints according to each step.

We have also shown that the proposed algorithm, only assuming known angles, provides calibration parameters whose accuracy is comparable with the Tsai calibration algorithm. One limitation of the proposed algorithm is that it needs a priori 3D scene information. However, we envision that this method may be effective to scenes consisting of man-made objects.

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