Propagation of Shock Waves in the Two-Phase Media

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이상유동에서의 충격파 전파 특성

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A mathematical formulation based on two-phase, two-fluid hyperbolic conservation laws is developed to investigate propagation of shock waves in one- and two-dimensions. We used a high resolution upwind scheme called the split-coefficient matrix method. Two extreme cases are computed for validation of the computer code: the states of a pure gas and a pure liquid. Computed results agreed well with the previous experimental and numerical results. It is studied how the shock wave propagation pattern is affected by the void fraction in the two-phase flow. The shock structure in a two-phase flow turned out, in fact, much deviated from the shape well known in the gas only phase.

Key Words: 이상유동(Two-Phase Flow), 이유체모델(Two-Fluid Model), 충격파(Shock Wave), 표면장력항(Surface Tension Force Terms)

1. Introduction

The subject of a two-phase flow has been of great importance in a wide variety of engineering problems. Despite its importance in engineering applications, understanding on the two phase phenomena is not sufficient due to complexities and uncertainties.

Mathematical formulation of a time-averaging two-phase flow uses models of two categories: a *two-fluid* model and a *mixture* (diffusion) model [1]. The mixture model considers that the two-phase flow has representative properties just like a single-phase. A set of governing equations are therefore formulated using the mixture properties. In contrast, the two-fluid model deals with each phase separately, and two sets of governing equations are needed in this case. More closure models are necessary in the two-fluid model than in the mixture model. For transient two-phase flow problems, it is regarded that the two-fluid model offers more general and detailed solution than the mixture model, even though experiments for a reliable closure model is insufficient so far.

Major feature of the two-fluid model is that the equation system is non-conservative and ill-posed due to the interfacial pressure terms; for example see [2]. Numerically it makes difficult to employ reliable and efficient conservative upwind schemes, for instance, those referred in Toro[3]. In addition, it is well known that the governing equations have complex eigenvalues [4], making the initial value problem ill posed and the numerical method unstable. Many remedies have appeared to rescue

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the numerical methods. They are categorized by two approaches: development of better physical closure models and improvement of numerical methods [5]. The virtual mass terms is well known and have been very successful in improving stability of the governing equation system; today, various virtual mass terms are employed in the commercial computer codes such as RELAP5 and CATHARE [5].

On the other hand, Lee and Chung [6] have analytically obtained the real eigenvalues by including the surface tension force terms in the governing equations. It remarkably improved the system stability. In their work, the system eigenvalues were in good agreement with the experimentally measured speed of sound in the two-phase flow. In the present work, we adopt the surface tension model by Lee and Chung to compute shock wave propagation in the two phase flow.

Various numerical schemes have been used to solve a two-phase flow: Roe-type schemes by Toumi *et al.* [2], flux vector splitting method by Chung *et al.* [6], hybrid-flux splitting scheme by Evje and Flatten [7], and split-coefficient matrix method by Tiselj & Petelin [5] and Ouyang & Aziz [8][9]. Here we employ the split-coefficient matrix method. It is efficient and easy to implement for a complicated non-conservative system. We study numerically in one- and two-dimensions the shock wave propagation in a two-phase flow with sample void fractions α, $0 \le \alpha \le 1$.

2. Governing equations

The governing equations including the surface tension effect were given by Chung *et al.* [6] in one-dimension. Here we expand the formulation to the two dimensions:

Mass

$$\frac{\partial}{\partial t}(\alpha_k\rho_k) + \frac{\partial}{\partial x}(\alpha_k\rho_k u_k) + \frac{\partial}{\partial y}(\alpha_k\rho_k v_k) = 0 \qquad (1)$$

x Momentum:

$$\frac{\partial}{\partial t}(\alpha_k\rho_k u_k) + \frac{\partial}{\partial x}(\alpha_k\rho_k u_k^2) + \frac{\partial}{\partial y}(\alpha_k\rho_k u_k v_k) + \alpha_k\frac{\partial p_k}{\partial x} + \Delta p_k^i\frac{\partial \alpha_k}{\partial x} = 0 \qquad (2)$$

y Momentum:

$$\frac{\partial}{\partial t}(\alpha_k\rho_k v_k) + \frac{\partial}{\partial x}(\alpha_k\rho_k u_k v_k) + \frac{\partial}{\partial y}(\alpha_k\rho_k v_k^2) + \alpha_k\frac{\partial p_k}{\partial y} + \Delta p_k^i\frac{\partial \alpha_k}{\partial y} = 0 \qquad (3)$$

Internal energy

$$\frac{\partial}{\partial t}(\alpha_k\rho_k e_k) + \frac{\partial}{\partial x}(\alpha_k\rho_k u_k e_k) + \frac{\partial}{\partial x}(\alpha_k\rho_k v_k e_k) + p_k\left[\frac{\partial \alpha_k}{\partial t} + \frac{\partial}{\partial x}(\alpha_k u_k) + \frac{\partial}{\partial y}(\alpha_k v_k)\right] = 0 \qquad (4)$$

where α_k, ρ_k, u_k, v_k, e_k, and p_k denotes void fraction, density, x-velocity, y-velocity, internal energy, and common pressure of the k-phase, respectively; the term $\Delta p_k^i \equiv (p_k - p_k^i)$ is the interfacial pressure jump. The subscript k is written as $k=g$ for the gas and $k=l$ for the liquid phase. The void fraction obeys $\alpha_g + \alpha_l = 1$.

Here we neglect the various source terms, such as a mass transfer, energy transfer and friction at an interface of the two phases, the eigenvalues of the governing equations being not changed by their presence. We will consider the interfacial pressure jump term. For more information on the source terms, the readers may refer to Tiselj & Petelin [5] or Ishii [1].

3. Numerical method

3.1 Split-coefficient matrix scheme

The governing equations (1)-(4) can be transformed into a compact matrix form using primitive variables, $W = (\alpha_g, p, u_g, u_l, v_g, v_l, e_g, e_l)^T$. If we apply the split-coefficient matrix scheme, it gives

$$\frac{\partial W}{\partial t}+\Phi^{+}\frac{\partial W}{\partial x}+\Phi^{-}\frac{\partial W}{\partial x}+\Psi^{+}\frac{\partial W}{\partial y}+\Psi^{-}\frac{\partial W}{\partial y}=00 \tag{5}$$

When discretized,

$$\begin{aligned}&\frac{W_{i,j}^{n+1}-W_{i,j}^{n}}{\Delta t}+\Phi_{i-1/2,j}^{+}\frac{W_{i,j}^{n}-W_{i-1,j}^{n}}{\Delta x}\\&+\Phi_{i+1/2,j}^{-}\frac{W_{i+1,j}^{n}-W_{i,j}^{n}}{\Delta x}+\Psi_{i,j-1/2}^{+}\frac{W_{i,j}^{n}-W_{i,j-1}^{n}}{\Delta y}\\&+\Psi_{i,j+1/2}^{-}\frac{W_{i,j+1}^{n}-W_{i,j}^{n}}{\Delta y}=0\end{aligned} \tag{6}$$

where split-coefficient matrices are defined as

$$\begin{aligned}\Phi_{i+1/2,j}^{\pm}&=R(W_{i+1/2,j})\Lambda^{\pm}(W_{i+1/2,j})R^{-1}(W_{i+1/2,j})\\\Psi_{i,j+1/2}^{\pm}&=K(W_{i,j+1/2})\Omega^{\pm}(W_{i,j+1/2})K^{-1}(W_{i,j+1/2})\end{aligned} \tag{7}$$

Here the eigenvalues and the eigenvectors are numerically obtained by the LAPACK subroutines [10]. The interfacial states $W_{i+1/2,j}$ and $W_{i,j+1/2}$ may be obtained simply by an algebraic average of the left and right cells.

3.2 Second-order extension

A second-order extension of the scheme can be achieved by the Lax-Wendroff method. In order to prevent a spurious oscillation, we employ a flux limiter. The final form then becomes

$$\begin{aligned}&\frac{W_{i,j}^{n+1}-W_{i,j}^{n}}{\Delta t}+\Phi_{i-1/2,j}^{++}\frac{W_{i,j}^{n}-W_{i-1,j}^{n}}{\Delta x}\\&+\Phi_{i+1/2,j}^{--}\frac{W_{i+1,j}^{n}-W_{i,j}^{n}}{\Delta x}+\Psi_{i,j-1/2}^{++}\frac{W_{i,j}^{n}-W_{i,j-1}^{n}}{\Delta y}\\&+\Psi_{i,j+1/2}^{--}\frac{W_{i,j+1}^{n}-W_{i,j}^{n}}{\Delta y}=0\end{aligned} \tag{8}$$

The coefficient matrices are

$$\begin{aligned}\Phi_{i+1/2,j}^{\pm\pm}&=R(W_{i+1/2,j})\Lambda^{\pm\pm}(W_{i+1/2,j})R^{-1}(W_{i-1/2,j})\\\Psi_{i,j+1/2}^{\pm\pm}&=K(W_{i,j+1/2})\Omega^{\pm\pm}(W_{i,j+1/2})K^{-1}(W_{i,j-1/2})\end{aligned} \tag{9}$$

where the eigenvalue matrices are defined as

$$\begin{aligned}\Lambda^{\pm\pm}&=diag(\lambda_1^{\pm\pm},\lambda_2^{\pm\pm},\dots,\lambda_8^{\pm\pm})\\\Omega^{\pm\pm}&=diag(w_1^{\pm\pm},w_2^{\pm\pm},\dots,w_8^{\pm\pm})\end{aligned} \tag{10}$$

Here the elements of the diagonal matrices are given by

$$\begin{aligned}\lambda_\alpha^{++}&=\max(0,\lambda_\alpha)+\frac{\xi_\alpha\lambda_\alpha}{2}\left(\lambda_\alpha\frac{\Delta t}{\Delta x}-sign(\lambda_\alpha)\right)\\\lambda_\alpha^{--}&=\min(0,\lambda_\alpha)-\frac{\xi_\alpha\lambda_\alpha}{2}\left(\lambda_\alpha\frac{\Delta t}{\Delta x}-sign(\lambda_\alpha)\right)\\w_\alpha^{++}&=\max(0,w_\alpha)+\frac{\eta_\alpha w_\alpha}{2}\left(w_\alpha\frac{\Delta t}{\Delta y}-sign(w_\alpha)\right)\\w_\alpha^{--}&=min(0,w_\alpha)-\frac{\eta_\alpha w_\alpha}{2}\left(w_\alpha\frac{\Delta t}{\Delta y}-sign(w_\alpha)\right)\end{aligned} \tag{11}$$

where $\alpha=1,2,\dots,8$; ξ_α and η_α represent the flux limiters. The function $sign(x)$ gives +1 for $x\geq 0$ and -1 for $x<0$. The minmod (minbee) limiters are given by

$$\begin{aligned}\xi_\alpha&=max(0,min(1,r_\alpha))\\\eta_\alpha&=max(0,min(1,s_\alpha))\end{aligned} \tag{12}$$

where r_α and s_α are the ratios of an *upwind* change of the primitive variable vector $W=(W_1,\dots,W_8)^T$.

$$\begin{aligned}(r_\alpha)_{i+1/2,j}&=\frac{(W_\alpha)_{i+1-m,j}-(W_\alpha)_{i-m,j}}{(W_\alpha)_{i+1,j}-(W_\alpha)_{i,j}}\\(s_\alpha)_{i,j+1/2}&=\frac{(W_\alpha)_{i,j+1-k}-(W_\alpha)_{i,j-k}}{(W_\alpha)_{i,j+1}-(W_\alpha)_{i,j}}\end{aligned} \tag{13}$$

where

$$m=sign((\lambda_\alpha)_{i+1/2,j})\ \text{and}\ k=sign((w_\alpha)_{i,j+1/2}) \tag{14}$$

4. Results

4.1 Shock wave diffraction over a 90-degree sharp corner in gas only phase

This test problem has been well known numerically and experimentally: see [11]. We here set the (gas) void fraction at a unit minus an infinitely small value, say, $\alpha = 1 - 10^{-9}$. Uniformly fine mesh made of 675,000 nodes are used in the computational domain. Figure 1 shows that the computed flow field appears similar to other researchers' experimental and numerical results. The present scheme is observed relatively more dissipative than other upwind schemes, for instance, Roe scheme, AUSM-family schemes, etc. But the result is quite satisfactory, considering that the present computation is based on the two-phase, two-fluid formulation to investigate a single-phase problem.

4.2 Underwater shock explosion

Underwater explosion has been investigated a lot experimentally by many researchers to study shock wave generation and propagation in water. Kira *et al.* [12] investigated the attenuation of an underwater shock wave using spherical explosives

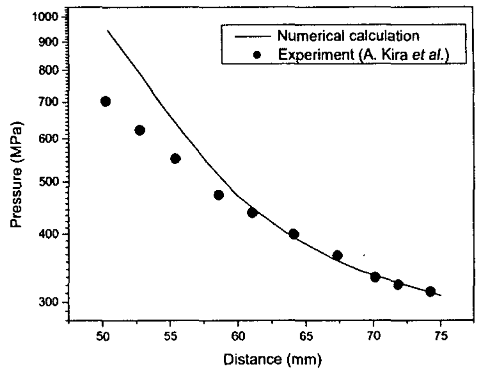

Fig.2 Comparison between numerical and experimental data of underwater shock explosion of spherical explosive

(silver azide pellets) in water. Present numerical computation is carried out in one-dimensional spherical coordinates and the result is compared with Kira's for the case having no gauze layer. Void fraction is set to an infinitely small value, say, $\alpha = 10^{-9}$. Figure 2 shows underwater shock attenuation, in comparison with the Kira's experiment for the radius R=18*mm* of explosives. The numerical calculation overestimates the experiment for a short distance. This error is believed mainly due to the ideal gas-type equation of state (EOS) used for water, i.e., $\gamma = 1.0005$.

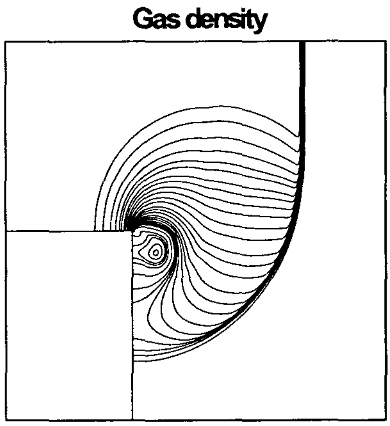

Fig.1 Density contours in gas-only phase

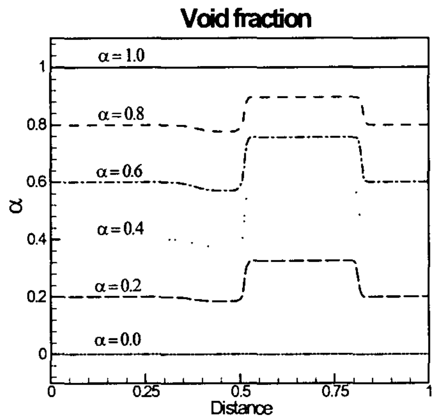

Fig.3 Void fraction for Sod-type two-phase shock tube problem

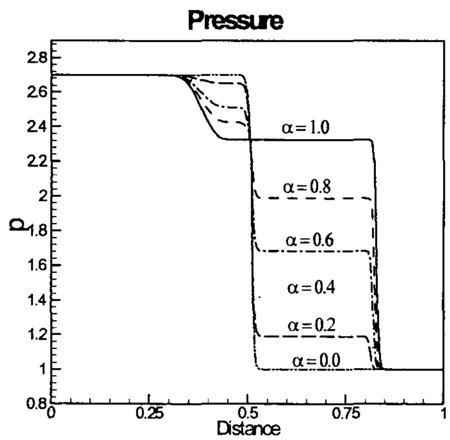

Fig.4 Pressure for Sod-type two-phase shock tube problem

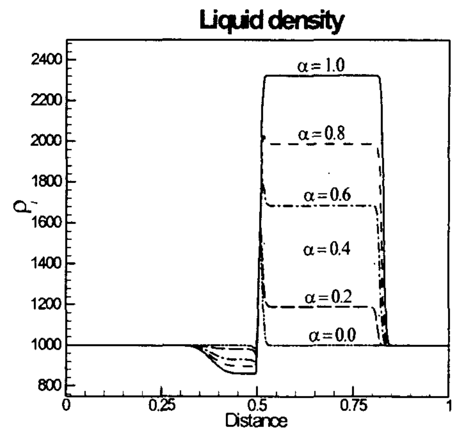

Fig.6 Liquid density for Sod-type two-phase shock tube problem

More accurate EOS may improve the results, for example, the Tammann EOS [13]. For a long distance the numerical computation well estimates the attenuation of the shock wave in water.

4.3 Sod-type two-phase shock tube problem

This test problem is analogous to the Sod test of a single phase. The Sod problem is very popular and has been used as a benchmark by many authors since it generates all possible waves simultaneously; a shock wave, a rarefaction wave, and a contact discontinuity. For $\alpha=1$, the problem is the same as the single-phase Sod test. Here we examine the effect of void fraction by using discrete values from $\alpha=1$ to $\alpha=0$. The initial discontinuity (physically a diaphragm) is located at $x=0.5$.

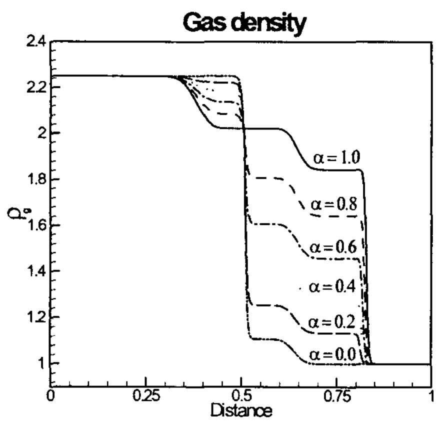

Fig.5 Gas density for Sod-type two-phase shock tube problem

Here the initial liquid velocity u_l on the left is relatively depressed while the influence of the volume fraction on gas is clearer . In Figures 3-6, the shock strength gradually decreases with increasing void fraction; this result is in accordance with our expectation that the strength of a shock wave in two-phase flows is increasingly attenuated as the flow regime gradually changes from the pure gas to the pure liquid.

5. Conclusions

The two-phase code has been validated by computing the two single phases, i.e., the pure gas and the pure liquid states. Our numerical results agreed well with previous experimental and numerical results. The code was robust and reliable.

One-dimensional test cases have shown the major features of a shock wave in the two-phase flow. The structure of shock waves was considerably influenced by the void fraction and the slip velocity. The two-dimensional structure of shock wave is also dependent on the void fraction of the two phase flow. Considerable experiments are necessary, however, to produce results that can be benchmarked by the computer code developers.

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