# A New Analytic Method for IEEE 802.11 Distributed Coordination Function 

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#### Abstract

SUMMARY In this paper, we consider a network of $N$ identical IEEE 802.11 DCF (Distributed Coordination Function) terminals with RTS/CTS mechanism, each of which is assumed to be saturated. For performance analysis, we propose a simple and efficient mathematical model to derive the statistical characteristics of the network such as the inter-transmission time of packets in the network and the service time (the inter-transmission time of successful packet transmissions) of the network. Numerical results and simulations are provided to validate the accuracy of our model and to study the performance of the IEEE 802.11 DCF network.


key words: wireless local area network, IEEE 802.11, distributed coordination function, performance analysis, medium access delay

## 1. Introduction

During the past few years we have witnessed an evergrowing interest in wireless technologies and the IEEE 802.11 MAC (Medium Access Control) protocols [1] have been proposed as standards for WLANs (Wireless Local Area Networks).

In the IEEE 802.11, the main mechanism to access the medium is the DCF (Distributed Coordination Function), which is a random access scheme based on the CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) protocol. The DCF describes two access techniques for packet transmission: the basic access mechanism and the RTS/CTS mechanism. In the basic access mechanism, when a terminal has a new packet to transmit, it generates a random backoff counter, and monitors the channel. The backoff counter of the terminal is decremented by 1 if the channel is idle for a pre-defined slot time, and frozen otherwise. In the latter case, when the channel is idle for an interval of time that exceeds the DIFS (Distributed Inter-Frame Space) after the channel busy period, the terminal resumes the decrement of the backoff counter for every idle slot. When the backoff counter of the terminal becomes 0 , the packet of the terminal is transmitted. After the transmission, if the terminal experiences a collision, it generates a random backoff counter for an additional deferral time before the next retransmission. This collision avoidance feature of the protocol intends to

[^0]minimize collisions during contention among multiple terminals.

The RTS/CTS mechanism involves the transmission of the RTS (Request-To-Send) and CTS (Clear-To-Send) control frames prior to the transmission of the data packet. A successful exchange of RTS and CTS frames attempts to reserve the channel for the time duration needed to transmit the packet under consideration. The rules for the transmission of an RTS frame are the same as those for a packet under the basic access scheme. Hence, the analysis of the basic access mechanism is the same as that of the RTS/CTS mechanism except that the packet transmission times are different in both mechanisms. This is one of the reasons why we consider only the RTS/CTS mechanism in this paper.

Many different works dealing with the performance of the IEEE 802.11 DCF network can be found in the literature. They have focused primarily on its throughput and capacity [2]-[4], adaptive backoff schemes [5]-[7], statistical analysis such as packet service times (or packet delay) of a terminal and related queueing analysis [8]-[13].

Among them, Carvalho and Garcia-Luna-Aceves [8] have introduced an analytical model to characterize the packet service time in a saturated IEEE 802.11 ad hoc network under the ideal channel condition. In [9], by considering the Rician fading channel, they have obtained a more realistic computation of a terminal's service time and throughput. Özdemir and McDonald [14] have derived the packet service time distribution of the terminal and have developed a queueing theoretic model for an IEEE 802.11 DCF terminal using the RTS/CTS mechanism. Tickoo and Sikdar [11] have computed the service time distribution and have proposed a queueing model for an IEEE 802.11 DCF terminal using the RTS/CTS mechanism. However, they have used many simplifying assumptions which result in various inaccuracies of their results. In [12], they have improved the model of [11] by including explicitly modelling the impact of the network load on the loss rates and thus the delays. Recently, Zanella and Pellegrini [13] have derived the service time distribution (the inter-transmission time distribution of consecutive successful transmissions) of the IEEE 802.11 network as in this paper. However, [11]-[13] have computed the service time of the network by using the PGF (Probability Generating Function), so that they need the IDFT (Inverse Discrete Fourier Transform) to get the actual distribution of the service time, which increases the complexity in the numerical analysis.

In this paper, we consider a network of $N$ identical

IEEE 802.11 DCF terminals with the RTS/CTS mechanism, each of which is assumed to be saturated. To analyze the performance of the network, we propose a new mathematical model based on renewal theory. In our model, we first characterize the packet transmission process of each terminal by a renewal process, and using the property of the forward recurrence time distribution in renewal theory we analyze the detailed packet transmission processes of all terminals, and compute the distribution of the inter-transmission time of packets in the network and the service time (the inter-transmission time of successful packet transmissions) of the network.

The main differences of our model from the previous models and works, e.g., [15] are as follows:

- Our model is based on the renewal approximation of the packet transmissions in each terminal, while most previous models are based on two dimensional Markov chain model [15] developed by Bianchi.
- Our model considers the detailed packet transmission processes and the backoff freezing processes of transmitting terminals and non-transmitting terminals separately, which is not considered in many previous works.
- Our model provides a direct computation for the probability distributions and performance measures regarding the network behavior, and accordingly the complexity in the numerical analysis can be reduced.

For the reasons listed above our model improves the previous models based on Bianchi's model. The accuracy and validity of our analysis will be checked through simulations and numerical studies later.

The organization of this paper is as follows. In Sect. 2, we propose an analytic model to derive the intertransmission time distribution of packets of the IEEE 802.11 DCF network. In Sect. 3, we analyze the service time of the network and obtain the expectation and variance of the service time. In Sect. 4, simulation results are provided to validate our model and to see the performance of the IEEE 802.11 DCF network. We give our conclusions in Sect. 5.

## 2. Analysis of Inter-Transmission Times

We consider a network of $N$ identical IEEE 802.11 terminals with DCF, each of which is assumed to be saturated, and analyze the inter-transmission times of packets in the network. Here, the inter-transmission times of packets in the network are defined by the time intervals between two consecutive packet transmissions of the network. Note that two consecutive packets may be transmitted from two different terminals in the network. Our analysis in this section is two-fold: we first model the packet transmission process of each terminal by a renewal process and then derive the distribution of the inter-transmission times of packets in the network by using renewal theory.

### 2.1 The Packet Transmission Process of a Terminal

We consider an arbitrary terminal, called the tagged terminal, and focus ourselves on embedded time points where the backoff counter value of the tagged terminal is changed (i.e., the backoff counter value is decremented by 1 or a new backoff counter value is selected). A time interval between two consecutive embedded time points are defined by a virtual slot in the analysis. Refer to Fig. 1.

Let $R$ denote the number of virtual slots between two consecutive packet transmissions of the tagged terminal in steady state. To obtain the distribution of the random variable $R$, we first compute the steady state probability $\pi_{k}, 0 \leq$ $k \leq m$, that the tagged terminal is in backoff stage $k$ just after a packet transmission. To do this, we assume that packet collision occurs independently at each packet transmission with a fixed probability $p$ as in [15] in steady state. Then, if the packet transmission is successful with probability $1-p$, the backoff stage after the packet transmission is 0 . If the packet transmission is not successful with probability $p$, the backoff stage after the packet transmission is increased by one until the backoff stage reaches the maximum backoff stage $m$. Refer to Fig. 2. Hence, the steady state probability $\pi_{k}, 0 \leq k \leq m$, is computed by

$$
\pi_{k}=\left\{\begin{array}{cl}
p^{k}(1-p), & \text { if } 0 \leq k \leq m-1,  \tag{1}\\
p^{m}, & \text { if } k=m .
\end{array}\right.
$$

Next, let $B C$ denote the backoff counter value selected by the tagged terminal after an arbitrary packet transmission in steady state. When the backoff stage after the packet transmission is $k$, the tagged terminal selects a new backoff counter value uniformly from the window $\left[0, C W_{k}-1\right]$. Here, the window size $C W_{k}$ at backoff state $k$ is defined by

$$
C W_{k}=2^{k} C W_{0}, \quad 1 \leq k \leq m
$$

where $C W_{0}$ is the initial window size at backoff stage 0 .


Fig. 1 Virtual slots and actual slots of terminal B.


Fig. 2 Transitions of the backoff stage after packet transmissions.

Hence, the steady state probability $r_{i}, 0 \leq i \leq C W_{m}-1$, that the backoff counter $B C$ is equal to $i$ is given as follows. If we let $B S$ denote the backoff stage after an arbitrary packet transmission in steady state,

$$
\begin{align*}
r_{i} & \Delta P\{B C=i\} \\
& =\sum_{j=l}^{m} P\{B S=j\} P\{B C=i \mid B S=j\} \\
& =\sum_{j=l}^{m} \pi_{j} \frac{1}{C W_{j}}, \quad C W_{l-1} \leq i \leq C W_{l}-1, \tag{2}
\end{align*}
$$

for $l=0,1,2, \cdots, m$ where $C W_{-1}=0$.
Then, by the definition of $R$ we have $R=B C+1$ because $R$ includes the virtual slot containing a packet transmission, from which we obtain the distribution of $R$ as follows:

$$
\begin{equation*}
P\{R=i\}=P\{B C=i-1\}=r_{i-1}, \quad 1 \leq i \leq C W_{m} \tag{3}
\end{equation*}
$$

Now, let $\tau$ denote the steady state probability that the tagged terminal transmits a packet in an arbitrary virtual slot. It is worthwhile to note that, by the definition of $\tau$ and the random variable $R$, it follows

$$
\begin{equation*}
\tau=\frac{1}{E[R]} \tag{4}
\end{equation*}
$$

where $E[R]$ denotes the expectation of the random variable $R$. The computation of $E[R]$ can be done by its definition and using (3), but we have another simple way to compute $E[R]$ as follows:

$$
\begin{align*}
E[R] & =\sum_{j=0}^{m} P\{B S=j\} E[R \mid B S=j] \\
& =\sum_{j=0}^{m} \pi_{j} \sum_{k=1}^{C W_{j}} k \frac{1}{C W_{j}}=\sum_{j=0}^{m} \pi_{j} \frac{C W_{j}+1}{2} . \tag{5}
\end{align*}
$$

In addition, from the definitions of $p$ and $\tau$ we have

$$
\begin{equation*}
p=1-(1-\tau)^{N-1} \tag{6}
\end{equation*}
$$

Therefore, starting from (1) and (6) with an initial value of $\tau$, we obtain an updated value of $\tau$ from (4) and (5). Then, if the updated value of $\tau$ is not very close to the initial value of $\tau$, we perform the same procedure with the updated value of $\tau$ as the initial value of $\tau$. We do this recursive procedure until we get an exact value of $\tau$.
Remark. It is worth mentioning that the values $\tau$ and $p$ obtained by combining (4) and (6) are exactly equal to the values obtained by Bianchi [15]. That is, using (1) and (5) $E[R]$ can be rewritten as

$$
\begin{aligned}
E[R] & =\sum_{j=0}^{m} \pi_{j} \frac{C W_{j}+1}{2} \\
& =\frac{C W_{0}}{2}\left[\sum_{j=0}^{m-1} p^{j}(1-p) 2^{j}+p^{m} 2^{m}\right]+\frac{1}{2} \\
& =\frac{C W_{0}\left[1+p \sum_{k=0}^{m-1}(2 p)^{k}\right]+1}{2}
\end{aligned}
$$

Hence, $\tau$ satisfies

$$
\tau=\frac{2}{1+C W_{0}\left[1+p \sum_{k=0}^{m-1}(2 p)^{k}\right]}
$$

which is identical to the equation $\tau(p)$ in page 539 of Bianchi [15].

For later use, we introduce a random variable $R^{(e)}$ which denotes the remaining number of virtual slots for the tagged terminal to perform the next packet transmission at an arbitrary virtual slot in steady state. By renewal theory [16], it can be shown that

$$
P\left\{R^{(e)}=k\right\}=\frac{P\{R \geq k\}}{E[R]}, \quad 1 \leq k \leq C W_{m}
$$

where $P\{R \geq k\}$ is obtained from (2) and (3).
Since all terminals are operated identically, the probabilistic characteristics of terminals are all identical, that is, the above results for $R$ and $R^{(e)}$ can be also used to model the packet transmission processes of non-tagged terminals in the network.

### 2.2 Inter-Transmission Time of Packets in the Network

In this subsection, we consider actual slot boundaries where the backoff counter value of the tagged terminal is either changed or frozen. A time interval between two consecutive actual slot boundaries is called an actual slot in the analysis. Note that the actual slot is different from the physical slot in the IEEE 802.11 DCF standard. In addition, the actual slot is also different from the virtual slot. The difference between actual and virtual slots occurs when there is a packet transmission from a non-tagged terminal in the virtual slot. In this case the virtual slot containing the packet transmission of the non-tagged terminal is divided into two actual slots, one of which is the packet transmission time of the non-tagged terminal and the other of which is the physical slot during which the backoff counter value of the tagged terminal is still frozen.

Let $H$ be the number of actual slots between two consecutive packet transmissions in the network. To get the distribution function of $H$ in steady state, we assume that there occurs at least one packet transmission at an arbitrary actual slot boundary, called the initial boundary, in steady state. We condition on the number $N_{0}$ of terminals which transmit packets simultaneously at the initial boundary. Then the probability mass function of $N_{0}$ is given as follows:

$$
\begin{equation*}
P\left\{N_{0}=j\right\}=\frac{\binom{N}{j} \tau^{j}(1-\tau)^{N-j}}{\sum_{l=1}^{N}\binom{N}{l} \tau^{l}(1-\tau)^{N-l}}, \quad 1 \leq j \leq N \tag{7}
\end{equation*}
$$

Here, the denominator is the probability that there is at least one terminal transmitting a packet at the initial boundary in steady state. Now, to compute the distribution of $H$, we should consider the backoff freezing procedure for nontransmitting terminals at the initial boundary. That is, a nontransmitting terminal with positive backoff counter value,
detecting a packet transmission needs one more actual slot to decrease its backoff counter value after the actual slot containing the packet transmission [1]. Then, if we consider a terminal transmitting a packet at the initial boundary in steady state, the next transmission time for the terminal is given by $R$. On the other hand, if we consider the terminal transmitting no packet at the initial boundary in steady state, the next transmission time is given by $R^{(e)}+1$. Here, one acutal time slot is added due to the backoff freezing procedure for the non-transmitting terminal explained above. Hence, the distribution of $H$ is given as follows:

$$
\begin{align*}
& P\{H \leq h\}=\sum_{j=1}^{N} P\left\{N_{0}=j\right\} \\
& \quad \times\left[1-(P\{R>h\})^{j}\left(P\left\{R^{(e)}>h-1\right\}\right)^{N-j}\right] \tag{8}
\end{align*}
$$

where $P\left\{N_{0}=j\right\}$ is given in (7). Note that

$$
(P\{R>h\})^{j}\left(P\left\{R^{(e)}>h-1\right\}\right)^{N-j}
$$

is the conditional probability that there will be no packet transmission in the network during $[0, h]$, given that there are $j$ terminals transmitting packets at the initial boundary.

Since the discrete time $H$ is not the real intertransmission time of packets in the network, our next step is to compute the p.m.f. of the real inter-transmission times of packets in the network from the p.m.f. of $H$. To do this, we introduce a conditional probability $q$ which is defined by the conditional probability that there occurs a packet collision, given that there is at least one packet transmission in the network. Note that the conditional probability $q$ is different from the conditional probability $p$. By its definition, the conditional probability $q$ is given by

$$
q=\frac{1-(1-\tau)^{N}-N \tau(1-\tau)^{N-1}}{1-(1-\tau)^{N}}
$$

Let $T_{s}$ and $T_{c}$ denote the times taken by a successful transmission and a collided transmission, respectively. That is, since we consider the RTS/CTS mechanism, $T_{s}$ and $T_{c}$ are given by

$$
\begin{aligned}
T_{s}= & R T S+S I F S+C T S+S I F S+\text { Packet } \\
& \quad+S I F S+A C K+D I F S \\
T_{c}= & R T S+D I F S
\end{aligned}
$$

where RTS, CTS, Packet and ACK are the respective packet transmission times of RTS, CTS, Packet and ACK, and SIFS denotes the Short Inter-Frame Space.

Consider the case where $H=h$. Then, since $H$ is the number of actual slots between two packet transmissions in the network, there is a packet transmission at the first actual slot and there is no packet transmission during the remaining $h-1$ slots. Hence, the real inter-transmission time of two packets in steady state is $(h-1) \sigma+T_{s}$ with probability $1-q$, and $(h-1) \sigma+T_{c}$ with probability $q$ where $\sigma$ denotes the physical slot time of the IEEE 802.11 DCF. That is,

$$
H^{(r)}=(H-1) \sigma+\chi_{s} T_{s}+\left(1-\chi_{s}\right) T_{c}
$$

where $H^{(r)}$ denotes the real inter-transmission time of packets in the network in steady state and $\chi_{s}$ is a random variable that takes 1 with probability $1-q$, and 0 with probability $q$. We will check the validity of our results through numerical studies in Sect. 4.

Before closing this section, it is worth giving a remark on our analysis. As mentioned in [3], the freezing procedure of the backoff counter for a non-transmitting terminal is not modelled for details in [15]. However, our analysis considers the freezing procedure of the backoff counter in detail.

## 3. The Service Time of the Network

In this section, we analyze the service time of the network, which is defined by the time interval of successful packet transmissions in the network.

Since the network experiences a collision with probability $q$ at each packet transmission, the service time of the network, denoted by $X$, is computed as

$$
\begin{equation*}
X=(H-1) \sigma+T_{s}+\sum_{i=1}^{Y}\left[\left(H_{i}-1\right) \sigma+T_{c, i}\right] \tag{9}
\end{equation*}
$$

where $Y$ is a geometric random variable with parameter 1 $q$, i.e., $P\{Y=y\}=q^{y}(1-q), y \geq 0$ and $H_{i}$ and $T_{c, i}$ are sequences of i.i.d. random variables whose distributions are the same as $H$ and $T_{c}$, respectively. From (9) we can get the expectation and variance of the service time, denoted by $E[X]$ and $\operatorname{Var}[X]$, respectively, in the network. From Wald's equation [16], it follows that

$$
\begin{align*}
& E[X]= E\left[(H-1) \sigma+T_{s}\right] \\
&+E[Y] E\left[(H-1) \sigma+T_{c}\right] \\
&=\sigma E[H](1+E[Y])+E\left[T_{s}-\sigma\right] \\
&+E[Y] E\left[T_{c}-\sigma\right] \tag{10}
\end{align*}
$$

where $E[Y]=q /(1-q)$ and $E[H]$, the expectation of $H$, can be obtained from (8).

To get $\operatorname{Var}[X]$, we first let $Z_{i}=\left(H_{i}-1\right) \sigma+T_{c, i}$ and use the conditional variance formula

$$
\begin{aligned}
\operatorname{Var}\left[\sum_{i=1}^{Y} Z_{i}\right]=\operatorname{Var} & {\left[E\left[\sum_{i=1}^{Y} Z_{i}|Y|\right]\right.} \\
+ & E\left[\operatorname{Var}\left[\sum_{i=1}^{Y} Z_{i} \mid Y\right]\right]
\end{aligned}
$$

to get

$$
\begin{equation*}
\operatorname{Var}\left[\sum_{i=1}^{Y} Z_{i}\right]=\operatorname{Var}[Z] E[Y]+E[Z]^{2} \operatorname{Var}[Y] \tag{11}
\end{equation*}
$$

where $Z$ denotes the generic random variable for the i.i.d. random variables $Z_{i}$. Then from (11) we finally get

$$
\begin{aligned}
\operatorname{Var}[X]= & \operatorname{Var}\left[(H-1) \sigma+T_{s}\right]+\operatorname{Var}\left[\sum_{i=1}^{Y} Z_{i}\right] \\
= & \sigma^{2} \operatorname{Var}[H]+\operatorname{Var}\left[T_{s}\right] \\
& +E[Y]\left(\sigma^{2} \operatorname{Var}[H]+\operatorname{Var}\left[T_{c}\right]\right) \\
& +\operatorname{Var}[Y]\left((E[H]-1) \sigma+E\left[T_{c}\right]\right)^{2}
\end{aligned}
$$

where $\operatorname{Var}[Y]=q /(1-q)^{2}$ and $\operatorname{Var}[H]$, the variance of $H$, is obtained from (8).

In Sect. 4, we simulate the IEEE 802.11 DCF network and compare the simulation results with the analytic results obtained above to check the validity of our analysis. In addition, to compare the accuracy of our results with others we also compute the expectation of the service time of the network by using Bianchi's model [15]. In Bianchi [15], he computes throughput $S$ of the network, e.g., equation (13) of [15]. Note that throughput $S$ can be also obtained from the expectation of the service time $E[X]$ obtained in (10) as follows:

$$
S=\frac{E[P]}{E[X]}
$$

where $E[P]$ denotes the expectation of the size of a packet. Hence, we can compute the expectation of the service time of the network by using Bianchi's model as follows:

$$
\begin{align*}
E[X]= & E[P] / S \\
= & \frac{(1-\tau)^{N} \sigma+N \tau(1-\tau)^{N-1} T_{s}}{N \tau(1-\tau)^{N-1}} \\
& +\frac{\left[1-(1-\tau)^{N}-N \tau(1-\tau)^{N-1}\right] T_{c}}{N \tau(1-\tau)^{N-1}} . \tag{12}
\end{align*}
$$

The comparison between the value of $E[X]$ in (10) from our model and the value of $E[X]$ in (12) from Bianchi's model will be given in Sect. 4 .

Based on the above results, we can easily obtain the expected medium access delay of a packet in the tagged terminal, which is defined by time needed for the packet to be successfully transmitted after it is positioned in the transmission buffer of the tagged terminal for the first time, as follows. Since all terminals are identical, the expected medium access delay of a packet in the tagged terminal is given by $N E[X]$. We will verify our result of the expected medium access delay through numerical studies in Sect. 4.

## 4. Model Validation and Numerical Studies

In this section, we use simulations to validate our analytic model. In simulation studies where the RTS/CTS mechanism is considered, we use system parameters given in Table 1. The simulator is written in the $\mathrm{C}++$ programming language and the simulation time for each run is 700 sec onds.

In the study, we change the number $N$ of terminals in the network. $N=20$ and $N=50$ are used in Fig. 3 and Fig. 4, respectively. Figures 3 and 4 show the frequency of real inter-transmission times of packets in the network. In

Table 1 System parameters.

| Payload Size | 8000 bits |
| :---: | :---: |
| Phy Header <br> (including preamble) | 192 bits |
| Mac Header <br> (including CRC bits) | 272 bits |
| RTS Frame | Phy Header +160 bits |
| CTS Frame | Phy Header +112 bits |
| ACK Frame | Phy Header +112 bits |
| Time Slot | $20 \mathrm{e}-6$ sec |
| SIFS | $10 \mathrm{e}-6$ sec |
| DIFS | $50 \mathrm{e}-6 \mathrm{sec}$ |
| Data Rate | $1 \mathrm{e} 6 \mathrm{bits} / \mathrm{sec}$ |
| $C W_{0}$ | 32 |
| $m$ | 5 |

the figures, we see that the peaks between 0 to 1 ms are those with collided transmissions and the peaks between 9 ms to 10 ms are those with successful transmissions. As seen in the figures, there is almost no difference between the simulation results and the analytic results. It should be noted that, if we do not consider the packet transmissions of transmitting terminals and non-transmitting terminals separately as in Bianchi's model [15], we do not have the first small peaks in the region $[0,1] \mathrm{ms}$ and the region $[9,10] \mathrm{ms}$. Hence, we can conclude that our analytic method provides a good prediction of the real inter-transmission times of packets in the network. In addition, as expected, by comparing 3(b) and 3(c) with 4(b) and 4(c), respectively, we see that there are more collided transmissions and less successful transmissions when $N=50$ than when $N=20$.

Next, we compute the expectation and variance of the service time of the network obtained in Sect. 3 and compare them with simulation results. In the experiment, we use the maximum backoff stage $m=5$ and change the network size, i.e., $N=10,20,50$ as well as the initial window size, i.e., $C W_{0}=16,32,64$. The results are given in Fig. 5 and Fig. 6 where simulation results are obtained by averaging the expectations and variances of the service time from 7 simulation runs with simulation time 100 seconds. As seen in the figures, our analytic results are well matched with the simulation results. In addition, as the initial window size $C W_{0}$ is getting large, our analytic method predicts the service time more exactly and the effect of the network size $N$ on the service time becomes less significant.

To compare the accuracy of our analysis with that of Bianchi's analysis, we give in Table 2 the expected service times of the network obtained from simulation, equation (10) from our model and equation (12) from Bianchi's model. As seen in Table 2, our results from (10) are closer to simulation results than those from (12). The reason for this is that our analysis can consider the packet transmission process and the backoff freezing in more detail than Bianchi's analysis.

Finally, we compute the expected medium access delay of a packet. In this study, we use the maximum backoff stage $m=5$ and the initial window size $C W_{0}=32$. The results are given in Fig. 7 where we change the number of nodes from



Fig. 5 The expected service time of the network.


Fig. 6 The variance of the service time of the network.

Table 2 Comparison with Bianchi's model for the expected service time of the network.

| $C W_{0}=16$ <br> $m=5$ | Simulation | Our model (10) | Bianchi (12) |
| :---: | :---: | :---: | :---: |
| $N=10$ | 0.00967127309 | 0.00968106237 | 0.00965890961 |
| $N=20$ | 0.00972075335 | 0.00973360338 | 0.00970840370 |
| $N=50$ | 0.00981745813 | 0.00983943680 | 0.00980857374 |
| $C W_{0}=32$ <br> $m=5$ | Simulation | Our model (10) | Bianchi (12) |
| $N=10$ | 0.00965288376 | 0.00965548240 | 0.00963347059 |
| $N=20$ | 0.00968251370 | 0.00968775897 | 0.00966349959 |
| $N=50$ | 0.00975202356 | 0.00975849714 | 0.00973028177 |
| $C W_{0}=64$ <br> $m=5$ | Simulation | Our model (10) | Bianchi (12) |
| $N=10$ | 0.00965428325 | 0.00965489823 | 0.00963349095 |
| $N=20$ | 0.00966002986 | 0.00966082759 | 0.00963771679 |
| $N=50$ | 0.00970375749 | 0.00970470017 | 0.00967861819 |



Fig. 7 The expected medium access delay of a packet.

10 to 50 . As seen in the figure, our analytic results are well matched with simulation results and the expected medium access delay of a packet is linearly increased in the number of nodes.

## 5. Conclusions

The main contribution of this paper is to provide a simple and accurate analytic model to characterize statistical properties of the IEEE 802.11 DCF network such as the intertransmission time of packets in the network and the intertransmission time of successful packet transmissions in the network. To validate our analytic results, we use simulations and compare our analytic results with simulation results. Our comparison shows that our analytic model well predicts the statistical properties of the IEEE 802.11 DCF network. As for further study, we will modify the proposed model to consider the real network conditions such as fading and unsaturated condition.

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