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Continuous trajectory planning of mobile sensors for informative forecasting[☆]

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ABSTRACT

This paper addresses the planning of continuous paths for mobile sensors to reduce the uncertainty in some quantities of interest in the future. The mutual information between the measurement along the continuous path and the future verification variables defines the information reward. Two expressions for computing this mutual information are presented: the filter form extended from the state of the art and the smoother form inspired by the conditional independence structure. The key properties of the approach using the filter and smoother strategies are presented and compared. The *smoother form* is shown to be preferable because it provides better computational efficiency, facilitates easy integration with existing path synthesis tools, and, most importantly, enables correct quantification of the rate of information accumulation. A spatial interpolation technique is used to relate the motion of the sensor to the evolution of the measurement matrix, which leads to the formulation of the optimal path planning problem. A gradient-ascent steering law based on the concept of information potential field is also presented as a computationally efficient suboptimal strategy. A simplified weather forecasting example is used to compare several planning methodologies and to illustrate the potential performance benefits of using the proposed planning approach.

1. Introduction

One key problem for (mobile) sensor networks is to create plans for maneuvering/locating sensing resources in order to extract information from the environment. In this research, the plans are often generated to reduce uncertainty in some quantity of interest (e.g., position and velocity of targets (Grocholsky, 2002; Grocholsky, Keller, Kumar, & Pappas, 2006; Grocholsky, Makarenko, & Durrant-Whyte, 2003; Gupta, Chung, Hassibi, & Murray, 2006; Hoffmann & Tomlin, 2010; Martinez & Bullo, 2006; Ristic & Arulampalam, 2003; Williams, Fisher, & Willsky, 2007), the pose of the sensor platform itself, the forecast of weather over some region of interest (Choi & How, 2007; Choi, How, & Hansen, 2007; Majumdar, Bishop, Etherton, & Toth, 2002; Palmer, Gelaro, Barkmeijer, & Buizza, 1998), physical quantities under the ocean (Fiorelli et al., 2006; Hover, 2008), or the distribution of radioactive materials (Cortez et al., 2008)) at some point in time—called the *verification time* and denoted here as T. Note that, in most previous work (Grocholsky, 2002; Grocholsky et al., 2006, 2003; Gupta et al., 2006; Hoffmann & Tomlin, 2010; Hover, 2008; Martinez & Bullo, 2006; Ristic & Arulampalam, 2003; Williams et al., 2007), this verification time was taken to be the final time of the generated plan τ, so that $T = \tau$.

This paper investigates a similar planning problem, but the goal is modified to determine the best measurement plan to minimize the uncertainty in some verification variables *in the future*. In particular, the problem formulation is extended to the more general setting wherein the verification time can be significantly larger than the final time of the plan; i.e., $T \gg \tau$. In addition, the quantities of interest, called verification variables, can be variables distributed over a subset of the entire environment, not necessarily being the whole set of state variables. These two modifications enable the solution of additional classes of planning problems, e.g., planning to reduce uncertainty in forecasts, and also provide additional insights on the standard problems solved in the literature (Choi & How, 2009). One way to address this generalized planning problem is to simply augment a null measurement between τ and T to some measurement path, and optimize the solution over this augmented measurement space. This approach is considered, but several potential issues are discussed. An alternative approach that is inspired by the well-known conditional independence of the past and the future for a given present state (Cover & Thomas, 1991) is then proposed and several advantages, including improved computational efficiency and *correct* quantification of the amount of information accumulated, are clearly demonstrated in Section 3.2.

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1.1. Quantification of mutual information

Mutual information is used herein to define the uncertainty reduction of the quantity of interest. The mutual information between the verification variables (in the future) and some measurement sequence over the time window $[0, \tau]$ represents the difference between the prior and the posterior entropy of the verification variables when conditioned on this sequence of measurements. Thus, mutual information explicitly quantifies the impact of sensing on the entropy reduction of the verification variables. For the continuous trajectory planning problems of interest in this paper, this measurement sequence is represented by continuous random processes instead of a finite number of random variables.

In the information theory literature, there has been a well-established theory on computing the mutual information between the signal and observation in the continuous-time domain. Duncan (1970) showed that the mutual information between the signal history and observation history (i.e., signal during $[0, \tau]$ and observation during $[0, \tau]$) can be expressed as a function of the estimation error when the signal is Gaussian and the observation is taken through an additive Gaussian channel. Similar quantification is performed for non-Gaussian signals (Guo, Shamai (Shitz), & Verdu, 2005; Kadota, Zakai, & Ziv, 1971) and a fractional Gaussian channel (Duncan & Pasik-Duncan, 2007). On the other hand, Tomita, Omatu, and Soeda (1976) showed that the optimal filter for a linear system that maximizes the mutual information between the observation history for $[0, \tau]$ and the state value at τ is the Kalman–Bucy filter; Mayer-Wolf and Zakai (1984) related this mutual information to the Fisher information matrix. Mitter and Newton (2005) presented an expression for the mutual information between the signal path during $[t, \tau]$, $t < \tau$ and the observation history during $[0, \tau]$, with a statistical mechanical interpretation of this expression.

However, these previous results cannot be directly used to quantify the mutual information of interest in this work because they all consider the case where the verification time is equal to the final time, $T = \tau$ (or there is a verification interval $[t, T]$ with $T = \tau$) and the verification variables are the whole state variables. Newton (2006, 2007) recently extended his prior results (Mitter & Newton, 2005) to quantify the mutual information between the future signal path during $[\tau, T]$, $\tau \leq T$ and the past measurement history during $[0, \tau]$. But the results in Newton (2006, 2007) (combined with those in Mitter and Newton (2005)) can only be used when the verification interval overlaps with the measurement interval and the verification variables are the whole state variables. Therefore, a new expression of the mutual information is required to address the more general planning problem in this paper. Also, although there have been several studies on continuous-time decision making for sensors (Grocholsky, 2002; Hover, 2008; Lee, Teo, & Lim, 2001), none of these explicitly quantified the mutual information associated with a continuous measurement over a finite time period. In contrast, this work presents an explicit way of computing the mutual information for continuous-time planning problems.

The straightforward approach, which augments a null measurement after τ, results in the mutual information being computed by integrating matrix differential equations during the time window $[0, T]$, generalizing the expression of mutual information in Mayer-Wolf and Zakai (1984), Mitter and Newton (2005) and Tomita et al. (1976). The alternative approach based on the conditional independence that is presented here enables the quantification of the mutual information between the variables at T and a measurement path over $[0, \tau]$ as the difference between the unconditioned and conditioned mutual information between the state at τ and the respective measurement path. The expressions of the mutual information obtained by these two approaches will be called the *filter form* and the *smoother form*, respectively. These two forms are shown to be equivalent under the standard assumptions that the system dynamics are Markovian and that the sensing noise at a given time is independent of the future process noise. However, the smoother form will be shown to possess the following advantages over the filter form (Section 3.2):

(1) for the smoother form, the associated matrix differential equations are integrated over a much shorter time window than for the filter form, which reduces the computational cost;
(2) the smoother form projects the decision space from the forecast horizon onto the planning horizon which facilitates the integration of this technique into various path planning algorithms;
(3) analysis of the time derivatives of the presented expressions of mutual information (in Section 3.1) shows that only the smoother form correctly predicts the rate of information accumulation in the generalized planning problem in this work.

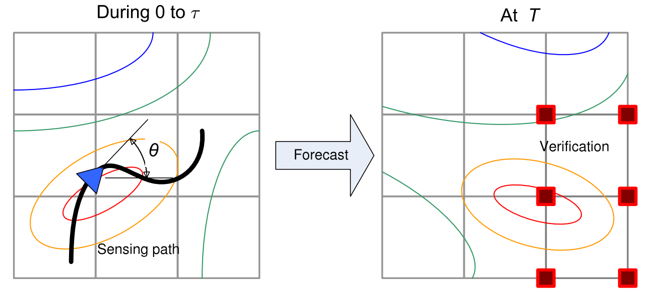

Fig. 1. Continuous motion planning of a sensor for informative forecast: a mobile sensor senses some environmental variable represented by the contours along the path designed to achieve the best forecast for the verification region in a two-dimensional space.

1.2. Continuous trajectory planning for environmental forecasting

This paper designs continuous trajectories for mobile sensors, specifically, in the context of environmental sensing. As illustrated in Fig. 1, a mobile sensor continuously observes environmental field variables (e.g., temperature, pressure, concentration of chemicals, diffusivity) along a continuous path during the time interval $[0, \tau]$. The goal is to reduce the uncertainty in the forecast of the environmental variables over the verification region (red squares) at the verification time T, with $T \gg \tau$. Weather forecasting is one motivating application of this planning problem, the goal being the design of an adaptive sensor network that supplements fixed observation networks (Choi & How, 2007; Choi et al., 2007; Majumdar et al., 2002; Palmer et al., 1998).

The rest of the paper is organized as follows. Section 2 presents two formulae for quantification of mutual information, whose properties are analyzed and discussed in Section 3. Section 4 describes spatial interpolation techniques used to develop a tractable representation of continuous trajectories. In Section 5, the optimal trajectory planning problem and a gradient-ascent steering law are presented. The paper concludes with numerical studies using a simplified weather forecasting problem.

2. Quantification of information

2.1. Linear system model

Consider the linear (time-varying) dynamics of state variables $X_t \in \mathbb{R}^{n_X}$ subject to additive process noise $W_t \in \mathbb{R}^{n_X}$:

$$\dot{X}_t = A(t)X_t + W_t. \tag{1}$$

W_t is zero-mean Gaussian, independent of X_t, and $\mathbb{E}[W_t W_s'] = \Sigma_W \delta(t - s)$, $\Sigma_W \succeq 0$, where the prime sign ($'$) denotes the transpose of a matrix. The initial condition of the state X_0 is normally distributed as $X_0 \sim \mathcal{N}(\mu_0, P_0)$, $P_0 \succ 0$. In the trajectory planning problem described in Section 1.2, the state vector X_t represents the environmental variables at a finite number of grid points. In the context of environmental forecasting, the linear time-varying dynamics in (1) are often used to approximate the propagation of a perturbation through the nonlinear dynamics (Palmer et al., 1998).

Also, consider a linear measurement model for $Z_t \in \mathbb{R}^m$ with additive sensing noise $N_t \in \mathbb{R}^m$:

$$Z_t = C(t)X_t + N_t. \tag{2}$$

N_t is zero-mean Gaussian, independent of X_t and W_s, $\forall s$, and $\mathbb{E}[N_t N_s'] = \Sigma_N \delta(t-s)$, $\Sigma_N \succ 0$. This linear sensing model is a good representation of the observations of directly measurable environmental variables (e.g., temperature, pressure) that are distributed in the field. Later in this paper, the time argument will be omitted if no confusion is expected.

With this environmental system model, this work determines the impact of a measurement path in the near future on the expected uncertainty reduction of some verification variables in the far future. A measurement path up to time t is defined as

$$\mathcal{Z}_t = \{Z_\sigma : \sigma \in [0, t]\}. \tag{3}$$

The verification variables are a subset of the state variables that can be expressed as

$$V_t = M_V X_t \in \mathbb{R}^{n_V}, \tag{4}$$

where $M_V \in \{0, 1\}^{n_V \times n_X}$, $n_V \le n_X$ with every row-sum of M_V being unity. Although this work is specifically focused on the case where entries of M_V are zero or one, the results can be easily extended to a general $M_V \in \mathbb{R}^{n_V \times n_X}$.

Entropy is employed as a metric of uncertainty because it represents the degree of randomness of generic random entities (e.g., random variables (Cover & Thomas, 1991), random processes (Kolmogorov, 1956), stochastic systems (Zames, 1979)). Then, the uncertainty reduction of one random quantity by another random quantity is expressed as the mutual information between them. Therefore, the information reward by a measurement path is defined as

$$\mathcal{J}_V(T, \tau) \triangleq \mathcal{I}(V_T; \mathcal{Z}_\tau), \quad 0 \le \tau < T, \tag{5}$$

where $\mathcal{I}(A_1; A_2)$ denotes the mutual information between two random entities A_1 and A_2. This reward represents the entropy reduction of the verification variables in the far future time T by the measurement history up to the near future time τ. Results for cases where $T = \tau$ can be found in Mayer-Wolf and Zakai (1984), Mitter and Newton (2005) and Tomita et al. (1976), and this work particularly focuses on the case where $T > \tau$.

2.2. Filter form

For linear Gaussian systems, there are known expressions for the mutual information between the state variables at a given time and a measurement history up to that time (Mayer-Wolf & Zakai, 1984; Mitter & Newton, 2005; Tomita et al., 1976). Therefore, one way to compute the information reward is to consider the *filtering* problem that estimates X_T based on the measurement history up to time T denoted as $\mathcal{Z}_T \triangleq \mathcal{Z}_\tau \cup \emptyset_{(\tau,T]}$, where $\emptyset_{(\tau,T]}$ means that no measurement is taken during $(\tau, T]$. Then, $\mathcal{I}(X_T; \mathcal{Z}_\tau) = \mathcal{I}(X_T; \mathcal{Z}_T)$, because no information is gathered by a null measurement. This procedure of obtaining $\mathcal{I}(X_T; \mathcal{Z}_\tau)$ can be extended to computing $\mathcal{I}(V_T; \mathcal{Z}_\tau)$, as outlined in the following proposition.

Proposition 1 (*Filter Form*)**.** *For the linear system described by (1) and (2), the information reward can be computed as*

$$\mathcal{J}_V^F(T, \tau) \triangleq \mathcal{I}(V_T; \mathcal{Z}_\tau, \emptyset_{(\tau,T]}) = \frac{1}{2}\text{ldet}(M_V P_X(T) M_V') - \frac{1}{2}\text{ldet}(M_V Q_X(T) M_V'), \tag{6}$$

where ldet *denotes* log det *of a symmetric positive definite matrix, and $P_X(T)$ and $Q_X(T)$ are obtained by integrating the following matrix differential equations for the covariance matrices $P_X(t) \triangleq \mathbb{E}[(X_t - \mathbb{E}[X_t])(X_t - \mathbb{E}[X_t])']$ and $Q_X(t) \triangleq \mathbb{E}[(X_t - \mathbb{E}[X_t|\mathcal{Z}_t])(X_t - \mathbb{E}[X_t|\mathcal{Z}_t])']$:*

$$\dot{P}_X(t) = A(t)P_X(t) + P_X(t)A'(t) + \Sigma_W \tag{7}$$

$$\dot{Q}_X(t) = A(t)Q_X(t) + Q_X(t)A'(t) + \Sigma_W - \mathbb{I}_{[0,\tau]}(t)Q_X(t)C(t)'\Sigma_N^{-1}C(t)Q_X(t) \tag{8}$$

with initial conditions $P_X(0) = Q_X(0) = P_0 \succ 0$, where $\mathbb{I}_{[0,\tau]}(t)$ is the indicator function that is unity if $t \in [0, \tau]$ and zero otherwise. Also, Eqs. (7) and (8) are well defined for finite T with $P_0 \succ 0$.

Proof. Note that $\mathcal{I}(V_T; \mathcal{Z}_\tau) = \mathcal{I}(V_T; \widehat{V}_T)$, where $\widehat{V}_T = \mathbb{E}[V_T|\mathcal{Z}_\tau]$, because $\widehat{V}_T$ is the sufficient statistic that captures all information contained in $\mathcal{Z}_\tau$ about V_T (Mitter & Newton, 2005). Since the remaining randomness in V_T for a given $\widehat{V}_T$ is the estimation error $\widetilde{V}_T \triangleq V_T - \widehat{V}_T$, the mutual information $\mathcal{I}(V_T; \mathcal{Z}_\tau) = \mathcal{H}(V_T) - \mathcal{H}(\widetilde{V}_T)$, where $\mathcal{H}(A_1)$ denotes the entropy of some quantity A_1. For a linear system with Gaussian noise, both V_T and $\widetilde{V}_T$ are normally distributed with covariances: $\text{Cov}(V_T) \triangleq \mathbb{E}[(V_T - \mathbb{E}[V_T])(V_T - \mathbb{E}[V_T])'] = M_V P_X(T) M_V'$, $\text{Cov}(\widetilde{V}_T) = M_V Q_X(T) M_V'$, respectively, when $P_X(T)$ and $Q_X(T)$ are obtained from (7) and (8). The entropy of a Gaussian random vector V_T (or $\widetilde{V}_T$) is computed as $\mathcal{H}(V_T) = \frac{1}{2}[\text{ldet}\text{Cov}(V_T) + n_V \log(2\pi e)]$ (or $\widetilde{V}_T$ in place of V_T). This results in the expression in (6). □

2.3. Smoother form

This section presents a *smoother form* of the information reward, which is equivalent to the filter form but offers many advantages over the filter one. The foundation of the smoother form is the following information identity based on conditional independence.

Proposition 2 (*Information Identity*)**.** *If the state dynamics satisfy the Markov property, i.e. the future state is conditionally independent of the past state given the present state, and the measurement noise is independent of future process noise, then*

$$\mathcal{I}(V_T; \mathcal{Z}_\tau) = \mathcal{I}(X_\tau; \mathcal{Z}_\tau) - \mathcal{I}(X_\tau; \mathcal{Z}_\tau|V_T). \tag{9}$$

Proof. By the chain rule of the mutual information (Cover & Thomas, 1991), the following holds:

$$\mathcal{I}(V_T, X_\tau; \mathcal{Z}_\tau) = \mathcal{I}(V_T; \mathcal{Z}_\tau) + \mathcal{I}(X_\tau; \mathcal{Z}_\tau|V_T) \tag{10}$$

$$= \mathcal{I}(X_\tau; \mathcal{Z}_\tau) + \mathcal{I}(V_T; \mathcal{Z}_\tau|X_\tau). \tag{11}$$

By equating the right-hand sides of (10) and (11), $\mathcal{I}(V_T; \mathcal{Z}_\tau)$ can be written as

$$\mathcal{I}(V_T; \mathcal{Z}_\tau) = \mathcal{I}(X_\tau; \mathcal{Z}_\tau) - \mathcal{I}(X_\tau; \mathcal{Z}_\tau|V_T) + \mathcal{I}(V_T; \mathcal{Z}_\tau|X_\tau).$$

Notice that $\mathcal{I}(V_T; \mathcal{Z}_\tau|X_\tau) = 0$, because $\mathcal{Z}_\tau$ and V_T are conditionally independent of each other for a given X_τ (Cover & Thomas, 1991). This leads to (9). □

Based on this proposition, $\mathcal{I}(V_T; \mathcal{Z}_\tau)$ can be interpreted as the difference between the information about X_τ contained in $\mathcal{Z}_\tau$, before and after V_T is revealed.

For linear systems with Gaussian noise described in Section 2.1, $\mathcal{I}(X_\tau; \mathcal{Z}_\tau)$ can be easily computed by using a known expression (Mayer-Wolf & Zakai, 1984; Mitter & Newton, 2005; Tomita et al., 1976). The conditional mutual information $\mathcal{I}(X_\tau; \mathcal{Z}_\tau|V_T)$ can be quantified by posing a fixed-interval smoothing problem that incorporates the continuous measurement history $\mathcal{Z}_\tau$ and the discrete noise-free measurement of the verification variables at time T.

Proposition 3 (*Smoother Form*)**.** *Suppose that $P_{0|V} \triangleq \text{Cov}(X_0|V_T) \succ 0$ is given. Then, for the linear system described by (1) and (2), the information reward can be computed as*

$$\mathcal{J}_V^S(T,\tau) \triangleq \mathcal{I}(X_\tau;\mathcal{Z}_\tau) - \mathcal{I}(X_\tau;\mathcal{Z}_\tau|V_T) \quad (12)$$

$$= \mathcal{J}_0(\tau) - \frac{1}{2}\,\mathrm{ldet}(I + Q_X(\tau)\Delta_S(\tau)), \quad (13)$$

where $\mathcal{J}_0(\tau) \triangleq \frac{1}{2}\mathrm{ldet}S_{X|V}(\tau) - \frac{1}{2}\mathrm{ldet}S_X(\tau)$, $\Delta_S(\tau) \triangleq S_{X|V}(\tau) - S_X(\tau)$, *and* $S_X(\tau) \triangleq \mathrm{Cov}(X_\tau)^{-1}$, $S_{X|V}(\tau) \triangleq \mathrm{Cov}(X_\tau|V_T)^{-1}$, *and* $Q_X(\tau) \triangleq \mathrm{Cov}(X_\tau|\mathcal{Z}_\tau)$ *are determined by the following matrix differential equations:*

$$\dot{S}_X = -S_XA - A'S_X - S_X\Sigma_WS_X \quad (14)$$

$$\dot{S}_{X|V} = -S_{X|V}(A + \Sigma_WS_X) - (A + \Sigma_WS_X)'S_{X|V} + S_{X|V}\Sigma_WS_{X|V} \quad (15)$$

$$\dot{Q}_X = AQ_X + Q_XA' + \Sigma_W - Q_XC'\Sigma_N^{-1}CQ_X \quad (16)$$

with initial conditions $S_X(0) = P_0^{-1}$, $S_{X|V}(0) = P_{0|V}^{-1}$ *and* $Q_X(0) = P_0$. *Dependency of* $\mathcal{J}_V^S(T,\tau)$ *on* T *is through* $P_{0|V}$, *which is assumed to be available.*

Proof. First, $\mathcal{I}(X_\tau;\mathcal{Z}_\tau) = \frac{1}{2}\mathrm{ldet}P_X(\tau) - \frac{1}{2}\mathrm{ldet}Q_X(\tau)$, where $P_X(\tau)$ is the solution to the Lyapunov equation $\dot{P}_X = AP_X + P_XA' + \Sigma_W$ with $P_X(0) = P_0$, and $Q_X(t)$ is the solution to the Riccati equation in (16). Regarding the conditional mutual information term, note that

$$\mathcal{I}(X_\tau;\mathcal{Z}_\tau|V_T) = \frac{1}{2}(\mathrm{ldet}P_1 - \mathrm{ldet}P_2), \quad (17)$$

where P_1 is the covariance of $\widetilde{X}_1 \triangleq X_\tau - \mathbb{E}[X_\tau|V_T]$ and P_2 is the covariance of $\widetilde{X}_2 \triangleq X_\tau - \mathbb{E}[X_\tau|V_T,\mathcal{Z}_\tau]$. Namely, P_2 is the error covariance of the fixed-interval smoothing with the past measurement $\mathcal{Z}_\tau$ and the future measurement V_T. Wall, Willsky, and Sandell (1981) derived the following expression for the error covariance for fixed-interval smoothing:

$$Q_{X|V}^{-1}(t) \triangleq \mathrm{Cov}(X_t|V_T,\mathcal{Z}_t)^{-1} = Q_X^{-1}(t) + P_{X|V}^{-1}(t) - P_X^{-1}(t),$$

where $P_{X|V}(t)$ is the estimation error covariance accounting for the future measurement plus a priori information. $P_{X|V}(t)$ is computed as a solution of a Riccati-like equation that is integrated backwards. For this work, the only future measurement is a discrete measurement at T, so $P_{X|V}(\tau)$ is the same as P_1 and can be computed by integrating the following Lyapunov-like equation backwards:

$$\dot{P}_{X|V} = (A + \Sigma_WP_X^{-1})P_{X|V} + P_{X|V}(A + \Sigma_WP_X^{-1})' - \Sigma_W \quad (18)$$

from terminal condition $P_{X|V}(T) = \mathrm{Cov}(X_T|V_T)$ to time τ. Note that $\mathrm{Cov}(X_T|V_T)$ is all zero except the part corresponding to $\mathrm{Cov}(X_T \setminus V_T|V_T)$. While the original formulation required a backwards integration of (18), that step is not needed in this work as every quantity on the right-hand side is available at any time by the past knowledge. Thus, instead, (18) can be integrated forward with initial condition $P_{X|V}(0) = P_{0|V}$, which is assumed to be available. Thus, P_2 and the information reward $\mathcal{J}_V^S(T,\tau)$ can be computed by the forward integration of three matrix differential equations: a Lyapunov equation for P_X, a Riccati equation for Q_X, and a Lyapunov-like equation for $P_{X|V}$.

In addition, equations for P_X and $P_{X|V}$ can be written in terms of the information matrices, $S_X \equiv P_X^{-1}$ and $S_{X|V} \equiv P_{X|V}^{-1}$; this removes the need for performing matrix inversion in (18). Using $\frac{\mathrm{d}}{\mathrm{d}t}\left(M_1^{-1}\right) = -M_1^{-1}\left(\frac{\mathrm{d}}{\mathrm{d}t}M_1\right)M_1^{-1}$ for any non-singular square matrix M_1, equations for S_X and $S_{X|V}$ are obtained as in (14) and (15). Finally, using the properties of the determinant function: $\mathrm{ldet}M_1^{-1} = -\mathrm{ldet}M_1$ and $\det(M_1M_2) = \det M_1 \det M_2$ for square matrices M_1 and M_2, we have

$$\mathcal{J}_V^S(T,\tau) = \frac{1}{2}\left[\mathrm{ldet}S_{X|V}(\tau) - \mathrm{ldet}S_X(\tau)\right] - \frac{1}{2}\left[\mathrm{ldet}\left(Q_X(\tau)[Q_X^{-1}(\tau) + S_{X|V}(\tau) - S_X(\tau)]\right)\right],$$

which is identical to (13). □

Remark 1 (*Computation of Conditional Initial Covariance*)**.** To apply Proposition 3, $P_{0|V}$ must be available. For the linear setting as in this work, $P_{0|V}$ can be computed by the covariance update formula:

$$P_{0|V} = P_0 - P_0\Phi_{(T,0)}'M_V'\left[M_VP_X(T)M_V'\right]^{-1}M_V\Phi_{(T,0)}P_0,$$

where $\Phi_{(T,0)}$ is the state transition matrix from time 0 to T, which is e^{AT} in the linear time-invariant case. Note that the inverse on the right-hand side exists for finite T with $P_0 \succ 0$. For a time-varying case, a fixed-point smoothing using state augmentation can be easily applied to find $P_{0|V}$. When the linear system is used to approximate the short-term behavior of a nonlinear system whose long-term behavior is tracked by some nonlinear estimation scheme, $P_{0|V}$ can be provided by this nonlinear estimator. For instance, in the ensemble-based estimation framework, the ensemble augmentation technique presented by the present authors (Choi et al., 2007) can be used for this purpose.

Corollary 1. *The filter-form information reward* $\mathcal{J}_V^F(T,\tau)$ *in Proposition* 1 *and the smoother-form information* $\mathcal{J}_V^S(T,\tau)$ *in Proposition* 3 *are identical, because they are two different expressions for* $\mathcal{I}(V_T;\mathcal{Z}_\tau)$.

Notice that the smoother form utilizes an additional piece of backward information $S_{X|V}$, while the filter form only uses the forward information captured in P_X (or, equivalently S_X) and Q_X. One aspect for which this additional backward information plays a key role is the information available *on the fly*, which is discussed in Section 3.1.

3. Analysis and discussions of information forms

In this section, the two forms of mutual information developed in Section 2 are analyzed and compared, and advantages of the smoother approach are identified.

3.1. On-the-fly information and mutual information rate

This section discusses the on-the-fly information available in the process of computing the filter and the smoother form of mutual information, and identifies important features of the smoother form in terms of information supply and dissipation. This analysis also facilitates building an information potential field that is used to visualize the spatial distribution of information quantities and to develop a gradient-ascent steering law for a mobile sensor in Section 5.2.

Filter-form on-the-fly information (FOI)

Since the Lyapunov equation in (7) and the Riccati equation in (8) are integrated forward from time 0, $P_X(t)$ and $Q_X(t)$ are available at arbitrary $t < \tau$ in the process of computing the mutual information in (6). With these, the mutual information between the current (i.e., at t) state variables and the measurement thus far (FOI) can be evaluated as

$$\mathcal{I}(X_t;\mathcal{Z}_t) = \frac{1}{2}\mathrm{ldet}P_X(t) - \frac{1}{2}\mathrm{ldet}Q_X(t). \quad (19)$$

The expression for the time derivative of FOI was first presented in Mayer-Wolf and Zakai (1984), and its interpretation as information supply and dissipation was presented in Mitter and Newton (2005). The rate of FOI can be derived as

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{I}(X_t;\mathcal{Z}_t) = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}\mathrm{ldet}P_X(t) - \frac{1}{2}\mathrm{ldet}Q_X(t)\right]$$
$$= \frac{1}{2}\mathtt{tr}\left\{P_X^{-1}\dot{P}_X - Q_X^{-1}\dot{Q}_X\right\}$$
$$= \underbrace{\frac{1}{2}\mathtt{tr}\left\{\Sigma_N^{-1}CQ_XC'\right\}}_{\texttt{Info Supply}} - \underbrace{\frac{1}{2}\mathtt{tr}\left\{\Sigma_W(Q_X^{-1} - P_X^{-1})\right\}}_{\texttt{Info Dissipation}}, \quad (20)$$

where tr denotes the trace of a matrix, and every matrix is evaluated at t. The first term in (20) depends on the measurement and represents the rate of information supply, while the second term depends on the process noise and represents the rate of information dissipation (Mitter & Newton, 2005). It can be shown that the supply and the dissipation terms are non-negative:

$$\mathrm{tr}\left\{\Sigma_N^{-1}CQ_XC'\right\}\geq 0,\qquad \mathrm{tr}\left\{\Sigma_W(Q_X^{-1}-P_X^{-1})\right\}\geq 0,$$

since $CQ_XC'\succeq 0$, $Q_X^{-1}-P_X^{-1}\succ 0$, and the trace of the product of two symmetric positive definite matrices is non-negative (Lasserre, 1995). Thus, measurement tends to increase FOI while the process noise tends to decrease it. Observe that FOI can be decreasing over time if the information dissipation dominates the information supply.

Remark 2. The approach in Grocholsky (2002) considered the entropy of the current state $\mathcal{H}(X_t)=-\frac{1}{2}\mathrm{ldet}J_X(t)+\frac{n_X}{2}\log(2\pi e)$, where $J_X(t)\triangleq Q_X^{-1}(t)$, rather than the mutual information as in (19). The resulting rate calculation is similar to (20) in that the first terms are the same (information supply), but the second terms (information dissipation) are quite different. Therefore, the procedure in Grocholsky (2002) cannot be used to accurately compute the rate of FOI.

Projected filter-form on-the-fly information (PFOI)

Similar to FOI, the mutual information between the current verification variables and the measurement thus far (PFOI) can also be computed on the fly, while computing the filter-form mutual information:

$$\mathcal{I}(V_t;\mathcal{Z}_t)=\frac{1}{2}\mathrm{ldet}P_V(t)-\frac{1}{2}\mathrm{ldet}Q_V(t),\tag{21}$$

where $P_V(t)\triangleq M_VP_X(t)M_V'$ and $Q_V(t)\triangleq M_VQ_X(t)M_V'$.

The time derivation of PFOI can also be expressed in terms of $P_X(t)$ and $Q_X(t)$ as follows.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{I}(V_t;\mathcal{Z}_t)&=\frac{1}{2}\mathrm{tr}\left\{P_V^{-1}\dot{P}_V-Q_V^{-1}\dot{Q}_V\right\}\\&=\underbrace{\frac{1}{2}\mathrm{tr}\left\{\Sigma_N^{-1}CQ_XM_V'Q_V^{-1}M_VQ_XC'\right\}}_{\texttt{Direct Supply}}+\beta(t),\end{aligned}\tag{22}$$

where $\beta(t)$ represents all the remaining terms that do not depend on the observation matrix C. The first term, underbraced as "Direct Supply", represents the immediate influence of the measurement on the current verification variables. The remaining term, $\beta(t)$, captures all of the correlated effect due to coupling in the dynamics on the information supply/dissipation. Observe that sign of $\beta(t)$ is *indefinite*, while the direct supply term is non-negative as $CQ_XM_V'Q_V^{-1}M_VQ_XC'\succeq 0$.

Smoother-form on-the-fly information (SOI)

In the smoother-form framework, the mutual information between the future verification variables V_T and the measurement up to the current time t (SOI) can be calculated as

$$\begin{aligned}\mathcal{I}(V_T;\mathcal{Z}_t)&=\mathcal{I}(X_t;\mathcal{Z}_t)-\mathcal{I}(X_t;\mathcal{Z}_t|V_T)\\&=\mathcal{J}_0(t)-\frac{1}{2}\mathrm{ldet}(I+Q(t)\Delta_S(t)).\end{aligned}\tag{23}$$

The values of matrices $\mathcal{J}_0(t)$, $Q(t)$, and $\Delta_S(t)$ are calculated in the process of the forward integration (14)–(16).

The temporal derivative of the smoother-form mutual information can be written as follows.

Proposition 4 (*Smoother-Form Information Rate*)**.** *For the temporal derivative of the smoother-form on-the-fly information, the following holds:*

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T,t)=\frac{1}{2}\mathrm{tr}\left\{\Sigma_N^{-1}C(t)\Pi(t)C(t)'\right\}\geq 0,\tag{24}$$

where $\Pi(t)\triangleq Q_X(t)(S_{X|V}(t)-S_X(t))[I+Q_X(t)(S_{X|V}(t)-S_X(t))]^{-1}Q_X(t)$.

Proof. Using the expression for the time derivative of ldet of a symmetric positive definite matrix,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T,t)&=\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}(\mathrm{ldet}S_{X|V}-\mathrm{ldet}S_X)-\frac{1}{2}\mathrm{ldet}(I+Q_X\Delta_S)\right]\\&=\frac{1}{2}\mathrm{tr}\left\{S_{X|V}^{-1}\dot{S}_{X|V}-S_X^{-1}\dot{S}_X\right\}\\&\quad-\frac{1}{2}\mathrm{tr}\left\{[I+Q_X\Delta_S]^{-1}\left[\dot{Q}_X\Delta_S+Q_X\dot{\Delta}_S\right]\right\},\end{aligned}$$

where $\dot{\Delta}_S\triangleq\dot{S}_{X|V}-\dot{S}_X$. Using the expressions of $\dot{S}_X$ and $\dot{S}_{X|V}$ in (14) and (15), and the cyclic property of the trace function $\mathrm{tr}(M_1M_2M_3)=\mathrm{tr}(M_2M_3M_1)=\mathrm{tr}(M_3M_1M_2)$, we have

$$\mathrm{tr}\left\{S_{X|V}^{-1}\dot{S}_{X|V}-S_X^{-1}\dot{S}_X\right\}=\mathrm{tr}\left\{\Sigma_W(S_{X|V}-S_X)\right\}.\tag{25}$$

In addition, utilizing expressions of $\dot{S}_X$ and $\dot{S}_{X|V}$, the cyclic property of the trace function, and the matrix inversion lemma (Horn & Johnson, 1985), we have

$$\begin{aligned}&\mathrm{tr}\left\{[I+Q_X\Delta_S]^{-1}\left[\dot{Q}_X\Delta_S+Q_X\dot{\Delta}_S\right]\right\}\\&\quad=\mathrm{tr}\left\{\Sigma_W\Delta_S-(I+Q_X\Delta_S)^{-1}Q_XC'\Sigma_N^{-1}CQ_X\Delta_S\right\}.\end{aligned}\tag{26}$$

From (25) and (26), $\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T,t)$ becomes

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T,t)&=\frac{1}{2}\mathrm{tr}\left\{(I+Q_X\Delta_S)^{-1}Q_XC'\Sigma_N^{-1}CQ_X\Delta_S\right\}\\&=\frac{1}{2}\mathrm{tr}\left\{\Sigma_N^{-1}CQ_X\Delta_S(I+Q_X\Delta_S)^{-1}Q_XC'\right\}.\end{aligned}$$

Wigner's theorem (Subramanian & Bhagwat, 1979) states that a product of three symmetric positive definite matrices is positive definite if the product is symmetric. Note that Π can be written as a product of three positive definite matrices:

$$\begin{aligned}\Pi&=Q_X\Delta_S(I+Q_X\Delta_S)^{-1}Q_X\\&=Q_X(\Delta_S^{-1}+Q_X)^{-1}Q_X,\end{aligned}\tag{27}$$

because Q_X and Δ_S are positive definite. Also, using the matrix inversion lemma (Horn & Johnson, 1985), it can be shown that Π is symmetric:

$$\Pi=Q_X\Delta_SQ_X-Q_X\Delta_S(Q_X^{-1}+\Delta_S)^{-1}\Delta_SQ_X=\Pi'.\tag{28}$$

From (27) and (28), $\Pi\succ 0$. This leads to $C\Pi C'\succeq 0$, and finally $\mathrm{tr}\{\Sigma_N^{-1}C\Pi C'\}\geq 0$, because the trace of the product of two positive definite matrices is non-negative (Lasserre, 1995). □

Since the influence of the future process noise has already been captured in $S_{X|V}$, the mutual information rate for the smoother form is non-negative regardless of the process noise, as stated in Proposition 4. If one stops taking measurement at time t, the information reward stays constant.

To summarize, the smoother-form information rate in (24) correctly accounts for the influence of the process noise and the coupling through dynamics occurring over $(t,T]$, and correctly identifies the net impact of sensing at time t on the entropy reduction of the verification variables at T. However, the two expressions in (20) and (22) based on the filter form only look at the information being accumulated in the current variables (state or verification) without accounting for their evolution and diffusion at the future time; therefore, these expressions do not correctly quantify the impact of the current measurement on the future verification variables.

Remark 3 (*Information Rate for Multiple Sensors*)**.** Consider the case when there are multiple sensor platforms, and the observation matrix of the i-th sensor is C_i, constituting the overall observation matrix of $C = [C_1' \cdots C_{n_S}']'$. Then, the smoother-form mutual information rate in Proposition 4 can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T,t) = \sum_{i=1}^{n_S} \frac{1}{2}\mathtt{tr}\left\{\Sigma_{N_i}^{-1} C_i(x_i, y_i)\Pi(t)C_i(x_i, y_i)'\right\}, \tag{29}$$

where Σ_{N_i} is the (i,i)-th block entry of Σ_N, and (x_i, y_i) is the location of the i-th sensor. In other words, the total rate of change of mutual information is the sum of the rate of change of mutual information of individual sensor platforms.

3.2. Comparison of filter and smoother forms

Correct on-the-fly information and information rate

One benefit of the smoother form is that on-the-fly information based on the smoother form correctly captures the amount and the rate of information gathering by pre-incorporating the effect of future process noise and the correlation via future dynamics, as discussed in Section 3.1. These properties are illustrated in the following example.

Example 1. Fig. 2 compares the time histories of three on-the-fly quantities: the smoother-form on-the-fly information $\mathcal{J}_V^S(T,t) = \mathcal{I}(V_T; \mathcal{Z}_t)$, the filter-form on-the-fly information $\mathcal{I}(X_t; \mathcal{Z}_t)$ and the projected filter-form on-the-fly information $\mathcal{I}(V_t; \mathcal{Z}_t)$. In this example, the following system matrices are used with $\tau = 2$ and $T = 5$:

$$A = \begin{bmatrix} 0.1 & 1 \\ -1 & -0.5 \end{bmatrix}, \quad \Sigma_W = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \quad \Sigma_N = 0.01, \quad M_V = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

There are three key points to discuss about Fig. 2. First, observe how each on-the-fly information changes over time. For $\mathcal{J}_V^S(T,t)$, it is found that information increases in the presence of measurement (before τ) and stays constant in the absence of measurement (after τ). In the history of $\mathcal{I}(X_t; \mathcal{Z}_t)$, the information supply over $[0, \tau]$ increases the accumulated information while the information dissipation over $(\tau, T]$ decreases the accumulated information. The history of $\mathcal{I}(V_t; \mathcal{Z}_t)$ is fluctuating; it can decrease with measurement (around $t = 0.5$) and can increase without measurement (at $t = 2$), because information can be supplied/dissipated from/to the other state variables $X_t \setminus V_t$ via the system dynamics.

Second, note that $\mathcal{J}_V^S(T,t)$ at $t = \tau$ agrees with $\mathcal{I}(V_t; \mathcal{Z}_t)$ at $t = T$ in Fig. 2, and this agreement numerically confirms the equivalence of the filter and smoother forms, i.e., $\mathcal{I}(V_T; \mathcal{Z}_\tau) = \mathcal{I}(V_T; \mathcal{Z}_T)$ with a null measurement during $(\tau, T]$. Third, when comparing three quantities at some arbitrary time t, it is found that $\mathcal{I}(X_t; \mathcal{Z}_t)$ overestimates $\mathcal{I}(V_T; \mathcal{Z}_t)$, and $\mathcal{I}(V_t; \mathcal{Z}_t)$ may overestimate or underestimate $\mathcal{I}(V_T; \mathcal{Z}_t)$ except when $t = T$. Thus, the filter form quantities, $\mathcal{I}(X_t; \mathcal{Z}_t)$ and $\mathcal{I}(V_t; \mathcal{Z}_t)$, are not good indicators of the accumulated information $\mathcal{I}(V_T; \mathcal{Z}_t)$; only the smoother-form quantity $\mathcal{J}_V^S(T,t)$ accurately represents the accumulated information.

Computational efficiency

A simplified analysis can be used to clearly demonstrate the computational efficiency of using the smoother form in the optimal planning process. The optimal trajectory planning typically requires the computation of the information reward for many different measurement choices, i.e., many options of $\mathcal{Z}_\tau$. Let $N_C \gg 1$ denote the total number of measurement options. The dominant portion of the time to compute the information reward involves integrating the matrix differential equations over various time windows. Of course, the actual computation time will depend on the details of the integration scheme. For a simplified analysis, it is enough to assume that the time complexity of integrating one of the matrix differential equations used in this work (i.e., Eqs. (7), (8) and (14)–(16)) is linear in the integration time interval but

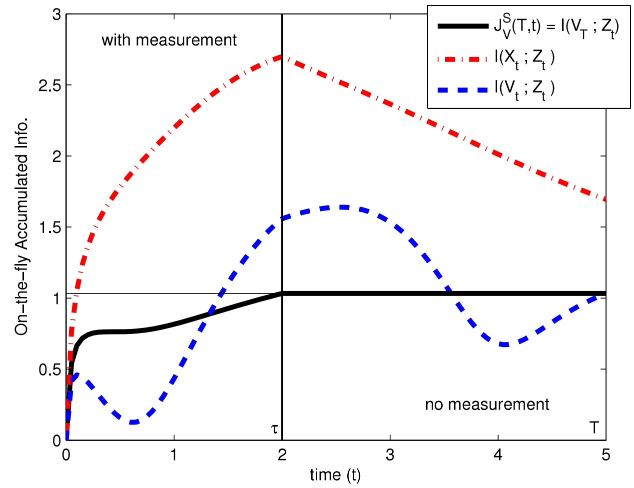

Fig. 2. On-the-fly information by a partial measurement path $\mathcal{Z}_t$.

independent of the type of the equation. For example, if integrating an $n_X \times n_X$ matrix differential equation during a unit time period takes δ_{n_X}, integration of the same equation over $[0, \sigma]$ takes approximately $\delta_{n_X}\sigma$.

If the filter form in (6) is used for the optimal planning process, $P_X(T)$ can be calculated independent of $\mathcal{Z}_\tau$, but $Q_X(T)$ must be calculated for each candidate measurement path. Thus, the Riccati equation in (8) must be integrated over the time window $[0, T]$ for every $\mathcal{Z}_\tau$. Therefore, calculation of the information reward for total of N_C measurement candidates takes $\mathtt{time}_F \approx T\delta_{n_X} + N_C T\delta_{n_X}$.

On the other hand, when computing the information rewards with the smoother form for N_C measurement options, the following computations must be done: (a) integration of the Lyapunov equation in (7) during $[0, T]$ to calculate the conditioned initial covariance $P_{0|V}$, (b) integration of the Lyapunov equation in (14) and the Lyapunov-like equation in (15) during $[0, \tau]$ to calculate $\mathcal{J}_0(\tau)$ and $\Delta_S(\tau)$; and (c) integration of the Riccati equation in (16) over $[0, \tau]$ for every measurement candidate to calculate $Q_X(\tau)$. Thus, with the smoother form calculation of the information rewards takes $\mathtt{time}_S \approx T\delta_{n_X} + 2\tau\delta_{n_X} + N_C\tau\delta_{n_X}$.

The difference and the ratio of the two computation times are

$$\mathtt{time}_F - \mathtt{time}_S \approx [N_C(T/\tau) - (N_C + 2)]\tau\delta_{n_X} \tag{30}$$

$$\mathtt{time}_F/\mathtt{time}_S \approx [\frac{1}{N_C} + 1]/[\frac{1}{N_C}(1 + 2\tau/T) + \tau/T]. \tag{31}$$

With $N_C \gg 1$, the following observations can be made: (a) from (30), if $T/\tau > 1 + 2/N_C \approx 1$, using the smoother form takes less computation time; and (b) from (31), the smoother form reduces the computation cost of evaluating rewards by a factor of approximately T/τ. Thus, if T is significantly bigger than τ, using the smoother form offers significant computational benefits.

Easy integration with existing synthesis techniques

The smoother form facilitates easy integration with existing path synthesis algorithms (established to solve the problem of $T = \tau$), since it projects the decision space from the forecast time window onto the planning time window. Previous sensor planning work (Grocholsky, 2002; Hoffmann & Tomlin, 2010) has established synthesis techniques for the conventional problem of $\min \mathrm{ldet} Q_X(\tau)$. With the smoother form, the optimal forecasting problem is written as $\min \mathrm{ldet}(I + Q_X(\tau)\Delta_S(\tau))$. Thus, the forecasting problem can be treated as a similar type of optimization problem with a possibly different weighting in the objective function. Of course, depending on $\Delta_S(\tau)$, the solutions of the two problems can be very different (see Section 6). However, existing synthesis tools for the conventional decision-making problem can be easily integrated into the forecasting problem, because the only change needed is a slight modification of the objective function. This provides flexibility in the choice of a synthesis method in the planning problem.

4. Continuous path representation

Section 2 presented a formula to quantify the information reward for a continuous measurement path in a finite-dimensional linear system framework. This section shows how to relate the motion of a sensor in continuous space to a measurement trajectory in the time domain.

The approach employs spatial interpolation techniques (e.g. Kriging (Cressie, 1990), Gaussian process regression (GPR) (Willams & Rasmussen, 1996)) that are used to describe continuous environmental variables at an arbitrary location in terms of a finite number of variables at some specified grid points. These techniques assume that the environmental variables at location $\mathbf{r}$ can be represented as a linear combination of those at a finite number of grid points $\mathbf{r}_i$'s:

$$\phi_t(\mathbf{r}) = \sum_{i=1}^{n_G} \lambda_i(\mathbf{r}, \mathbf{r}_i)\phi_t(\mathbf{r}_i), \tag{32}$$

where n_G is the number of grid points, $\phi_t(\mathbf{r}_i) \in \mathbb{R}^{n_E}$ represents the environmental variables at $\mathbf{r}_i$ at time t for $i \in \{1, 2, \dots, n_G\}$, and n_E denotes the number of environmental variables associated with a single grid point. In determining the coefficients λ_i's, this paper considers the zero-mean GPR method (Willams & Rasmussen, 1996) with squared exponential covariance functions, which leads to

$$\lambda_i(\mathbf{r}, \mathbf{r}_i) = \sum_{j=1}^{n_G} \alpha_{ij}\rho(\mathbf{r}, \mathbf{r}_j), \tag{33}$$

where $\rho(\mathbf{r}, \mathbf{r}_j) \triangleq \exp\left[-\frac{1}{2l_x^2}(x - x_j)^2 - \frac{1}{2l_y^2}(y - y_j)^2\right]$ in the two-dimensional space, and α_{ij} is the (i, j)-th element of the matrix $[\rho(\mathbf{r}_i, \mathbf{r}_j)]^{-1}$. The parameters l_x and l_y represent the correlation length scales in each direction.

Under the assumption in (32), the environmental dynamics over the whole continuous space can be fully described by the dynamics of the finite number of variables at grid points. The state vector $X_t \in \mathbb{R}^{n_X}$, where $n_X = n_G \times n_E$, is defined as

$$X_t = \left[\phi_t(\mathbf{r}_i)' \cdots \phi_t(\mathbf{r}_{n_G})'\right]',$$

and this work considers linear dynamics for X_t as in (1). Consider a sensor located at $\mathbf{r}$ at time t that receives measurement of $\phi_t(\mathbf{r})$. Since $\phi_t(\mathbf{r})$ is a linear combination of the $\phi_t(\mathbf{r}_i)$'s, the observation equation for this sensor can be expressed as $Z_t = C(t)X_t + N_t$, where

$$C(t) = \left[\lambda_1(\mathbf{r}, \mathbf{r}_1)I_{n_E} \cdots \lambda_{n_G}(\mathbf{r}, \mathbf{r}_{n_G})I_{n_E}\right] \in \mathbb{R}^{n_E \times n_X}.$$

If a sensor is continuously moving, its motion is fully described by the time history of the location vector $\mathbf{r}(t)$. Thus, the effect of the sensor's motion on the uncertainty dynamics is through the evolution of the observation matrix $C(t)$ due to changes in the $\lambda_i(\mathbf{r}, \mathbf{r}_i)$'s in time. Consider a sensor moving along a specified path $\mathbf{p}_\tau = \{\mathbf{r}(t) : t \in [0, \tau]\}$, where $\mathbf{r}(t)$ is known for all $t \in [0, \tau]$. Then, the evolution of observation matrix $\mathcal{C}_\tau \triangleq \{C(t) : t \in [0, \tau]\}$ can be derived by relating $C(t)$ and $\mathbf{r}(t)$. The information reward associated with this path, denoted as $\mathcal{J}_V(T, \tau; \mathbf{p}_\tau)$, can be computed by evaluating $Q_X(\tau; \mathcal{C}_\tau)$, which is the final value of the Riccati equation corresponding to observation matrix history $\mathcal{C}_\tau$, while $\mathcal{J}_0(\tau)$ and $\Delta_S(\tau)$ in (13) have been computed in advance independently of $\mathbf{p}_\tau$.

To account for the limited mobility of the sensor, the path is, in general, represented as a set of equations of the location vector and its time derivatives: $g_{\mathrm{dyn}}(\mathbf{r}(t), \dot{\mathbf{r}}(t), \ddot{\mathbf{r}}(t), \mathbf{u}(t)) = \mathbf{0}$, where $\mathbf{u}$ is the control input for the sensor motion. For instance, a two-dimensional holonomic motion of a sensor platform with constant speed v can be written as

$$\dot{x}(t) = v\cos\theta(t), \qquad \dot{y}(t) = v\sin\theta(t), \tag{34}$$

where $\theta(t)$ is the heading angle, which is treated as a control input in this model.

5. Path planning formulations

5.1. Optimal path planning

The optimal trajectory planning determines the path $\mathbf{p}_\tau$, or equivalently the time history of the control input, that maximizes the smoother form information reward $\mathcal{J}_V^S(T, \tau) = \mathcal{J}_0(\tau) - \frac{1}{2}\,\mathrm{ldet}(I + Q_X(\tau; \mathcal{C}_\tau)\Delta_S(\tau))$. The prior and posterior initial covariance P_0 and $P_{0|V}$, respectively, are computed first, which enables calculation of $\mathcal{J}_0(\tau)$ and $\Delta_S(\tau)$. Then, an optimization problem involving only the computation of $Q_X(\tau; \mathcal{C}_\tau)$ is posed. This optimization problem is indeed a nonlinear *optimal control* problem (OCP) with a terminal cost functional. The control variables for this OCP are the controls for the sensor motion (e.g., $\theta(t)$ for the two-dimensional holonomic motion), while there are two types of state variable: the vehicle position variables, x and y, and the entries of the $Q_X(t)$ matrix. The optimal path planning problem for a two-dimensional holonomic mobile sensor is stated as

$$\theta^\star(t) \in \arg\max_{\theta(t)} \mathcal{J}_0(\tau) - \mathrm{ldet}(I + Q_X(\tau; \mathcal{C}_\tau)\Delta_S(\tau)) \tag{35}$$

subject to

$$\dot{Q}_X = AQ_X + Q_XA' + \Sigma_W - Q_XC(x, y)'\Sigma_N^{-1}C(x, y)Q_X$$
$$\dot{x} = v\cos\theta, \qquad \dot{y} = v\sin\theta, \qquad Q_X(0) = P_0,$$
$$x(0) = x_0, \qquad y(0) = y_0,$$

where $C(x, y)$ is expressed as a function of x and y to emphasize that its dependency on time is only through the evolution of x and y. Regarding the size of this OCP, there is one control variable, and the number of state variables is $n_X(n_X + 1)/2 + 2$. Constraints in the sensor's motion such as specified final locations and those induced by non-holonomy can be easily incorporated by modifying the vehicle's dynamics and imposing additional constraints. Also, multiple sensor problems can be dealt with by adding associated dynamic/kinematic constraints and by modifying the expression of the observation matrix.

5.2. Gradient-ascent steering

Optimal path planning gives a motion plan for maximum information reward, but solving a nonlinear optimal control problem can take a substantial amount of computational effort, especially when n_X is large. Thus it is beneficial in practice to devise a computationally efficient suboptimal steering law. One approach is to build some potential field and to move along the gradient of that field. The mutual information rate discussed in Section 3.1 can be utilized to construct an *information potential field*, which provides a visualization of how information is distributed or concentrated. This type of information potential field extends a similar notion presented in Grocholsky (2002), which derived the expression of time derivative of entropy (as discussed in Remark 2), and neglected terms unrelated to the observation matrix to build a potential field. Note that since the potential field in Grocholsky (2002) was developed for a different problem formulation that focused on reducing the entropy of the state at a given time, it cannot be directly used for the problem formulation in this work. Thus, this section builds a potential field using the smoother-form information rate in (24) that correctly identifies the influence of a measurement taken at time t on the entropy reduction of the verification variables at T.

For the two-dimensional holonomic sensor motion in (34), the gradient-ascent steering law is

$$\theta_G(t) = \mathtt{atan2}\left\{\frac{\partial}{\partial y}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T, t)\right), \frac{\partial}{\partial x}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{J}_V^S(T, t)\right)\right\},$$

where $\frac{d}{dt}\mathcal{J}_V^S(T,t)$ is the smoother-form mutual information rate, and atan2 denotes the four-quadrant inverse tangent function. Since the relationship between $C(x,y)$ and (x,y) is known, the mutual information rate in (24) can be written as a function of spatial coordinates, and its gradient can be evaluated accordingly. When $C(x(t),y(t)) \in \mathbb{R}^{1\times n_X}$, namely, there is only one environmental variable of interest, the spatial derivative can be written as

$$\frac{\partial}{\partial p}\left(\frac{d}{dt}\mathcal{J}_V^S(T,t)\right) = \Sigma_N^{-1}C(x(t),y(t))\Pi(t)\mathbf{d}(p), \quad p = x, y,$$

where $\mathbf{d}(p)$ is an n_X-dimensional column vector whose i-th element is $\mathbf{d}(p)_i = -l_p^{-2}\sum_j \alpha_{ij}\rho(\mathbf{r},\mathbf{r}_j)(p-p_j),\ p = x, y$. When C is not a row vector, the relation in (29) can be used to derive the expressions for the mutual information rate and its gradient by treating each row of the observation matrix as a separate sensor.

6. Numerical examples

This section compares several planning methodologies that are based on different quantifications of the information reward. The results of the simulation study show the potential advantages of the new technologies proposed in this paper for forecasting problems when compared with traditional planning techniques.

6.1. Scenarios

A simplified weather forecasting problem is considered for numerical simulations. The two-dimensional Lorenz-2003 model (Choi & How, 2007) is employed to describe the nonlinear environmental dynamics. The system equations are

$$\dot{\phi}_{ij} = -\phi_{ij} - \zeta_{i-4,j}\zeta_{i-2,j} + \frac{1}{3}\sum_{k\in[-1,1]}\zeta_{i-2+k,j}\phi_{i+2+k,j} - \mu\eta_{i,j-4}\eta_{i,j-2} + \frac{\mu}{3}\sum_{k\in[-1,1]}\eta_{i,j-2+k}\phi_{i,j+2+k} + \phi_0,$$

where $\zeta_{ij} \triangleq \frac{1}{3}\sum_{k\in[-1,1]}\phi_{i+k,j}$, $\eta_{ij} \triangleq \frac{1}{3}\sum_{k\in[-1,1]}\phi_{i,j+k}$ for $(i,j) \in \{1,2,\ldots,L_i\}\times\{1,2,\ldots,L_j\}$. The subscript i denotes the west-to-east grid index, while j denotes the south-to-north grid index. The boundary conditions of $\phi_{i+L_i,j} = \phi_{i-L_i,j} = \phi_{i,j}$ and $\phi_{i,0} = \phi_{i,-1} = 3$, $\phi_{i,L_j+1} = 0$, in advection terms, are applied to model the mid-latitude area of the northern hemisphere as an annulus. The parameter values are $L_i = 72$, $L_j = 17$, $\mu = 0.66$ and $\phi_0 = 8$. The size of the 1×1 grid corresponds to 347 km $\times$ 347 km in real distance, and 1 time unit in this model is equivalent to 5 days in real time. The overall system is tracked by a nonlinear estimation scheme, specifically an ensemble square-root filter (EnSRF) (Whitaker & Hamill, 2002) data assimilation scheme, that incorporates measurements from a fixed observation network of size 186.

The path planning problem is posed for the linearized model over some 4×3 local region (therefore, $n_G = n_X = 12$) in the entire $L_i \times L_j$ grid space. A linear time-invariant model is obtained by deriving the Jacobian matrix of the dynamics around the nonlinear estimate for the ϕ_{ij}'s at the grid points in the local region. Thus, the state vector X_t represents the perturbation of the ϕ_{ij} from the ensemble mean. In this linear model, the dependence of the local dynamics on the evolution of the external dynamics is ignored in deriving the Jacobian matrix (or A matrix). Instead, this effect is incorporated in the process noise term, i.e., the states on the boundary of the local region, which may be affected by external dynamics more substantially, are assumed to be subject to larger process noise. The goal is to design a 6 h flight path ($\tau = 6$ h) for a single UAV sensor platform to improve the forecast over the three grid points (red squares in Fig. 3) in the eastern part of the local region in 72 h ($T = 72$ h). The motion of the sensor is described as two-dimensional holonomic motion in (34) and it flies at constant speed v grid/h (=347 V km/h). The prior and posterior initial covariance matrices, P_0 and $P_{0|V}$ are provided by the EnSRF data assimilation scheme, where $P_{0|V}$ is computed by the ensemble augmentation method in Choi et al. (2007). Two scenarios with different local region, correlation length scale parameters ($(l_x, l_y) = (1, 0.7)$ or $(1.5, 1)$), and vehicle speed ($v = 1/3$ or $1/2$) are considered, and the sensing noise intensity is set at $\Sigma_N = 0.0025$.

6.2. Comparison of strategies for two scenarios

Two proposed path planning methods, optimal path planning and gradient-ascent steering, are compared with the shortsighted versions of them. Shortsighted path planning takes into account $\mathcal{I}(X_\tau; \mathcal{Z}_\tau)$ instead of $\mathcal{I}(V_T; \mathcal{Z}_\tau)$ to represent the implementation of the traditional planning approach (i.e., $T = \tau$ and $M_V = I$) to a forecasting problem. The optimal shortsighted solution minimizes ldet$Q_X(\tau)$, and the shortsighted gradient-ascent law utilizes the filter-form information rate in (20) to construct an information potential field. Since the information dissipation term of the filter-form information rate in (20) is not an explicit function of (x,y), the shortsighted gradient-ascent law results in a formula that has Q_X instead of Π in the gradient expression.

Each of the two optimal control problems is formulated as a nonlinear program (NLP) by parameterizing the control history as a piecewise linear function consisting of 12 linear segments of equal time span. TOMLAB/SNOPT (2008, v6.0) is used to solve the NLPs; gradient-ascent solutions, and various straight-line solutions are used as the initial guess for the optimization. Both optimized solutions are obtained within two minutes (per initial guess) and satisfy first-order optimality criteria with tolerance of 10^{-4}. Also, as a reference, the best and the worst straight-line paths are also considered. The best straight line solves an NLP to find a constant θ_0 that maximizes the smoother-form information reward, assuming the vehicle dynamics of $\dot{x} = v\cos\theta_0$, $\dot{y} = v\sin\theta_0$.

Table 1 represents the information rewards $\mathcal{J}_V^S(T,\tau)$ for all the methods considered. It is first found that the gradient-ascent steering provides relatively good performance for both scenarios, while the two shortsighted strategies provide very poor performance in scenario 2. Fig. 3(a) and (b) illustrate the sensor trajectories from the six strategies for scenario 1 overlaid with the snapshots of the smoother-form and the filter-form information potential field at the initial time. In both potential fields, a dark region represents an information-rich area; it is generally beneficial to move toward dark regions to gather information. But, since the shape of the potential fields change over time as measurements are taken, the shape of an optimal trajectory can be quite complex, depending on the dynamics and the correlation structure.

In Fig. 3(a) and (b), the shape of the smoother-form information and filter-form information fields are similar in terms of the locations of information-rich regions: two information peaks in the middle, and information ridges in the far east and far west. It can be seen that the gradient-ascent, the shortsighted optimal, the shortsighted gradient-ascent strategies lead the sensor toward the northern information peak in the middle, resulting in similar total amounts of information gathered. The best straight-line trajectory steers the sensor toward the southern information peak, and turns out to gather more information than the aforementioned three strategies. The optimal solution first heads toward to the southern peak, and then turns back to the north in order to gather the information around the north peak. This turning maneuver results in the maximum information by effectively visiting both information peaks. These figures are complex to interpret since the information potential field evolves with time as measurements are taken along the path. However, Fig. 3 just shows snapshots of the information field at the initial time (showing the time evolution of the information field is unfortunately not practical, but see Choi (2009, Section 5.5.2) for more detailed discussions). Of course, the peaks represent

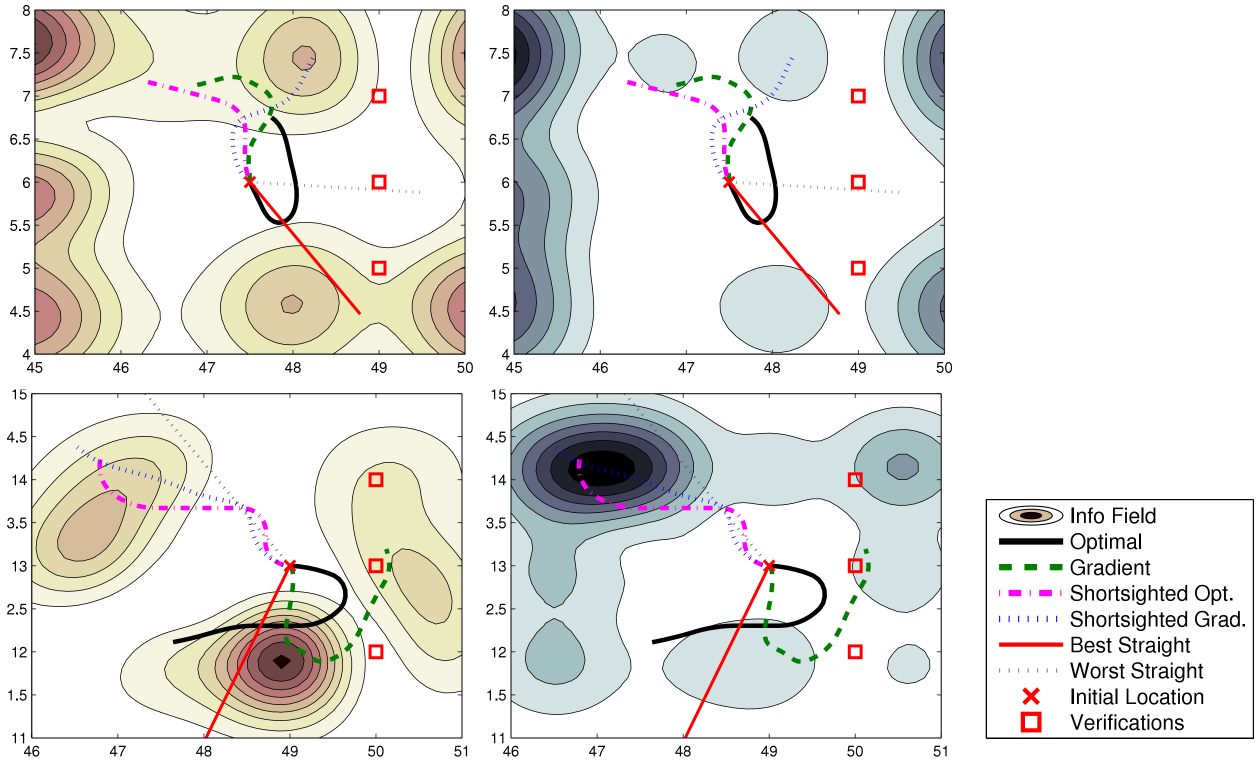

Fig. 3. Sensor trajectories during the planning horizon for different strategies overlaid with snapshots of the information field at the initial time: (a) top left: scenario 1 on smoother-form field; (b) top right: scenario 1 on filter-form field; (c) bottom left: scenario 2 on smoother-form field; (d) bottom right: scenario 2 on filter-form field; the x-axis and y-axis indicate the longitudinal and latitudinal grid indices; the smoother-form and the filter-form information potential fields are significantly different from each other for scenario 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1
Information rewards for different strategies.

	Optimal	Gradient	Shortsighted optimal	Shortsighted gradient	Best straight	Worst straight
Scenario 1	1.04	0.85	0.86	0.79	0.93	0.29
Scenario 2	0.69	0.62	0.20	0.14	0.43	0.14

key sources of information and are likely targets for the optimized paths, but with constrained length paths it might be beneficial to just approach many of the peaks rather than actually visit a limited number. This tendency can be seen in the optimal solution for scenario 1. Also note that the two shortsighted strategies provide reasonable performance compared to the far-sighted strategies in this scenario, as the locations of information peaks are relatively similar in both information fields.

In contrast, Fig. 3(c) and (d) illustrate the large difference between the initial smoother-form and filter-form information fields in scenario 2. The smoother-form information potential field indicates that a large amount of information is concentrated in the south-middle area, while the filter-form information potential field suggests that information is concentrated in the northwest area. As a consequence, while the paths generated based on the smoother form (i.e., optimal, gradient-ascent, and best straight-line) head southwards, those for the two shortsighted decisions head north, leading to significant performance deterioration.

6.3. Choice of verification variables

Monte Carlo simulations are performed to further investigate the differences in the information potential fields, and ensuing performance deterioration of shortsighted strategies, discussed in the previous section. Recall that the shortsighted and the far-sighted decisions concern $\mathcal{I}(X_\tau; \mathcal{Z}_\tau)$ and $\mathcal{I}(V_T; \mathcal{Z}_\tau)$, respectively; and there are two main factors that distinguish these quantities: (i) $M_V \neq I$, and (ii) $T > \tau$. While the second factor is discussed in Choi and How (2009) in the context of receding-horizon sensor planning, this section illustrates the importance of the first factor with a weather forecasting example, using $T = 72$ h and $\tau = 6$ h.

For 36 different local regions of size 4×3 grids (i.e., 1041 km $\times$ 694 km), four choices of verification variables are considered: whole local region ($n_V = n_X = 12$), eastern half of the local region ($n_V = n_X/2 = 6$), eastern quarter of the local region ($n_V = n_X/4 = 3$), and a single point in the east ($n_V = 1$). For a total of 144 scenarios, information rewards for the (far-sighted) gradient-ascent and the shortsighted gradient-ascent strategies were compared. Table 2 compares the average ratio of the information reward of the far-sighted and shortsighted strategies for the four different choices of verification variables. Note that the performance degradation of the shortsighted strategy tends to be more significant as the verification variables are a smaller subset of the whole state variables (ratio increases to 1.46 for a single point).

6.4. Summary

In summary, the numerical results illustrate that (a) for some problems, the difference between planning solutions based on the smoother form $\mathcal{I}(V_T; \mathcal{Z}_\tau)$ and those based on the filter approach $\mathcal{I}(X_\tau; \mathcal{Z}_\tau)$ can be quite large; and (b) one key factor causing this difference is that the information gather focuses on a verification region that is a particular subset of the state variables.

Table 2
Average ratio of information reward for the (far-sighted) gradient-ascent to the shortsighted gradient-ascent for different choices of the scope of the verification region.

Verification variables	Whole state	Eastern half	Eastern quarter	Single point
Info. reward ratio	1. 10	1. 12	1.21	1.46

7. Conclusions

A methodology for continuous motion planning of sensors for informative forecasting was presented. The key contribution of this work is to provide a framework for quantifying the information obtained by a continuous measurement path to reduce the uncertainty in the long-term forecast for a subset of state variables. The smoother form of the information reward that projects the decision space from the long forecast horizon onto a short planning horizon is shown to improve computational efficiency, enables correct evaluation of the rate of information gathering, and offers flexibility in the choice of path synthesis tools. An optimal path planning formulation and a gradient-ascent steering law were presented using spatial interpolation for path representation. A numerical example for a simplified weather forecast compared several planning methodologies and illustrated the performance degradation of the traditional approach for some scenarios.

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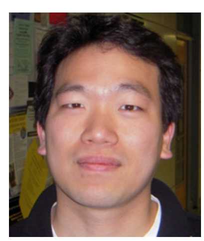

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