A Precise Analytical Estimation Method of Data-dependent Jitter for High Speed Serial Links with the Consideration of Finite Slew Rate of Input Signal

Jeonghyeon Cho, Eakhwan Song, Jongjoo Shim, Gawon Kim, Jiseong Kim, and Joungho Kim
Terahertz Interconnection and Package Laboratory, Department of Electrical Engineering, KAIST
373-1 Guseong-dong, Yuseong-gu, Daejeon, 305-701, Korea

1. INTRODUCTION

As data rate sharply increases, the non-ideality of a channel is becoming a main obstacle to this speed evolution [1]. To overcome this limitation, a lot of interconnection technologies are introduced, nowadays, most of them are in the form of a high speed serial link. The performance of a high speed serial link can be evaluated by its achievable maximum data rate, which is mainly restricted by the data-dependent jitter (DDJ) caused by the insufficient channel bandwidth [2]. As a result, there have been a lot of works to estimate the DDJ in simple analytical way [3]-[5]. In [4], Buckwalter et al. proposed an analytical method based on two simple approximations; a channel as a left-hand side single pole (LHSSP) system, the infinite slew rate for an input signal. This method provided an efficient way to estimate the DDJ under given conditions and it also correlated some physical parameters of channel to its DDJ value. However, two assumptions in this method seem to be somewhat too excessive to apply them for a real channel.

In this paper, as a first step to extend the conventional analytical way to a true world high speed serial link, an analytical method similar to [4] but with consideration of finite slew rate of input signal is proposed. In section II, first, we closely examine the worst-case '1' and the worst-case '0' as worst-case data patterns. In section III, based on the consideration of these worst-case data patterns, we revisit the same situations as in [4]. In section IV, the step function as an input signal is replaced by a ramp function. With this minor modification, we effectively consider the effects of the finite slew rate of input signal and can get some additional advantages unique in this method. Section V summarizes this work and briefly discusses some future works.

2. WORST-CASE '1' AND WORST-CASE '0' AS WORST-CASE DATA PATTERNS

As shown in Figure 1, the worst-case '1' is a data pattern which has just a single '1' data in its total period. This single '1' happens to be after sufficiently long '0' stay. On the other hand, the worst-case '0' is a data pattern which has just a single '0' data in its total period. This single '0' also happens to be after sufficiently long '1' stay. The long stays at the opposite sign to both data transition make the transitions start from bottom-most (or top-most) positions. Hence, the transitions manage to reach the lowest (or highest) high (or low) level, which determine the inner contour of eye-opening. In addition, these two data patterns include the fastest data transitions as well as the slowest data transitions. The transitions from bottom-most (or top-most) position reflect on the slowest data transitions, while the transitions from the lowest (or highest) high (or low) level do the fastest data transitions. Hence, these four data transition are gathered together, the timing jitter can be also obtained. This is the fundamentals of the worst-case analysis.

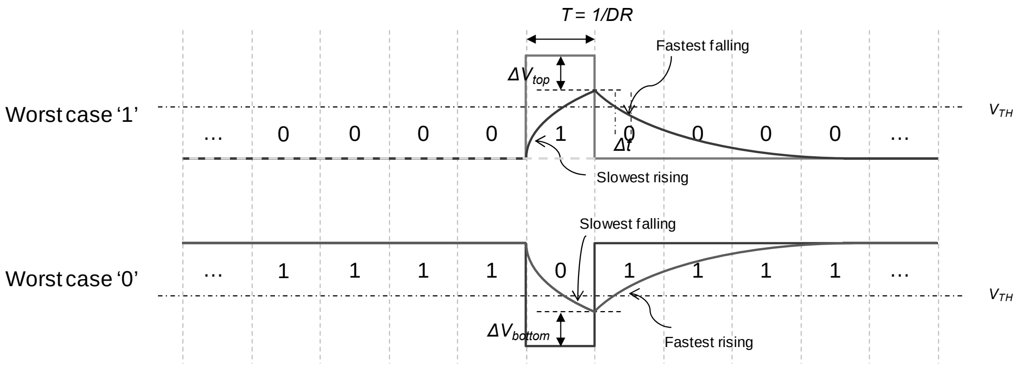


Figure 1 Worst-case '1' and worst-case '0' as worst-case data patterns

3. REVISIT TO THE SAME APPROXIMATIONS IN [4]

Two approximations in [4] are considerably far from the real situations of a data transmission system. However, it presents the foundations of the analytical estimation of DDJ, moreover, it clear up the relationship between the channel bandwidth and the data rate. Under these assumptions, the input signal can be modeled as a step function, which can be written in frequency domain as $1/s$ and a LHSSP system can be expressed in the form of $a/(s+a)$, where a is the reciprocal of the time constant of a channel, or $a=1/\tau$. In frequency domain, the output response is obtained by merely multiplying these values and the output response in time domain is obtained by inverse Laplace transform (ILT) of its frequency domain counterpart, which yields $(1-e^{-at})\cdot u(t)$.

Figure 2 shows the channel response to worst-case '1' signal input. Over there, the rising part of the signal is no other than the time response of a channel. Thus, it can be expressed as

$$g(t) = 1 - e^{-\frac{t}{\tau}} \tag{1}$$

For worst-case 1 signal, the rising part of the signal lasts for only a unit interval ($=T=1/$(data rate)). Thus, the voltage difference between top-most point of the output response and '1' – the normalized ideal logic '1' level – is given by

$$\Delta V = 1 - g(T) = 1 - \left(1 - e^{-\frac{T}{\tau}}\right) = e^{-\frac{T}{\tau}} \tag{2}$$

Then, the output response begins to decline from this point, which can expressed as $(1-\Delta V)e^{-(t/\tau)}$. This is the case of the fastest logic transition. On the contrary, the slowest logic '0'transition starts from the ideal logic '1' level, that is, '1', which given by $e^{-(t/\tau)}$. As aforementioned, the timing jitter can be extracted from the timing difference between these two functions, which can be written by

$$\Delta t = -\tau \ln(1 - \Delta V) = -\tau \ln\left(1 - e^{-\frac{T}{\tau}}\right) \tag{3}$$

In equations (2) and (3), it can be easily seen that both of eye-opening and timing jitter are the function of channel bandwidth and data rate.

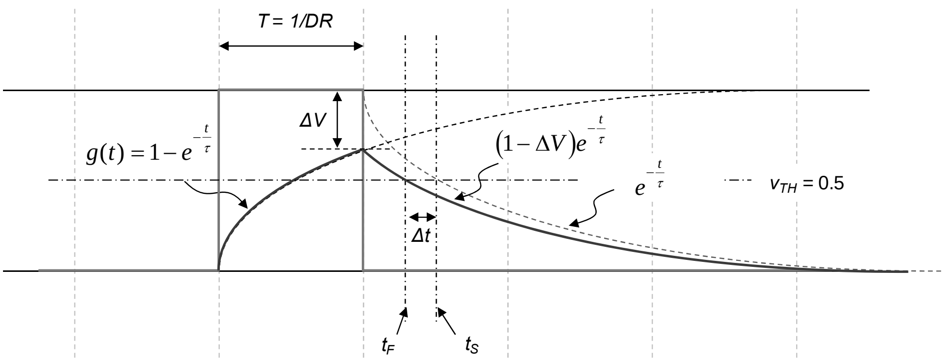


Figure 2 Estimation of DDJ$_{p\text{-}p}$ with the same approximations in [4]

4. CONSIDRATION OF THE FINITE SLEW RATE OF INPUT SIGNAL

A real signal has finite slew rate, which can be effectively modeled as a ramp function. The Laplace transform of it is given by $1/t_r\cdot(1\text{-}e^{-t_r s})\cdot 1/s^2$. Then, to get the output response in frequency domain, multiplying this equation by the transfer function of the same channel in [4] gives $1/t_r\cdot(1\text{-}e^{-t_r s})\cdot[\ 1/s^2\text{-}1/a\cdot\{1/s\text{-}1/(s+a)\}]$. Once again, to get the output response in time domain, applying ILT to this equation gives

$$g'(t)=\frac{1}{t_r}\left[\left\{t-\frac{1}{a}\left(1-e^{-at}\right)\right\}\cdot u(t)-\left\{\left(t-t_r\right)-\frac{1}{a}\left(1-e^{-a(t-t_r)}\right)\right\}\cdot u\left(t-t_r\right)\right] \qquad (4)$$

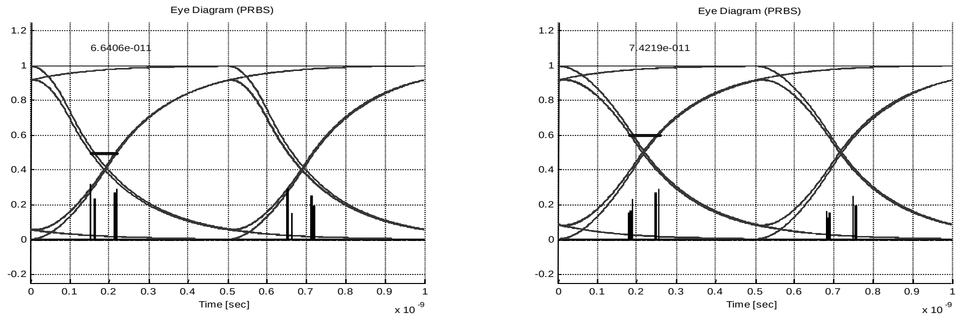
Now, let us derive the analytical equations for eye-opening and timing jitter in this case. The extracting procedure is almost the same as the case of the previous section except for the complexity of its computation. For simplicity, omitting the halfway gives

$$\begin{cases}\Delta V_t=\dfrac{\tau}{t_r}\left(e^{\frac{t_r}{\tau}}-1\right)e^{-\frac{(T+t_\Delta)}{\tau}}+\dfrac{1}{t_f}\left\{t_\Delta-\tau\left(1-e^{-\frac{t_\Delta}{\tau}}\right)\right\}\\ \Delta V_b=\dfrac{\tau}{t_f}\left(e^{\frac{t_f}{\tau}}-1\right)e^{-\frac{(T+t'_\Delta)}{\tau}}+\dfrac{1}{t_r}\left\{t'_\Delta-\tau\left(1-e^{-\frac{t'_\Delta}{\tau}}\right)\right\}\\ t_\Delta=\tau\ln\left\{1+\dfrac{t_f}{t_r}\left(e^{\frac{t_r}{\tau}}-1\right)e^{-\frac{T}{\tau}}\right\}\end{cases} \qquad (5)$$

$$\Delta t=\max\left(t_{fS}\,,t_{rS}\right)-\min\left(t_{fF}\,,t_{rF}\right)$$

$$\begin{cases}t_{rF}=-\tau\left[\ln\left(t_r\right)+\ln\left(1-v_{TH}\right)-\ln(\tau)-\ln\left\{\left(e^{\frac{t_r}{\tau}}-1\right)-\dfrac{t_r}{t_f}\left(e^{\frac{t_f}{\tau}}-1\right)e^{-\frac{T}{\tau}}\right\}\right]\\ t_{rS}=-\tau\left\{\ln\left(t_r\right)+\ln\left(1-v_{TH}\right)-\ln(\tau)-\ln\left(e^{\frac{t_r}{\tau}}-1\right)\right\}\\ t_{fF}=-\tau\left[\ln\left(t_f\right)+\ln\left(v_{TH}\right)-\ln(\tau)-\ln\left\{\left(e^{\frac{t_f}{\tau}}-1\right)-\dfrac{t_f}{t_r}\left(e^{\frac{t_r}{\tau}}-1\right)e^{-\frac{T}{\tau}}\right\}\right]\\ t_{fS}=-\tau\left\{\ln\left(t_f\right)+\ln\left(v_{TH}\right)-\ln(\tau)-\ln\left(e^{\frac{t_f}{\tau}}-1\right)\right\}\end{cases} \qquad (6)$$

The extracted equations look very hard to understand. However, by the aid of them along with their graphical demonstration, it can be found out that the finite slew rate of an input signal affects the output response with 1) smoothing the waveform, 2) changing the transition timing, and 3) bringing about skew with different rise/fall time. Among these effects, the third leads to another valuable consideration. Uneven rise/fall time inherently give rise to the duty-cycle-distortion (DCD) problem. Of course, DCD may come from the other causes. Paraphrasing this, DCD may result from non-half logic threshold voltage and timing deviation of each unit interval. However, the first thing is already considered in the process of extracting equations (4) – (6) and the other thing can be included by substituting *T* with *T'*, where *T* is the expected period of a unit interval and *T'* is the minimum value of the real period of a unit interval. In this manner, equations (5) and (6) predict $DDJ_{p\text{-}p}$ indeed both for eye-opening and timing jitter.



(a) (b)

Figure 3 DCD due to (a) the inequality of rise/fall time and (b) uneven logic threshold voltage

Besides, these equations also unveil that, with identical rise/fall time, the voltage degradations both for top and bottom sides are the same, and timing jitter is independent of rise/fall time.

5. CONCLUSION

In this paper, we proposed an analytical estimation method for high speed serial link. A main objective of this research is to present a fast and comprehensive estimation method for early stage design verification. Hence, the proposed DDJ estimation method is based on the worst-case analysis and the results are presented in the form of analytical equations. Another main objective of this research is to make up for the immoderate approximations of the previous works. Hence, in the proposed method, the finite slew rate of input signal is made into account by modeling the input signal as a ramp function. This minor modification gives a very intuitive tool for understanding the effects of the finite slew rate of input signal, moreover, additional advantages such as the consideration of DCD. The next step for this work will be considering the non LHSSP channel characteristics.

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