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Nonlinear behavior of slender RC columns (2). Introduction of design formula

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Abstract

In this paper, a simple design formula to predict the resisting capacity of slender reinforced concrete (RC) columns is introduced. Using the proposed numerical model in the companion paper, nonlinear time-dependent analyses of slender RC columns that consider the creep deformation of concrete and the P–Δ effect are conducted to determine their ultimate resisting capacity according to the variation of design variables. On the basis of the numerical analysis results obtained, the resisting capacity reduction factors, which represent the reduction rates of the resisting capacity in a long column corresponding to a short column on a P–M interaction diagram, are determined. In addition, the relationships between the resisting capacity reduction factors and design variables are established from regression and used to determine the ultimate resisting capacity of slender RC columns without any rigorous numerical analysis. The ultimate resisting capacities calculated from the regression formula are compared with those constructed from rigorous nonlinear time-dependent analyses and from the ACI formula with the objective of establishing the relative efficiencies of the proposed formula.

Keywords: RC Column; Slenderness ratio; P–Δ Effect; Creep; Resisting capacity reduction factor; Design formula

1. Introduction

Short RC columns, when overloaded, experience material failure prior to reaching a buckling mode of failure. Furthermore, the lateral deflections of short compression members subjected to bending moments are small and, thus, contribute little secondary bending moment by the P–Δ effect. Therefore, the ultimate resisting capacity of short reinforced concrete (RC) columns is generally determined on the basis of the assumption that the effects of buckling and lateral deflection on strength are negligibly small. Unlike a short RC column whose ultimate resisting capacity can be uniquely represented from a P–M interaction diagram for a typical section, however, a slender RC column has a considerable reduction in strength because of a secondary bending moment caused by the lateral deflection, and its strength is dominantly affected by the slenderness ratio. Consequently, the development of a correct P–M interaction diagram for a slender RC column would require an elaborate numerical analysis taking into account the time-dependent and cracking effects on deformation.

To permit the greatest flexibility in structural design, specifications must also provide for adequate determination of column strength with any slenderness ratio. Thus, most design codes [1,3] take into account the length effect on the resisting capacity of long columns by increasing the applied primary moment. In advance, the increase is based on the moment magnification factor derived from elastic stability theory. However, the

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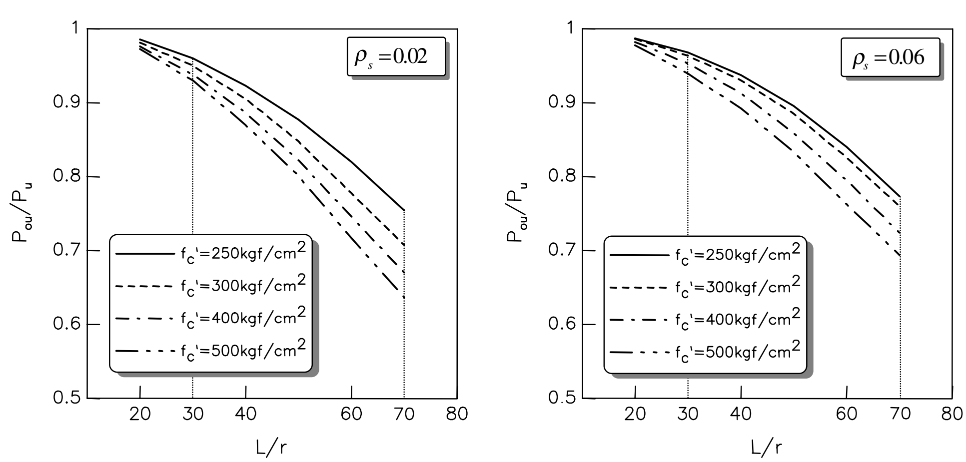
principal difficulty in using the moment magnification factor is that it requires an exact estimation of the bending stiffness (*EI*). Since cracking, time-dependent effects, and the nonlinearity of the concrete stress–strain curve cause decreases in *EI* as the applied load increases, a more exact strength prediction of slender RC columns fully depends on the precise calculation of *EI* according to the loading history. Nevertheless, most design codes propose simple expressions for the calculation of *EI* without any further consideration of the steel ratio and eccentricity. Consequently, the application of current design codes [1,3] may result in underestimation of the resisting capacity, and this may accelerate as the slenderness ratio increases. Hence, the construction of a correct *P–M* interaction diagram for RC columns with large slenderness ratios would require a rigorous analysis considering material and geometric nonlinearities due to concrete cracking and time-dependent deformation of concrete. However, rigorous analysis is time consuming, and its use in design practice requires much experience.

To solve this problem in practice and to consider all the nonlinear effects in design effectively, a simplified design method for slender RC columns is proposed in this paper. Many parameter studies for slender RC columns are conducted using the proposed numerical model in the companion paper, and more refined comparisons of numerical results follow. Moreover, the ultimate resisting capacity reduction factors, which represent the reduction rates of *P–M* interaction diagrams in slender RC columns to short RC columns, are proposed from regression. These can be used effectively in the preliminary design stage. Based on the proposed reduction factors, the required *P–M* interaction diagram of a slender RC column can be easily constructed without conducting additional sophisticated creep buckling analyses for a slender RC column. The ultimate resisting capacities calculated from the proposed design formula are compared with those calculated from rigorous nonlinear time-dependent analyses and from the ACI method with the objective of establishing the relative efficiencies of the method involving the proposed formula.

2. Behavior of slender RC columns

In the companion paper, it was shown that the numerical results for the short-term and long-term behavior of slender RC columns compared well with the experimental and analytical results. In advance, it was found that the strength of slender RC columns depends on many design variables and shows a considerable reduction as the slenderness ratio increases, especially in slender RC columns with a relatively high compressive strength of concrete and steel ratio. In this paper, more refined comparisons for the structural behavior according to the change in each design variable are made on the basis of the proposed numerical model in the companion paper.

A $h = 50$-cm square concrete column section reinforced symmetrically by steel at $0.1h$ distance from each of the two critical faces is used. By changing the variables from $f'_c = 250\ \text{kgf/cm}^2$ to $f'_c = 500\ \text{kgf/cm}^2$ for the compressive strength of concrete, from $\rho_s = 0.02$ to $\rho_s = 0.08$ for the steel ratio, and from $L/r = 10$ to $L/r = 70$ for the slenderness ratio, nonlinear analyses are conducted. Creep deformation of concrete is also taken into account on the basis of $\beta_d = P_D/(P_D + P_L) = 1$ because the most strength reduction appears at this upper limit condition in spite of a relatively small difference for β_d (see Fig. 9 in the companion paper), and the ultimate creep coefficient in the ACI model is assumed to be changed from $c_u = 1.0$ to $c_u = 3.0$. In advance, the following material properties generally used in design of RC columns are assumed: $E_s = 2.1 \times 10^6$ kgf/cm^2 and $f_y = 4000$ kgf/cm^2, and the slenderness ratio is defined by L/r (L = pin-ended column height and r = radius of gyration $= h/\sqrt{12}$).



(a) $\rho_s = 0.02$ (b) $\rho_s = 0.06$

Fig. 1. Resisting capacity reductions in accordance with the concrete compressive strength: (a) $\rho_s = 0.02$; (b) $\rho_s = 0.06$.

The effect of the compressive strength of concrete on the resisting capacity is shown in detail in Fig. 1. The numerical results are for the case of RC columns subjected to axial forces with a minimum eccentricity of e_{min} = 1.524 + 0.076h (cm) [6]. The ultimate axial force ratios of P_{ou}/P_u are plotted as the slenderness ratio

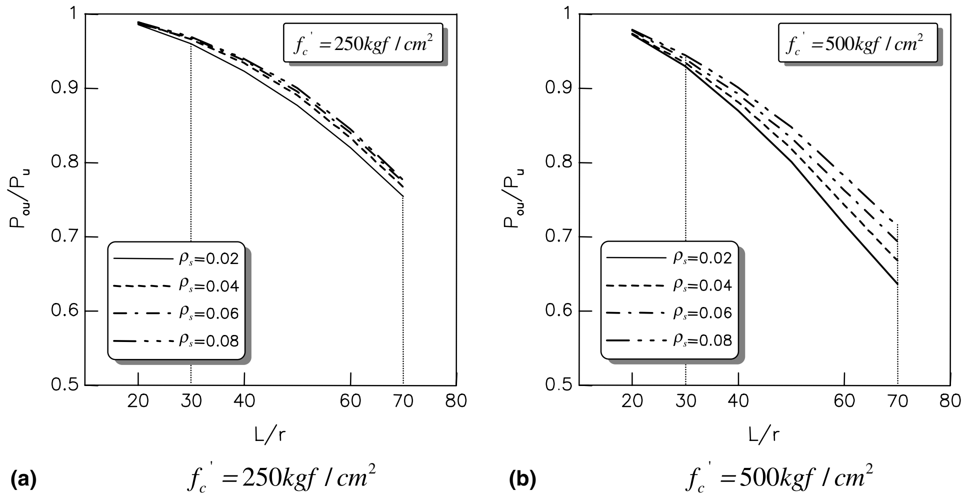


Fig. 2. Resisting capacity reductions in accordance with the steel ratio: (a) $f_c' = 250$ kgf/cm^2; (b) $f_c' = 500$ kgf/cm^2.

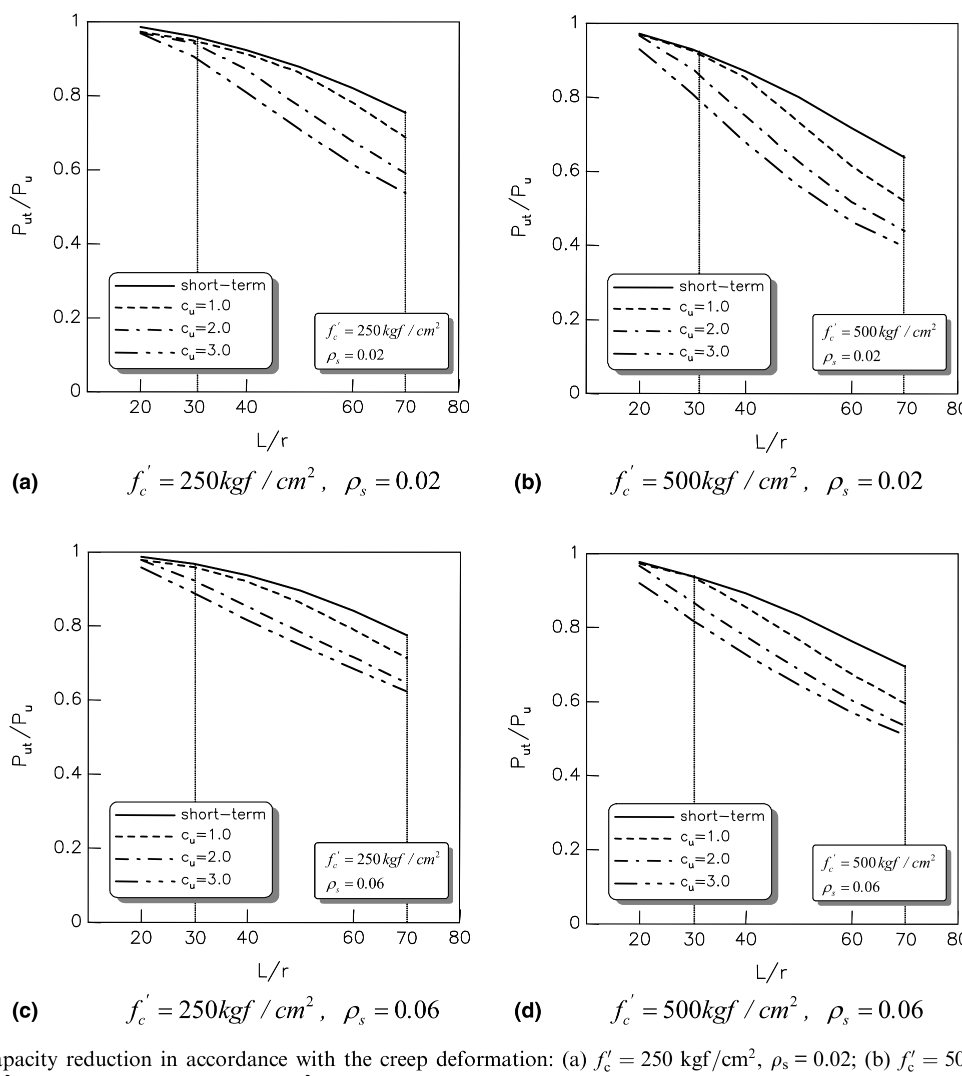


Fig. 3. Resisting capacity reduction in accordance with the creep deformation: (a) $f_c' = 250$ kgf/cm^2, ρ_s = 0.02; (b) $f_c' = 500$ kgf/cm^2, ρ_s = 0.02; (c) $f_c' = 250$ kgf/cm^2, ρ_s = 0.06; (d) $f_c' = 500$ kgf/cm^2, ρ_s = 0.06.

changes from $L/r = 20$ to $L/r = 70$, where P_u and P_{ou} are the short-term ultimate axial forces at a section and at a member, respectively.

As shown in Fig. 1, the strength of RC columns depends significantly on the compressive strength of concrete and steel ratio. The strength of slender RC columns decreases with an increase of the slenderness ratio, and this phenomenon is more significant in columns with relatively high compressive strengths of concrete. In the case of $\rho_s = 0.02$, the resisting capacity reduction for the ultimate axial force reaches 23% for $f'_c = 250$ kgf/cm^2 and 36% for $f'_c = 500$ kgf/cm^2 when the slenderness ratio $L/r = 70$, because of the P–Δ effect. On the other hand, these reduction ratios are slightly reduced as the steel ratio increases. This result seems to be caused by two factors; First of all, the maximum axial force capacity in a short column is directly proportional to the compressive strength of concrete [1], while the critical axial force in a long column, represented by the bending stiffness EI, is almost independent of the compressive strength of concrete. This implies that a more remarkable strength reduction rate according to the slenderness ratio may occur in a column with relatively high concrete compressive strength. Secondly, since a long column accompanies relatively large lateral deflection and a secondary moment due to the P–Δ effect, the column collapses when the moment resisting capacity corresponding to the applied axial force is insufficient. A slender RC column with lower steel ratio will represent a larger bending curvature at the fully cracked stage before the section failure. Hence, the more the steel ratio decreases, the more the strength reduction rate increases because of the increasing P–Δ effect. These trends seem to be maintained over the entire eccentricity range as shown in Figs. 10 and 11 in the companion paper, and it can be concluded that the resisting capacity reduction becomes larger in an RC column with a relatively high concrete compressive strength and small steel ratio as the slenderness ratio increases.

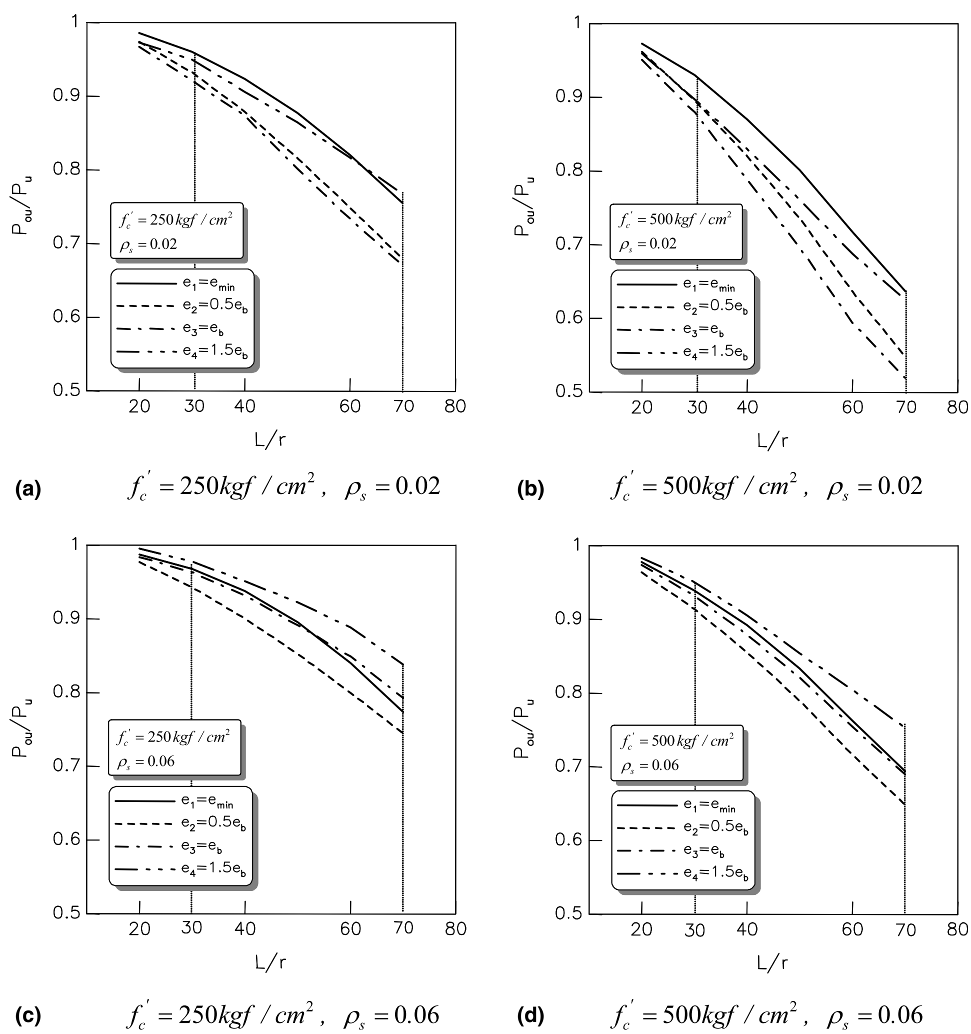


Fig. 4. Resisting capacity reductions in accordance with the eccentricity: (a) $f'_c = 250$ kgf/cm^2, $\rho_s = 0.02$; (b) $f'_c = 500$ kgf/cm^2, $\rho_s = 0.02$; (c) $f'_c = 250$ kgf/cm^2, $\rho_s = 0.06$; (d) $f'_c = 500$ kgf/cm^2, $\rho_s = 0.06$.

The numerical results obtained for the same example structure are rearranged with respect to the steel ratio. As shown in Fig. 2, the resisting capacity reduction for the ultimate axial force becomes larger as the steel ratio decreases, and a more remarkable reduction appears according to an increase of the concrete compressive strength, as mentioned above. Since the effect of the steel ratio on the resisting capacity of RC columns is gradually increased in proportion to the concrete compressive strength, the *P–Δ* effect dominantly appears in high strength concrete columns with relatively low steel ratios. In advance, from a comparison of Figs. 1 and 2, it can be inferred that the compressive strength of concrete has a more dominant influence on the resisting capacity of RC columns than the steel ratio.

In order to consider the effect of creep deformation, time-dependent analyses are conducted for the same example structure when the ultimate creep coefficients are 1.0, 2.0 and 3.0, respectively. The obtained numerical results are shown in Fig. 3.

As shown in Fig. 3, the creep deformation reduces the resisting capacity of slender RC columns and the most remarkable reduction of the resisting capacity appears in an RC column with relatively high concrete compressive strength and low steel ratio, as in the short-term loading case (see Figs. 1 and 2). This seems to arise from the fact that this column causes a relatively large *P–Δ* effect as mentioned above and accompanies larger creep deformation in proportion to the mechanical deformation. Nevertheless, it is almost impossible to find any relationship between the *P*-effect and the additional creep deformation because the creep deformation is influenced by aging effects. In advance, the failure of a slender RC column subjected to long-term loading may be caused by structural instability (see Fig. 1 in the companion paper).

As shown in Figs. 10–13 in the companion paper, which represent *P–M* interaction diagrams according to changes in the slenderness ratio, the resisting capacity of a slender RC column with a typical slenderness ratio does not maintain a uniform reduction at each eccentricity. Fig. 4 shows the strength reduction ratios of slender RC columns at four typical eccentricities. A column with a relatively low steel ratio demonstrates the most remarkable strength reduction at the balanced eccentricity e_b. Fig. 4 also shows that the eccentricity that exhibits the maximum strength reduction decreases with an increase in the steel ratio ($e = 0.5e_b$ when $\rho_s = 0.06$). This result seems to be originated from the characteristics in the resisting capacity of RC columns. An increase of steel ratio causes a larger increase of the moment resisting capacity than the axial force resisting capacity and leads to an increase of the balanced eccentricity. It means that the critical eccentricity corresponding to an applied axial force decreases. Therefore, it seems to be almost impossible to find any relationship between the eccentricity and the other design variables.

3. Proposed formula

3.1. Determination of strength reduction coefficient

Since the ultimate resisting capacity of RC columns is governed by many design variables and is gradually reduced as the slenderness ratio increases because of the *P–Δ* effect and accompanying creep deformation of concrete, it is absolutely necessary to conduct a sophisticated time-dependent nonlinear analysis considering material and geometric nonlinearities to exactly predict the ultimate resisting capacity of slender RC columns. However, nonlinear analyses is time consuming and costly. To circumvent these limitations and to estimate the ultimate resisting capacity of RC columns effectively in practice, a strength reduction coefficient for slender RC columns is introduced in this paper. If a rigorous nonlinear analysis is conducted at the final design stage after selecting the section in the preliminary design stage with the proposed formula below, then efficiency in design can be expected.

When the dimensions of the concrete section and the material properties of concrete and steel are determined, the *P–M* interaction diagram of a section representing the ultimate resisting capacity of a column is uniquely defined through a section analysis on the basis of the force equilibrium and compatibility condition. However, as a slender RC column accompanies *P–Δ* effect and creep deformation of concrete, the *P–M* interaction diagram cannot be easily determined. If a strength reduction coefficient *F* is defined as the ratio of the ultimate resisting capacity (P_n, M_n) in RC sections to that in a slender column, point A and point B in Fig. 5, at the same eccentricity, then the ultimate resisting capacity of a slender RC column can be easily determined from ($P_n(1-F)$, $M_n(1-F)$) without conducting additional sophisticated numerical analyses.

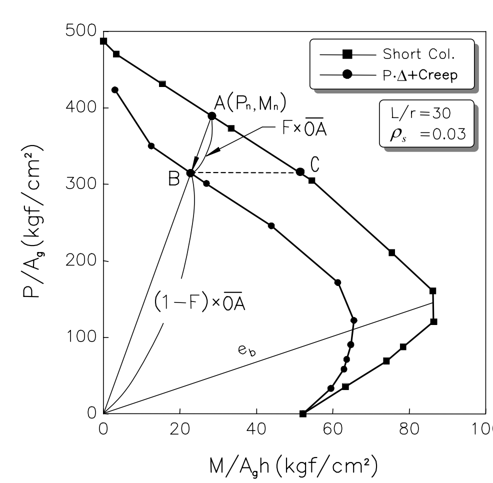
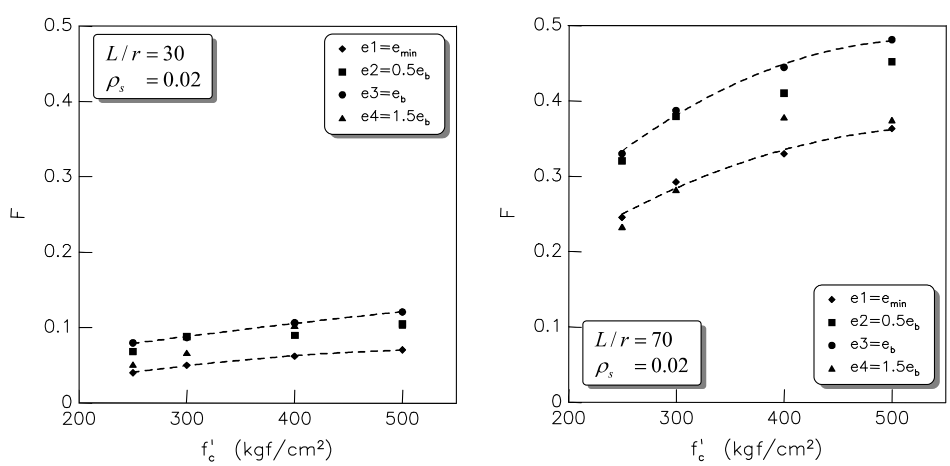


Fig. 5. Definition of strength reduction coefficient *F*.

To introduce a design formula for the strength reduction coefficient F, however, the following difficulties must be overcome: (1) The P–M interaction diagrams must be determined for a column section (equivalent to a short column) and for long columns with the same design variables; (2) the calculated strength reduction coefficient F does not maintain a constant value but changes according to the design variables such as the eccentricity, the slenderness ratio, the compressive strength of concrete, and the ultimate creep coefficients; and (3) unlike structural steel members, RC columns do not use the commercially available standard sections. Theoretically, an infinite number of RC column sections can be designed for the applied forces.

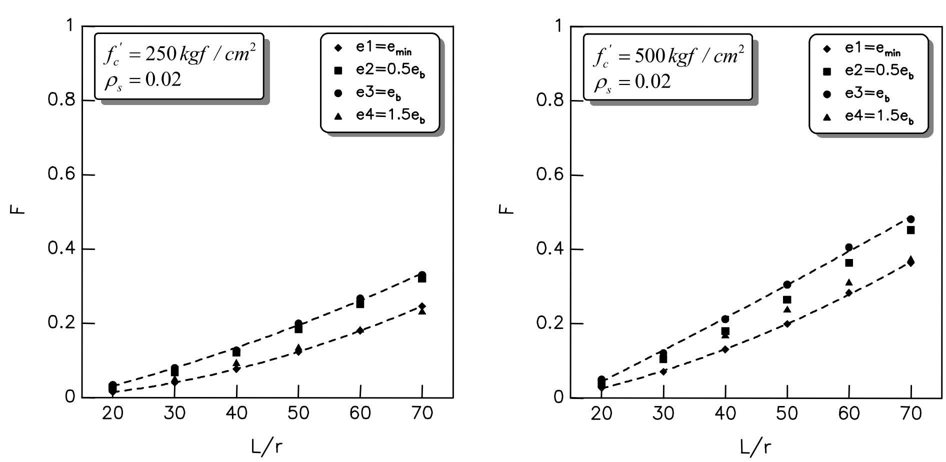
Hence, in determining P–M interaction diagrams of RC columns, all the variables need to be ranged and/or assumed on the basis of practical limitations and the design code requirements. The yielding stress of steel is $f_y = 4350$ kgf/cm^2 (Grade 60 steel) which is the generally used value in column design. In addition, the steel ratio ranges from $\rho_s = 0.02$ to $\rho_s = 0.08$, the compressive strength of concrete from $f'_c = 250$ kgf/cm^2 to $f'_c = 500$ kgf/cm^2, the slenderness ratio from $L/r = 20$ to $L/r = 70$, and four different ultimate creep coefficients of 0.0, 1.0, 2.0 and 3.0, and seven different eccentricities of e_{min}, 0.25 e_b, 0.5e_b, 0.75 e_b, e_b, 1.2e_b and 1.5e_b are selected for the numerical analyses conducted in this paper for developing the strength reduction coefficient F.

Six typical results of the strength reduction coefficients F calculated, when the ultimate creep coefficient C_u is 0.0, are represented with dots in Figs. 6–8. These figures show that the strength reduction coefficients F increase in proportion to the slenderness ratio L/r and become larger as the steel ratio decreases. The increase of coefficients F indicates a reduction of the ultimate resisting capacity. Since it may be very difficult to find a regulation formula for the eccentricity as shown in Fig. 4, it



(a) $\rho_s = 0.02$, $L/r = 30$ (b) $\rho_s = 0.02$, $L/r = 70$

Fig. 6. Strength reduction coefficient F in accordance with the compressive strength of concrete: (a) $\rho_s = 0.02$, $L/r = 30$; (b) $\rho_s = 0.02$, $L/r = 70$.



(a) $f'_c = 250 kgf/cm^2$, $\rho_s = 0.02$ (b) $f'_c = 500 kgf/cm^2$, $\rho_s = 0.02$

Fig. 7. Strength reduction coefficient F in accordance with the slenderness ratio: (a) $f'_c = 250$ kgf/cm^2, $\rho_s = 0.02$; (b) $f'_c = 500$ kgf/cm^2, $\rho_s = 0.02$.

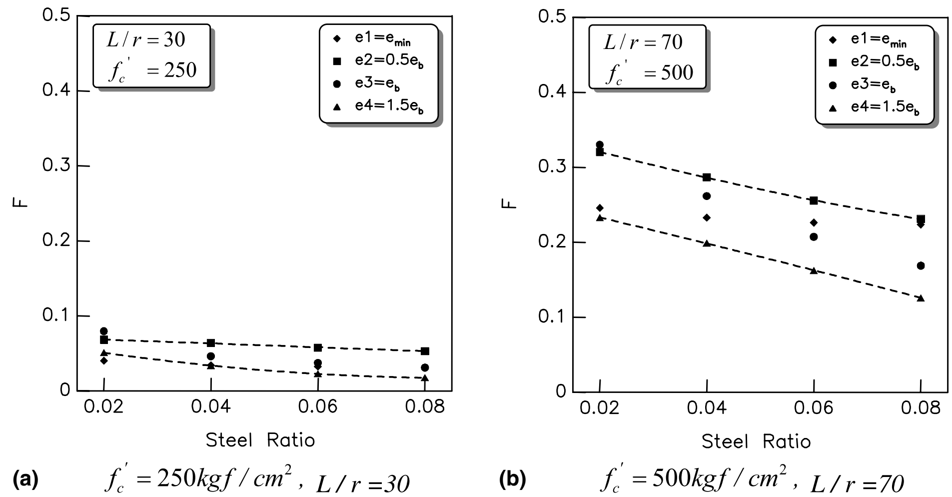


Fig. 8. Strength reduction coefficient F in accordance with the steel ratio: (a) $f_c' = 250$ kgf/cm^2, $L/r = 30$; (b) $f_c' = 500$ kgf/cm^2, $L/r = 70$.

is almost impossible to consider the variation of eccentricity in the regression formula.

Even though the strength reduction coefficients have a somewhat scattered distribution caused by the absence of regulation in the eccentricity, regulation can be found in the other design variables. To determine a reasonable regression formula, an overall review of the effect of each design variable was conducted (see Figs. 1–4). Consequently, it was found that the variation of steel ratio ρ_s, compressive strength of concrete f_c' and slenderness ratio L/r of all the design variables had the greatest effects on the strength reduction coefficient F of slender RC columns (see Figs. 6–8). Since the strength reduction coefficients are gradually increased or decreased according to change in each design variable and represent a nonlinear characteristic (see Figs. 6–8), a second order polynomial is assumed for each design variable. On the basis of the strength reduction coefficients F calculated according to the change in each design variable, the regression formula represented in Eq. (1) is finally chosen, and the regression results obtained at each eccentricity are listed in Tables 1–4. In particular, the correlation coefficient R^2 representing the values close to 1.0 in these Tables implies that the proposed regression formula can simulate the strength reduction coefficients effectively.

$$F = a1 + a2(f_c'/100) + a3\rho_s + a4(L/100r) + a5(f_c'/100)^2 + a6\rho_s(f_c'/100) + a7(\rho_s)^2 + a8(L/100r)(f_c'/100) + a9(L/100r)\rho_s + a10(L/100r)^2, \quad (1)$$

where f_c' is the compressive strength of concrete (kgf/cm^2), ρ_s is the steel ratio (A_{st}/A_c), and L/r is the slenderness ratio.

As shown in Tables 1–4, independent regression formula are defined at each eccentricity because of the difficulty in finding regulation formulas for the eccentricity and ultimate creep coefficient. Hence, determination of the strength reduction coefficients for a slender RC column subjected to an axial force with an arbitrary eccentricity can be achieved through a linear interpolation of the strength reduction coefficients calculated on the basis of Tables 1–4.

Table 1
Regression constants of strength reduction coefficient F ($c_u = 0.0$)

Eccentricity	e_{min}	$0.25e_b$	$0.5e_b$	$0.75e_b$	e_b	$1.2e_b$	$1.5e_b$
a1	−0.024	−0.045	−0.059	−0.049	−0.042	−0.070	−0.070
a2	0.007	0.007	0.009	0.008	0.007	0.023	0.022
a3	0.261	0.576	0.494	−0.111	−0.318	−0.206	0.333
a4	−0.115	0.050	0.017	0.210	0.203	0.188	0.085
a5	−0.001	−0.001	−0.002	−0.001	−0.001	−0.003	−0.002
a6	−0.112	−0.060	−0.089	−0.142	−0.181	−0.261	−0.274
a7	3.815	−0.837	3.560	11.817	17.080	16.998	11.058
a8	0.063	0.055	0.007	0.080	0.085	0.084	0.083
a9	−1.807	−1.379	−0.351	−4.711	−5.968	−5.798	−4.630
a10	0.547	0.443	0.004	0.334	0.367	0.342	0.301
R^2	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**	**0.99**	**0.99**

Table 2
Regression constants of strength reduction coefficient F ($c_u = 1.0$)

Eccentricity	e_{min}	$0.25e_b$	$0.5e_b$	$0.75e_b$	e_b	$1.2e_b$	$1.5\ e_b$
a1	−0.011	0.010	0.010	0.065	0.058	−0.012	0.000
a2	0.015	−0.015	−0.015	−0.035	−0.032	−0.007	−0.005
a3	0.421	0.961	0.494	−0.809	−0.551	0.414	0.451
a4	−0.258	0.021	0.181	0.398	0.492	0.453	0.291
a5	−0.003	0.001	0.001	0.003	0.004	0.003	0.003
a6	−0.145	−0.149	−0.196	−0.095	−0.241	−0.442	−0.441
a7	4.688	1.183	8.728	16.027	16.696	14.710	16.138
a8	0.101	0.088	0.089	0.097	0.102	0.097	0.083
a9	−2.060	−2.746	−4.492	−5.098	−5.482	−5.633	−5.325
a10	0.766	0.559	0.435	0.136	−0.050	−0.026	0.115
$\mathbf{R^2}$	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**	**0.98**

Table 3
Regression constants of strength reduction coefficient F ($c_u = 2.0$)

Eccentricity	e_{min}	$0.25e_b$	$0.5e_b$	$0.75e_b$	e_b	$1.2e_b$	$1.5e_b$
a1	−0.121	−0.063	−0.035	−0.012	0.116	−0.080	−0.084
a2	0.033	0.008	0.005	−0.004	−0.032	0.045	0.049
a3	0.963	0.678	0.082	−1.048	−0.551	−0.921	−0.356
a4	0.328	0.341	0.350	0.591	0.492	0.681	0.433
a5	−0.004	−0.001	0.000	0.000	0.004	−0.003	−0.003
a6	−0.189	−0.164	−0.293	−0.200	−0.241	−0.401	−0.397
a7	5.379	6.116	15.692	21.094	16.696	21.540	17.344
a8	0.089	0.097	0.111	0.108	0.102	0.098	0.089
a9	−3.675	−4.245	−5.309	−5.761	−5.482	−5.561	−4.964
a10	0.370	0.348	0.284	−0.027	−0.050	−0.272	−0.084
$\mathbf{R^2}$	**0.99**	**0.99**	**0.99**	**0.99**	**0.98**	**0.98**	**0.97**

Table 4
Regression constants of strength reduction coefficient F ($c_u = 3.0$)

Eccentricity	e_{min}	$0.25e_b$	$0.5e_b$	$0.75e_b$	e_b	$1.2e_b$	$1.5e_b$
a1	−0.176	−0.116	−0.090	−0.054	−0.040	−0.081	−0.083
a2	0.040	0.025	0.021	0.014	0.024	0.051	0.051
a3	1.046	0.125	−0.467	−1.433	−1.940	−1.510	−1.510
a4	0.764	0.638	0.621	0.832	0.887	0.799	0.799
a5	−0.004	−0.003	−0.002	−0.001	−0.001	−0.004	−0.004
a6	−0.196	−0.139	−0.298	−0.235	−0.293	−0.358	−0.358
a7	6.004	9.598	20.580	23.929	27.433	24.621	24.621
a8	0.079	0.094	0.109	0.105	0.098	0.095	0.095
a9	−4.762	−4.808	−5.848	−5.895	−5.936	−5.774	−5.774
a10	−0.017	0.084	0.052	−0.267	−0.394	−0.369	−0.369
$\mathbf{R^2}$	**0.99**	**0.99**	**0.99**	**0.99**	**0.99**	**0.98**	**0.98**

3.2. Application of the proposed formula

The related details in design steps are described in Fig. 9 to show the difference in long column design using the proposed formula in this paper as opposed to the ACI formula [1]. As shown in this figure, the ACI formula requires an increase in the applied ultimate moment M_u^0 by multiplying a magnification factor $\delta = 1/(1 - P_u^0/\phi_k P_{cr}) \geqslant 1.0 (M_u = \delta M_u^0)$, where P_{cr} a critical load of a slender RC column and φ_k is the stiffness reduction factor designed to consider the inevitable random variability of the materials. Then, a column section is designed to ensure that the modified ultimate loads of $P_u = P_u^0$ and M_u exist inside the P–M interaction diagram in which the nominal strength of a column section is reduced by the strength reduction factor ϕ (see Figs. 9 and 10(a)). Conversely, the proposed method does not magnify the applied ultimate loads, while the reduction of the P–M interaction diagram itself is taken into consideration according to the proposed formula (see Figs. 9 and 10(b)). As an example, the ultimate resisting capacity of a short RC column corresponding to point A in Fig. 5 moves to point B in Fig. 5 as the slenderness ratio increases while maintaining the eccentricity by the primary bending moment $M_u^0 = P_u^0 e$. However, the design steps proceeding from the application of the strength reduction factor ϕ for column members to the consideration of design code requirements are the same as those mentioned in the ACI formula.

In particular, in frame structures accompanying sidesway, the bending moment of an RC beam connected to a slender RC column must also be magnified by the moment magnification factor δ because of the force equilibrium at the considering joint. However, it is not subjected to a resisting capacity reduction due to the P–Δ effect. Hence, if a slender RC column is designed by the proposed formula in this paper, it may be necessary to increase the applied moment for the design of the RC beam. In this case, the moment magnification factor δ noted in the ACI formula can be used, or the distance between the points B and C in Fig. 5 can be directly calculated since the P–M interaction diagrams for the short RC column and for the corresponding slender RC column can be uniquely determined without any rigorous nonlinear analysis.

3.3. Extension of the proposed formula to the columns subjected to moment gradient

Unlike an ideal RC column bent in a single uniform curvature, most RC columns used in practice will likely be subjected to moment gradient. Many design codes adopting the moment magnifier method (the ACI Code [1] and the AISC Specification [2], etc.) introduce an equivalent uniform moment diagram factor C_m to determine the equivalent uniform end moment which gives the same maximum internal moment for a column subjected to unequal end moments, in spite of different locations of the maximum moment. Currently, the ACI Code uses a simplified Austin expression [4] that

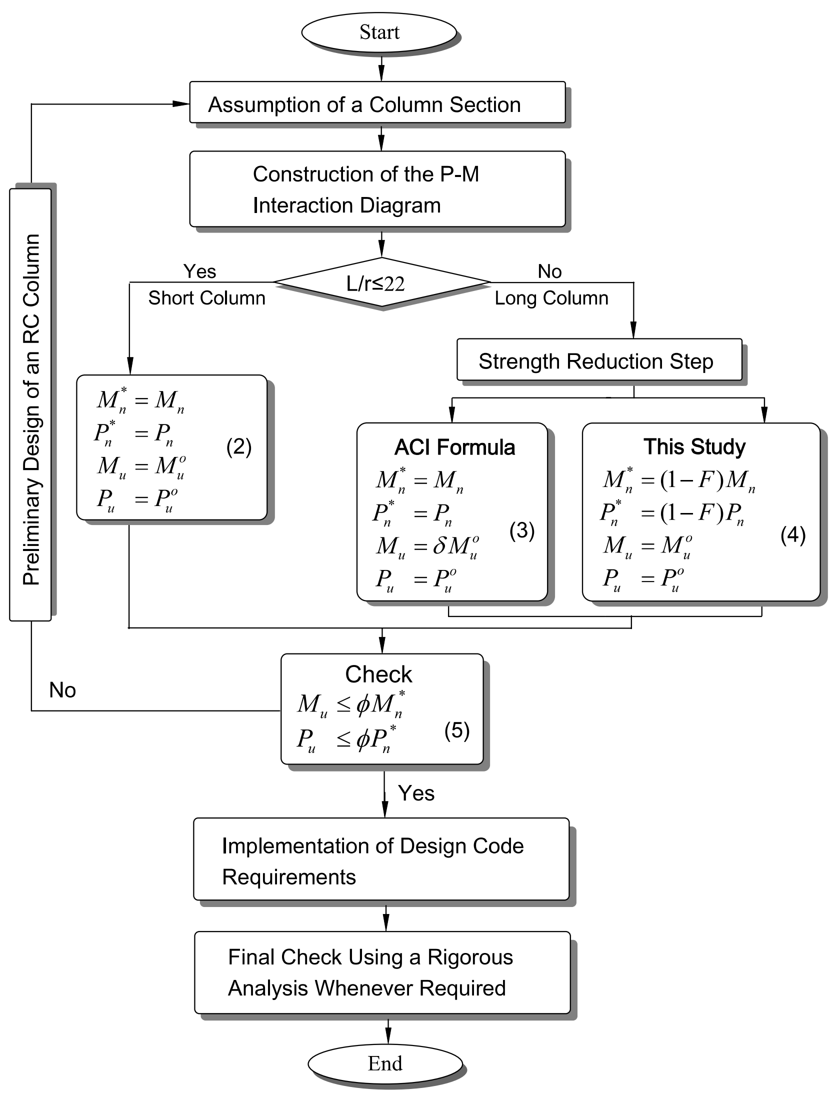


Fig. 9. Comparison of design procedure.

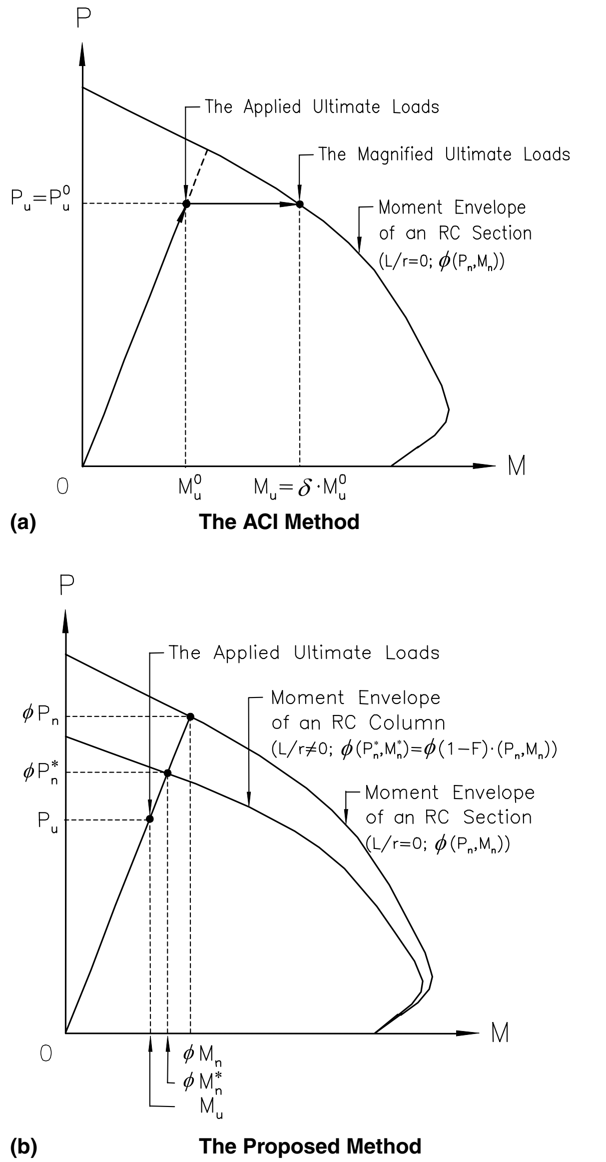


Fig. 10. Difference in design procedure of both methods: (a) the ACI method; (b) the proposed method.

eliminates its dependency on the axial force $C_m = 0.6 - 0.4(M_A/M_B) \geqslant 0.4$, where M_A/M_B is negative if the column is bent in a single curvature (see Fig. 11).

As opposed to the moment magnifier method, in which only an increase in the applied ultimate moment is magnified by a magnification factor δ to consider the P–Δ effect without any change in the applied axial force, the proposed formula is based on a strength reduction coefficient F (see Fig. 5) at the same eccentricity.

Hence, no direct adoption of the same moment correction factor C_m used in the ACI Code is possible because the axial force P and end moments M_A and M_B which act on the member (as shown in Fig. 11) must be replaced by the equivalent axial force and end moments to maintain the consistency with a strength reduction coefficient F. This means that the magnitude of the equivalent axial force and end moments are such that the maximum moment M_{max} produced by them will be equal to that produced by the actual axial force and end moments M_A and M_B.

From the equality condition for the maximum moments of the two systems in Fig. 11,

$$\begin{aligned} M_{max} &\equiv |M_B|\sqrt{\frac{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}{\sin^2 kL}} \\ &\equiv C_m^{EQ}|M_B|\sqrt{\frac{2(1-\cos k_{EQ}L)}{\sin^2 k_{EQ}L}} \\ &= C_m^{EQ}|M_B|\sec\frac{k_{EQ}L}{2} = C_m^{EQ}|M_B|\frac{1}{1-\frac{C_m^{EQ}P}{P_e}}. \end{aligned} \tag{6}$$

The equivalent uniform moment diagram factor C_m^{EQ} in Fig. 11(b) can be written as

$$\frac{1}{C_m^{EQ}} = \frac{P}{P_e} + \sqrt{\frac{\sin^2 kL}{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}}, \tag{7}$$

or approximately on the basis of the Austin expression,

$$\frac{1}{C_m^{EQ}} = \frac{P}{P_e} + \frac{1 - P/P_e}{0.6 - 0.4M_A/M_B}, \tag{8}$$

because $\sqrt{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1} \cong [0.6 - 0.4(M_A/M_B)]\sqrt{2(1-\cos kL)} = [0.6 - 0.4(M_A/M_B)]2\sin(kL/2)$, where $kL = \pi\sqrt{P/P_{cr}}, k_{EQ}L = kL\sqrt{C_m^{EQ}}$, $P_{cr} = \pi^2 EI/(\beta L)^2$, $EI = 0.2E_c I_g + E_s I_{se}$, where I_g and I_{se} denote the moment of inertia of a gross concrete section and reinforcement about the centroidal axis as proposed in the ACI Code, and βL the effective span length considering the end boundary condition [1].

In advance, the estimated ultimate resisting capacity of a slender RC column subjected to unequal end moments of M_A and M_B as shown in Fig. 11(a) can finally be represented by $\left((1-F)/C_m^{EQ}P_n, (1-F)/C_m^{EQ}M_n\right) = (1-F)/C_m^{EQ}(P_n, M_n) = (1-F)/C_m^{EQ} \times \overline{OA}$ in Fig. 5 in the case of the proposed formula because the transformed equivalent forces of $\left(C_m^{EQ}P, C_m^{EQ}M_B\right)$ in Fig. 11(b) correspond to point B in Fig. 5 represented by $(1-F)(P_n, M_n)$. Especially, the reduction rate $(1-F)/C_m^{EQ}$ must be less than or equal to unity to have physical meaning as the inverse of a moment magnification factor $1/\delta$ in the ACI Code.

In advance, most RC members subject to combined axial compression and bending moment occur as parts of rigid frames, rather than as isolated members. A correct rational analysis and design of long columns in such frames must include the actual end restraints by adjoining members. Nevertheless, in design aspect, a column may be considered as an isolated member removed from the frame and replaced by an equivalent pin-end column

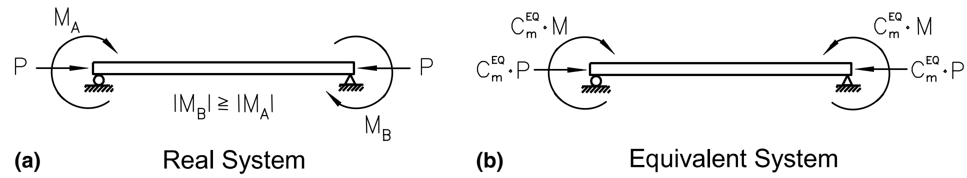


Fig. 11. Schematic representation of equivalent forces: (a) real system; (b) equivalent system.

whose length is equal to the effective length βL for axial compression on the real column. The equivalent column is then analyzed for compression plus the end moments carried by the member.

The most commonly used procedure for obtaining the equivalent pin-end effective length is to use the alignment charts from the Structural Stability Research Council Guide [5], which has also been adopted by ACI318 for the approximate evaluation of effective length factor β. Since the proposed regression formula of strength reduction factors F is proposed for use in determining an initial section in the preliminary design stage, and the effective length factor β has been introduced to replace an end-restrained column by an equivalent pin-end column, the same procedure for calculation of β, adopted in the ACI318 Code, can be used in this study to consider the effective length βL. Namely, the variable L in Eq. (1) can be replaced by βL on the basis of β calculated from the ACI Code. In advance, more detailed analyses of preliminary designed RC columns may be followed in the final design stage on the basis of the proposed numerical model in this paper.

4. Verification of the proposed formula

To verify the effectiveness of the proposed formula, typical RC columns with different steel and slenderness ratios are analyzed for short-term and long-term loadings. For convenience of comparison, the reduced ultimate moments of $M_n\delta$ from Eq. (3) in Fig. 9 are used in case of the ACI formula as the moment corresponding to P_n, and the obtained results can be found in Figs. 12–15. In addition, the stiffness reduction factor φ_k and the strength reduction factor φ are assumed to be 1.0 for the purpose of comparison with the numerical results.

In order to review the structural responses according to the compressive strength of concrete and steel ratios, RC columns with $f'_c = 250\ \text{kgf/cm}^2$ and $f'_c = 400\ \text{kgf/cm}^2$, which are the generally adopted compressive design strengths in small to medium building structures, are analyzed and the obtained results can be found in Fig. 12. The ACI model represents very close results with sophisticated P–Δ analysis in RC columns with a small slenderness ratio, even though there is still a large difference from the rigorous nonlinear analyses results in RC columns with large slenderness ratio. Furthermore, the proposed formula provides a uniform safety margin, which means the uniform difference between the results predicted by the proposed formula and the results calculated by a rigorous analysis over an entire eccentricity, regardless of the compressive strength of concrete and steel ratio.

As shown in Fig. 12, the ACI formula effectively simulates the P–Δ effect when the slenderness ratio is relatively small regardless of the compressive strength of concrete and steel ratio. However, in columns with large steel and slenderness ratios, the ACI formula appears to be slightly on the unsafe side, such that the ultimate resisting capacity of RC columns may be overestimated. Meanwhile, the ACI formula gives very conservative results with a great difference between the ACI envelopes and the sophisticated P–Δ analysis as the slenderness ratio increases ($L/r = 70$), and the difference seems to be larger in high strength concrete columns with relatively small steel ratios.

In advance, Figs. 13–15 show that the ACI formula is limited in its ability to calculate the ultimate resisting capacity of slender RC columns subjected to long-term loadings and may not be applicable to an RC column with a slenderness ratio greater than 30. This means that the time-dependent deformation of concrete cannot be effectively considered with one parameter β_d representing the ratio of dead load to total load. In fact, the term $1 + \beta_d$ in the ACI formula [1] implies that a fixed value of the ultimate creep coefficient is assumed, and an improvement of the ACI formula may be required. As shown in Figs. 13–15, the ACI formula gives very close results to those obtained by the rigorous time-dependent analysis in the case of slender RC columns with small slenderness ratios, regardless of the ultimate creep coefficient. Overall, it may be observed that the ACI formula is conservative in most cases, and consequently it does not achieve the objective of a uniform safety margin. On the other hand, the proposed formula presents good agreement with sophisticated P–Δ analysis through the entire eccentricity range.

To verify the extensibility of the proposed formula to a slender RC column subjected to unequal end moments, typical slender RC columns with $f'_c = 250\ \text{kgf/cm}^2$ and $f'_c = 400\ \text{kgf/cm}^2$ are analyzed, and representative numerical results for RC columns which show a remarkable P–Δ effect can be found in Figs. 16 and 17. As

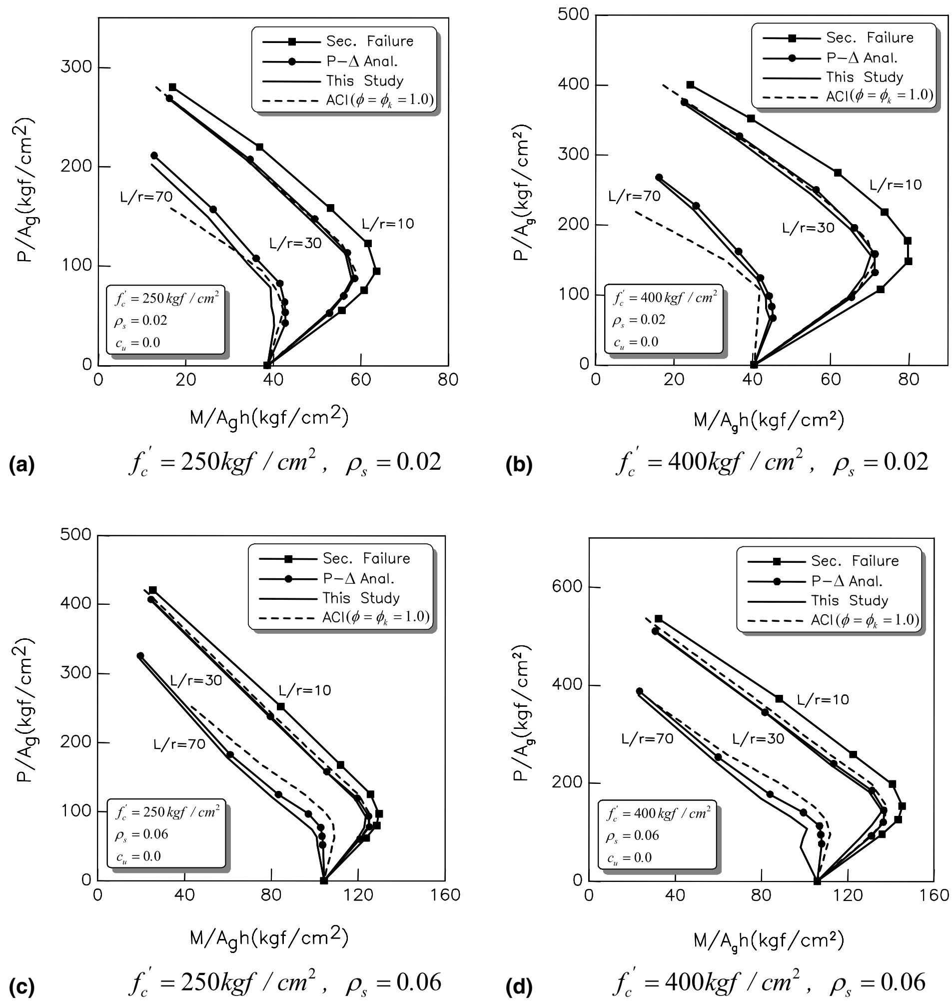


Fig. 12. *P*–*M* interaction diagrams (short-term loading): (a) $f_c' = 250$ kgf/cm^2, $\rho_s = 0.02$; (b) $f_c' = 400$ kgf/cm^2, $\rho_s = 0.02$; (c) $f_c' = 250$ kgf/cm^2, $\rho_s = 0.06$; (d) $f_c' = 400$ kgf/cm^2, $\rho_s = 0.06$.

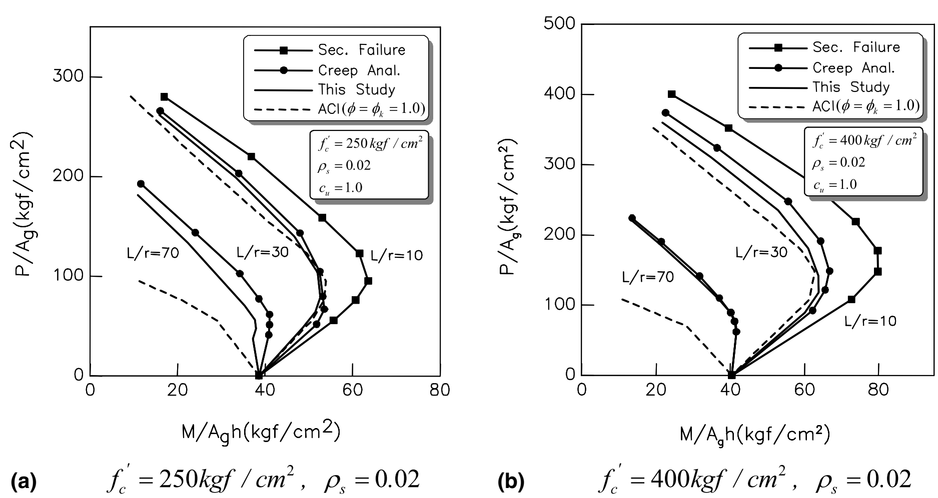


Fig. 13. *P*–*M* interaction diagrams ($c_u = 1.0$): (a) $f_c' = 250$ kgf/cm^2, $\rho_s = 0.02$; (b) $f_c' = 400$ kgf/cm^2, $\rho_s = 0.02$.

shown in these figures, the proposed formula can effectively estimate the ultimate resisting capacity of slender RC columns subjected to unequal end moments in cooperation with the equivalent uniform moment diagram factor C_m^{EQ} in Eq. (8) and shows more improved results than the ACI method even though there is still not

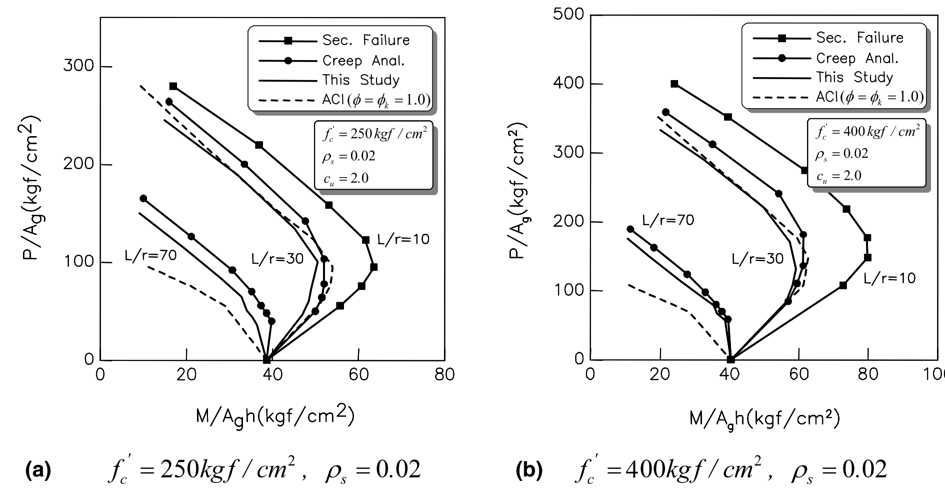


Fig. 14. *P–M* interaction diagrams ($c_u = 2.0$); (a) $f_c' = 250$ kgf/cm², $\rho_s = 0.02$; (b) $f_c' = 400$ kgf/cm², $\rho_s = 0.02$.

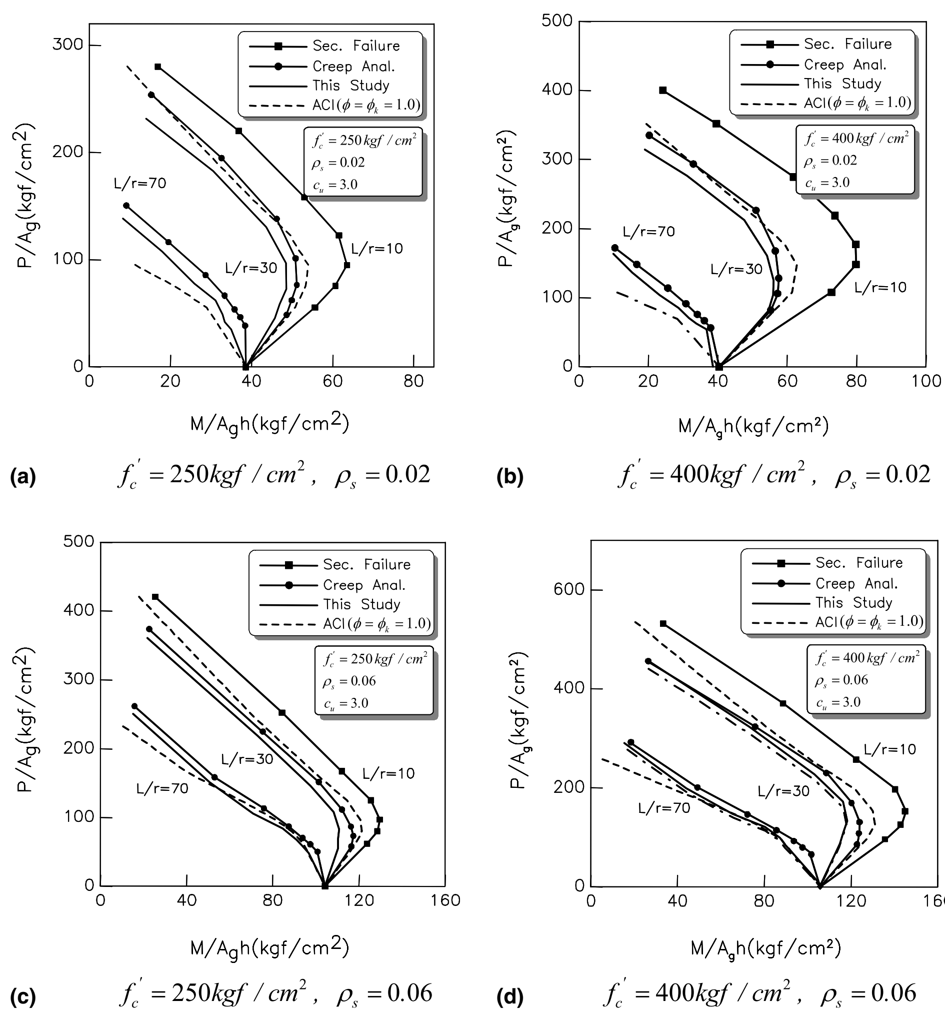


Fig. 15. *P–M* interaction diagrams ($c_u = 3.0$): (a) $f_c' = 250$ kgf/cm², $\rho_s = 0.02$; (b) $f_c' = 400$ kgf/cm², $\rho_s = 0.02$; (c) $f_c' = 250$ kgf/cm², $\rho_s = 0.06$; (d) $f_c' = 400$ kgf/cm², $\rho_s = 0.06$.

so a small difference between the numerical results obtained by rigorous nonlinear analysis and the results obtained using the proposed formula. The difference seems to be a little larger in slender RC columns with a small steel ratio in particular. These differences seem to be caused by adopting the approximate expression

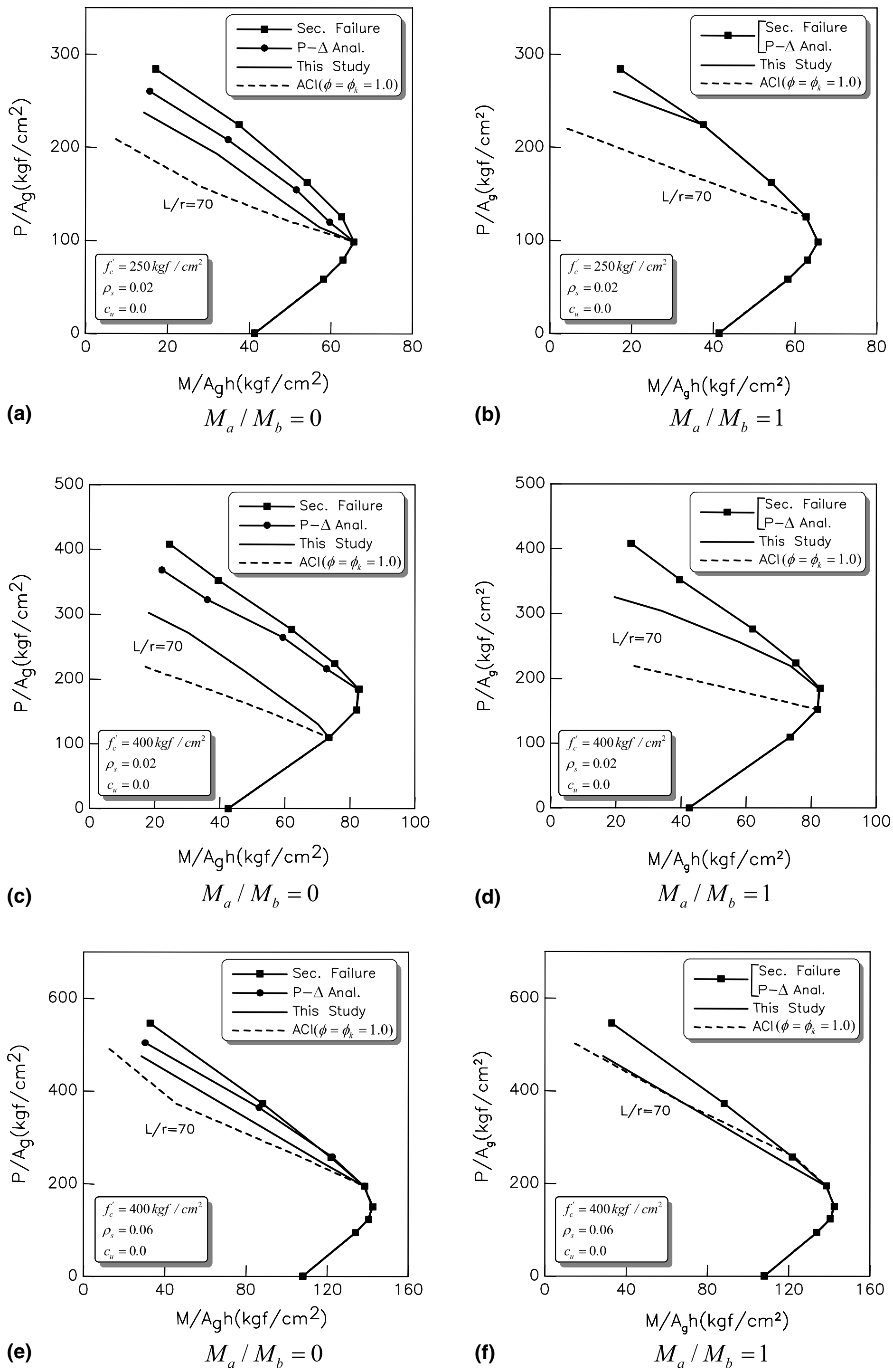


Fig. 16. *P–M* interaction diagrams of RC columns subject to unequal end moments (short term behavior): (a) $M_a/M_b = 0$; (b) $M_a/M_b = 1$; (c) $M_a/M_b = 0$; (d) $M_a/M_b = 1$; (e) $M_a/M_b = 0$; (f) $M_a/M_b = 1$.

of C_m^{EQ} (see Eq. (8)). Nevertheless, the proposed formula can be effectively used in determining an initial section of slender RC columns.

Moreover, as shown in Figs. 16 and 17, the ACI formula gives a conservative estimation of slender RC columns subjected to unequal end moments, and this trend endures regardless of the ultimate creep coefficient C_u. Specifically, an influence of the time-dependent deformation of concrete on the resisting capacity of slender RC columns cannot be effectively considered with one parameter β_d, as in the RC columns subjected to equal end moments. On the other

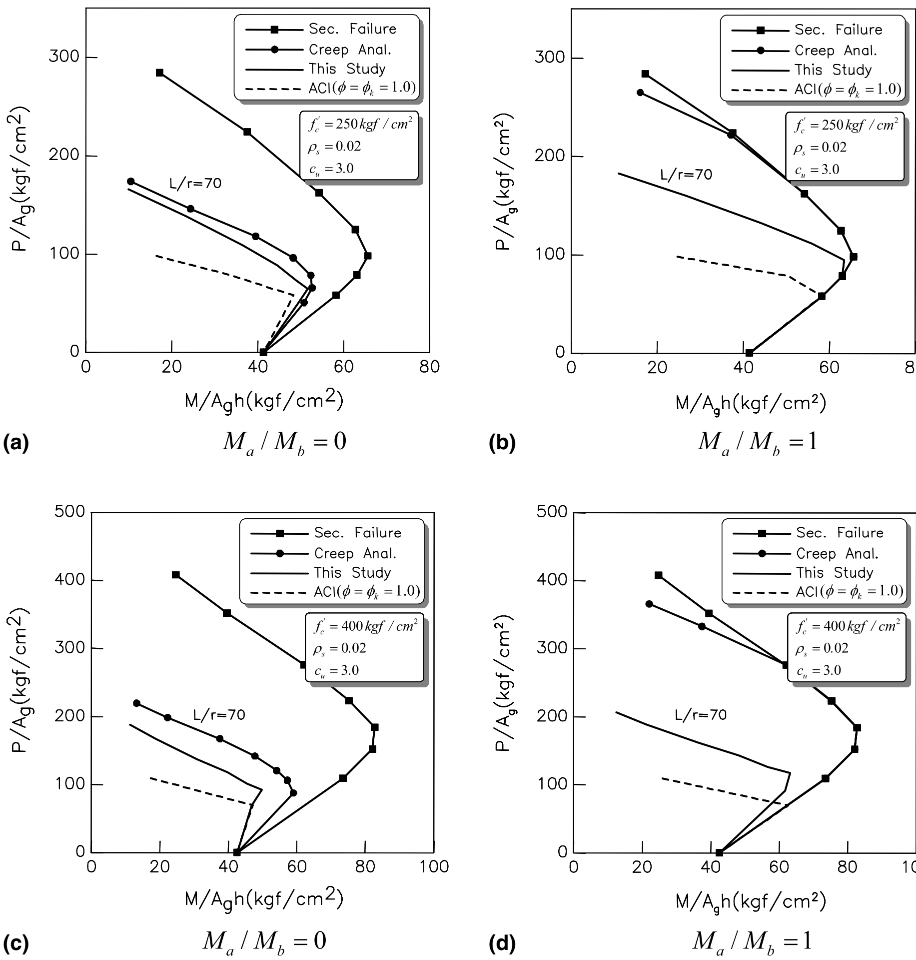


Fig. 17. *P–M* interaction diagrams of RC columns subject to unequal end moments (long term behavior): (a) $M_a/M_b = 0$; (b) $M_a/M_b = 1$; (c) $M_a/M_b = 0$; (d) $M_a/M_b = 1$.

hand, since the proposed formula has adopted the direct regression of the rigorous analysis results obtained for each ultimate creep coefficient C_u as was done in the short-term loading case ($C_u = 0$), an additional loss of accuracy in the proposed formula caused by introducing an another parameter C_u representing no regulation seems to be minimized.

5. Conclusions

In this paper, numerous nonlinear time-dependent analyses are performed using the proposed numerical model in the companion paper. Through time-dependent and *P–Δ* analyses of slender RC columns, a simple and effective regression formula for the design of a slender RC column is introduced. Based on the results in this limited investigation, the following conclusions are obtained: (1) The ACI formula gives good agreement with the *P–Δ* analysis of RC columns subjected to short-term loading when the compressive strength of concrete, slenderness ratio and steel ratio are relatively small; but (2) it gives very conservative results as the slenderness ratio increases; and (3) the ACI formula needs to be improved for more effective simulation of the time-dependent effect; (4) a reasonable estimation of the ultimate resisting capacity by the ACI formula may not be expected for an RC column with a slenderness ratio greater than 30 that is subjected to long-term loading; and (5) the proposed formula shows good agreement with the *P–Δ* analysis and gives a uniform safety margin for any slender RC column.

Although sophisticated numerical methods considering material and geometric nonlinearities and time-dependent deformation will play an increasingly important role and will become the standard for final design checks, the proposed formula can be effectively used in determining an initial section of slender RC columns at the preliminary design stage. Moreover, to reach a more rational approach, extensive studies for reliability assessment, including experimental studies, need to be undertaken.

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