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Study of the Propagation of an Intense Laser Beam in a Collisional Plasma Channel by Using a Source-Dependent Expansion Method

Hua TANG
LP01 Group, Laboratory of Light Transmission Optics, South China Normal University, Guangzhou 510631, P. R. China
and
LP01 Group, Key Laboratory for Quantum Information and Measurements of Ministry of Education, School of Electronics Engineering & Computer Science, Peking University, Beijing 100871, P. R. China

Guo HONG*
LP01 Group, Key Laboratory for Quantum Information and Measurements of Ministry of Education, School of Electronics Engineering & Computer Science, Peking University, Beijing 100871, P. R. China

Hong Jin KONG
Department of Physics, Korea Advanced Institute of Science and Technology, Taejon 305-701

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In this paper, the propagation of an intense laser beam in a collisional plasma channel is studied analytically, and the investigation is focused on the self-focusing due to the Kerr nonlinearity, the relativistic nonlinearity, and the electron density perturbation. The propagation of laser, including the effects of diffraction, the plasma channel, the 3rd Kerr nonlinearity, the relativistic self-focusing, the electron density perturbation, and the defocusing due to electron collisions, is analyzed. The source-dependent expansion (SDE) method for analyzing the wave equation is introduced and employed in the paper.

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I. INTRODUCTION

The propagation of ultrashort intense laser pulses has received considerable attention due to its wide range of applications, such as plasma-based acceleration, X-ray generation, optical harmonics generation, ultrabroadband radiation generation, and "fast ignition" schemes in laser fusion. A long propagation distance in the medium is desirable in these applications. There are many theoretical, numerical, and experimental works [1], and there are many instabilities and phenomena, such as self-modulation [2], the filamentation instability [3], plasma waves [4], group-velocity dispersion (GVD), finite pulse effects [5], relativistic self-focusing effects [6], *etc.*, in this field.

In this paper, we will investigate the propagation of intense laser pulses in a plasma channel, in which the relativistic self-focusing associated with the relativistic factor $\gamma = \sqrt{1+a^2}$ ($a = 8.5\times10^{-10} I^{1/2}\lambda_0(\mathrm{W}\mu\mathrm{m/cm}^2)$]), the third-order intensity-dependent nonlinearity [7], and the electron density perturbation will be studied theoretically. The refractive index equation of an intense laser beam in a partially stripped, pre-formed plasma channel is derived and includes collisions in the plasma. We introduce the source-dependent expansion (SDE) method and use it to solve the refractive index equation. The solutions are the evolution equations of the electric parameters, *i.e.*, the wave curvature, the spot size, the amplitude, and the phase.

II. THE REFRACTIVE INDEX EQUATION

Starting from Maxwell's equations, ultrashort intense laser propagation in a partially stripped plasma medium

*E-mail: hongguo@pku.edu.cn;
Tel: +86-10-6275-7035; Fax: +86-10-6275-3208

is given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E}(\boldsymbol{r},t) = \frac{4\pi}{c^2}\left(\frac{\partial^2 \boldsymbol{P}}{\partial t^2} + \frac{\partial \boldsymbol{J}}{\partial t}\right), \quad (1)$$

where $\boldsymbol{E}(\boldsymbol{r},t)$ is the electric field, $\boldsymbol{P}$ is the polarization associated with bound electrons, and $\boldsymbol{J}$ is the plasma current density associated with free electrons. The electric field is assumed to be linearly polarized in the x direction and takes the form

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{1}{2}E(\boldsymbol{r},t)e^{ik_0 z - i\omega_0 t}\boldsymbol{e}_x + \text{c.c.} \quad (2)$$

Based on Eq. (2), we will derive the evolution equation for the electric field, *i.e.*, the refractive index equation.

The polarization and the plasma current density can be divided into linear and nonlinear parts. The linear parts are given by

$$\frac{\partial^2 \boldsymbol{P}_L}{\partial t^2} = -\frac{\omega_0^2}{4\pi}(\eta_L^2 - 1)\boldsymbol{E}(\boldsymbol{r},t),$$

where the refractive index of the plasma is divided into linear and nonlinear parts, *i.e.*, $\eta = \eta_L + \eta_{NL}$, $\eta_{NL} = \eta_2 I$, and

$$\frac{\partial \boldsymbol{J}_L}{\partial t} = \frac{\omega_{pe}^2}{4\pi}\cdot\frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\boldsymbol{E}(\boldsymbol{r},t),$$

respectively, where ω_{pe} is the unperturbed value of the electron plasma frequency $\omega_p^2 = 4\pi n_e e^2/m$, n_e being the electron density, and the second factor is the contribution of the collisions of electrons in terms of the collision time τ_e. The nonlinear polarization satisfies

$$\frac{\partial^2 \boldsymbol{P}_{NL}}{\partial t^2} = -\frac{c^3\eta_L}{8\pi p_a} \mid E \mid^2 \boldsymbol{E}(\boldsymbol{r},t),$$

where $p_a = 2\pi c^2/\omega_0^2\eta_L\eta_2$ is the critical power for third-order nonlinear self-focusing with $I = (c\eta_L/4\pi)\langle \boldsymbol{E}\cdot\boldsymbol{E}\rangle$ being the time-averaged laser intensity. The nonlinear plasma current density satisfies

$$\frac{\partial \boldsymbol{J}_{NL}}{\partial t} = \frac{\omega_{pe}^2}{4\pi}\cdot\frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left(\frac{\delta n_e}{n_e} - \frac{\delta m}{m}\right)\boldsymbol{E}(\boldsymbol{r},t), (3)$$

where the two terms on the right-hand side of Eq. (3) represent the perturbation of the electron plasma wave (associated with the ponderomotive force effect) and the change of the electron mass due to the relativistic effect, respectively. The electron density perturbation [1] and the relativistic mass change [2] are $\delta n_e/n_e = (c^2/\omega_{pe}^2)\nabla_\perp^2\gamma = (c^2/\omega_{pe}^2)\nabla_\perp^2(1+\boldsymbol{a}\cdot\boldsymbol{a})^{1/2}$ and $\delta m/m = \boldsymbol{a}\cdot\boldsymbol{a}/2$, respectively, where $\boldsymbol{a} = e\boldsymbol{A}/mc^2$ is the normalized vector potential. Using the critical power of plasma relativistic self-focusing, $p_p = 2c\eta_L(\omega_0^2/\omega_{pe}^2)(e/r_e)^2$, where $r_e = e^2/mc^2$ is the classical electron radius, one obtains

$$\frac{\partial \boldsymbol{J}_{NL}}{\partial t} = \frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left(-\frac{c^3\eta_L}{8\pi p_p}\cdot \mid E \mid^2 + \frac{c^2}{4\pi}\frac{1}{4\gamma}\frac{e^2}{m^2c^2\omega_0^2}\nabla_\perp^2 \mid E \mid^2\right)\boldsymbol{E}(\boldsymbol{r},t).$$

Therefore, with the approximations $\mid \partial E/\partial z \mid \ll \mid k_0 E \mid$ and $\mid \partial E/\partial t \mid \ll \mid \omega_0 E \mid$, the evolution Equation for the electric field, which we call the refractive index Equation, is

$$\begin{aligned}\left(\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial z}\right)E = k_0^2\Bigg\{1 - \Bigg[\eta_L^2 - \frac{\omega_{pe}^2}{\omega_0^2}\frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2} \\ +\frac{c^2}{\omega_0^2}\left(\frac{1}{p_a} + \frac{1}{p_p}\cdot\frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\right)\frac{c\eta_L}{2} \mid E \mid^2 \\ - \frac{1}{4\gamma}\frac{e^2}{m^2\omega_0^4}\nabla_\perp^2 \mid E \mid^2 \frac{\omega_0^2\tau_e^2 - i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\Bigg]\Bigg\}E, \quad (4)\end{aligned}$$

where the wave number $k_0 = \omega_0/c$. The real terms on the right-hand side of Eq. (4) represent the effects of the diffraction, the plasma defocusing, the third-order intensity-dependent self-focusing, the relativistic self-focusing, and the electron density perturbation, respectively, while the imaginary terms represent the plasma absorption.

III. THE SOURCE-DEPENDENT EXPANSION METHOD

The source-dependent expansion (SDE) method is an effective method for solving the paraxial wave equation with nonlinear source terms, *i.e.*, $(\nabla_\perp^2 + 2ik_0\partial/\partial z)E = k_0^2(1-\eta_r^2)E$, where η_r is the total refractive index. In the source-dependent expansion (SDE) method, the electric field is expanded in a complete set of orthogonal Laguerre-Gaussian functions. These functions are implicit functions of the propagation distance z through electric field parameters such as the spot size, the wave curvature, the amplitude, and the phase. The electric field can be described by four coupled differential equations in the field parameters.

In the following discussion, the electric field is described only by a single Laguerre-Gaussian mode. In general, the pulsed beam can be written in terms of a complete set of Laguerre-Gaussian functions

$$E(\boldsymbol{r},t) = \sum_n \hat{E}_n L_n(\chi)\exp[-(1-i\alpha_s)\chi/2], \quad (5)$$

where $n = 0, 1, 2, ...$, $\hat{E}_n$ is the complex amplitude, $\chi = 2r^2/r_s^2$, α_s is the wavefront curvature, and $L_n(\chi)$ is a Laguerre polynomial, *e.g.*, $L_0 = 1$ and $L_1 = 1-\chi$. After a calculation (the details are given in Ref. [8]), the equation for the complex amplitude is given by

$$\left(\frac{\partial}{\partial z} + A_n\right)\hat{E}_n - inB\hat{E}_{n-1} - i(n+1)B^*\hat{E}_{n+1} = -iH_n, \quad (6)$$

where

$$A_n = \frac{\dot{r}_s}{r_s} + i(2n+1)[\frac{1+\alpha_s^2}{k_0 r_s^2} - \alpha_s\frac{\dot{r}_s}{r_s} + \frac{\dot{\alpha}_s}{2}],$$

$$B=-\alpha_s\frac{\dot{r}_s}{r_s}-\frac{1-\alpha_s^2}{k_0r_s^2}+\frac{\dot{\alpha}_s}{2}-i\left(\frac{\dot{r}_s}{r_s}-\frac{2\alpha_s}{k_0r_s^2}\right),$$

$$H_n=\frac{k_0}{2}\int_0^\infty d\chi(1-\eta^2)EL_n\exp[-(1+i\alpha_s)\frac{\chi}{2}].$$

The dot denotes the operator $\partial/\partial z$ and the asterisk denotes the complex conjugate.

In the following, it is assumed that the fundamental Gaussian ($n=0$) mode can describe the laser dynamics sufficiently. Assuming $|\hat{E}_0|\gg|\hat{E}_n|$ for the SDE mode ($n\geq 1$), from Eq. (6) (with $n=0$, $n=1$), one obtains

$$B=\frac{H_1}{\hat{E}_0} \tag{7}$$

and

$$\left(\frac{\partial}{\partial z}+A_0\right)\hat{E}_0=-iH_0. \tag{8}$$

Using Eqs. (7) and (8) and setting $\hat{E}_0=E_s\exp(i\theta_s)$, one obtains

$$\alpha_s=\frac{k_0r_s\dot{r}_s}{2}+\frac{k_0r_s^2}{2}T_I, \tag{9}$$

$$\frac{\partial p}{\partial z}=2S_Ip, \tag{10}$$

$$\frac{\partial\theta_s}{\partial z}=-\frac{2}{k_0r_s^2}-S_R-T_R, \tag{11}$$

$$\frac{\partial^2r_s}{\partial z^2}=\frac{4}{k_0^2r_s^3}\left(1+k_0r_s^2T_R-\frac{k_0^2r_s^3\dot{r}_s}{2}T_I-\frac{k_0^2r_s^4}{4}\dot{T}_I-\frac{k_0^2r_s^4}{4}T_I^2\right), \tag{12}$$

where $S=H_0/\hat{E}_0$ and $T=H_1/\hat{E}_0$, *i.e.*,

$$S=\frac{k_0}{2}\int_0^\infty d\chi(1-\eta_r^2)\exp(-\chi), \tag{13}$$

and

$$T=\frac{k_0}{2}\int_0^\infty d\chi(1-\eta_r^2)(1-\chi)\exp(-\chi). \tag{14}$$

The laser power is given by $p=E_s^2r_s^2\cdot c\eta_L/16$, and the subscripts R and I represent the real and the imaginary parts of the functions, respectively.

IV. THE SOLUTION TO THE PROPAGATION EQUATION

From Eq. (4), one obtains

$$\eta_r^2=\eta_L^2-\frac{\omega_{pe}^2}{\omega_0^2}\frac{\omega_0^2\tau_e^2-i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}+\frac{c^2}{\omega_0^2}\left(\frac{1}{p_a}+\frac{1}{p_p}\frac{\omega_0^2\tau_e^2-i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\right)\frac{c\eta_L}{2}\mid E\mid^2-\frac{1}{4\gamma}\frac{e^2}{m^2\omega_0^4}\nabla_\perp^2\mid E\mid^2\frac{\omega_0^2\tau_e^2-i\omega_0\tau_e}{1+\omega_0^2\tau_e^2}, \tag{15}$$

where the plasma frequency satisfies a parabolic profile, *i.e.*, $\omega_{pe}^2=\omega_{pe0}^2(1+\triangle nr^2/n_0r_{\rm ch}^2)$, with $r_{\rm ch}$ being the channel width, $\triangle n$ the density width of the plasma channel, and $\omega_{pe0}=\omega_{pe}(0)$ the electron plasma frequency in terms of the on-axis electron density n_0, and $\mid E\mid^2=E_s^2\exp(-\chi)$. If Eq. (15) is substituted into Eqs. (13) and (14), the real and the imaginary parts of the functions S and T are given by

$$S_R=\frac{k_0}{2}\left[1-\eta_L^2+\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\cdot\frac{\omega_{pe0}^2}{\omega_0^2}\left(1+\frac{\triangle n}{n_0}\frac{r_s^2}{2r_{\rm ch}^2}\right)-\frac{4p}{k_0^2r_s^2}\left(\frac{1}{p_a}+\frac{1}{p_p}\cdot\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\right)-\frac{8}{\gamma}\frac{e^2}{m^2\omega_0^4c\eta_L}\frac{p}{r_s^4}\cdot\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\right],$$

$$S_I=\frac{k_0}{2}\cdot\frac{-\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left[\frac{\omega_{pe0}^2}{\omega_0^2}\left(1+\frac{\triangle n}{n_0}\frac{r_s^2}{2r_{\rm ch}^2}\right)-\frac{4}{k_0^2r_s^2}\frac{p}{p_p}-\frac{8}{\gamma}\frac{e^2}{m^2\omega_0^4c\eta_L}\frac{p}{r_s^4}\right],$$

$$T_R=\frac{k_0}{2}\left[-\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\cdot\frac{\omega_{pe0}^2}{\omega_0^2}\frac{\triangle n}{n_0}\frac{r_s^2}{2r_{\rm ch}^2}-\frac{2p}{k_0^2r_s^2}\left(\frac{1}{p_a}+\frac{1}{p_p}\cdot\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\right)-\frac{8}{\gamma}\frac{e^2}{m^2\omega_0^4c\eta_L}\frac{p}{r_s^4}\cdot\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\right],$$

$$T_I=\frac{k_0}{2}\cdot\frac{\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left(\frac{\omega_{pe0}^2}{\omega_0^2}\frac{\triangle n}{n_0}\frac{r_s^2}{2r_{\rm ch}^2}+\frac{2}{k_0^2r_s^2}\frac{p}{p_p}-\frac{8}{\gamma}\frac{e^2}{m^2\omega_0^4c\eta_L}\frac{p}{r_s^4}\right).$$

Then, according to Eqs. (9), (10), (11), and (12), the evolution Equations of the electric field parameters are

$$\alpha_s=Z_RR\dot{R}+\frac{\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left(\frac{p}{2p_p}+\frac{p}{p_d}\frac{k_0Z_R}{R^2}\right)+\frac{1}{\omega_0\tau_e}\frac{\triangle n}{2\triangle n_c}R^4, \tag{16}$$

$$\frac{\partial p}{\partial z} = \frac{-k_0\omega_0\tau_e}{1+\omega_0^2\tau_e^2}\left(\frac{\omega_{pe0}^2}{\omega_0^2} - \frac{2}{k_0 Z_R R^2}\frac{p}{p_p} - \frac{p}{p_d}\frac{2}{R^4}\right)p - \frac{p}{\omega_0\tau_e}\frac{\triangle n}{\triangle n_c}\frac{R^2}{Z_R}, \quad (17)$$

$$\frac{\partial \theta_s}{\partial z} = \frac{k_0}{2}\left(\eta_L^2 - 1 - \frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\frac{\omega_{pe0}^2}{\omega_0^2} + \frac{p}{p_d}\frac{4}{R^4}\cdot\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}\right) + \frac{1}{Z_R R^2}\left(\frac{3p}{p_{\text{crit}}} - 1\right), \quad (18)$$

$$\frac{\partial^2 R}{\partial z^2} = \frac{1}{Z_R^2 R^3}\left(1 - \frac{\triangle n}{\triangle n_c}R^4 - \frac{p}{p_{\text{crit}}} - \frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}A_1 - \frac{1}{\omega_0\tau_e}A_2 - \frac{1}{\omega_0^2\tau_e^2}A_3 - \frac{1}{1+\omega_0^2\tau_e^2}A_4 - \frac{\omega_0\tau_e}{1+\omega_0^2\tau_e^2}A_5 - \frac{\omega_0^2\tau_e^2}{(1+\omega_0^2\tau_e^2)^2}A_6\right), \quad (19)$$

where

$$A_1 = \frac{2p}{p_d}\frac{k_0 Z_R}{R^2},$$
$$A_2 = \frac{2\triangle n}{\triangle n_c}Z_R R^5 \dot{R},$$
$$A_3 = \left(\frac{\triangle n}{\triangle n_c}\right)^2\frac{R^8}{4},$$
$$A_4 = \frac{\triangle n}{\triangle n_c}\frac{p}{p_d}k_0 Z_R\left(R^2 + \frac{1}{R^2}\right),$$
$$A_5 = \frac{2p}{p_d}\frac{k_0 Z_R^2 \dot{R}}{R} - \frac{4p}{p_d}\frac{k_0 Z_R^2}{r_{s0}R^5} + \frac{p}{p_p}Z_R R\dot{R},$$

$$A_6 = \frac{5p^2}{4p_p^2} + \frac{p^2}{p_d^2}k_0^2 Z_R^2\left(\frac{1}{R^4} + \frac{2}{R^8}\right) + \frac{2p^2}{p_d p_p}k_0 Z_R\left(\frac{1}{R^2} + \frac{1}{R^6}\right) - \frac{\omega_{pe0}^2}{\omega_0^2}\left(\frac{p}{2p_p}k_0 Z_R R^2 + \frac{p}{p_d}\frac{k_0^2 Z_R^2}{R^4}\right).$$

The spot size $R = r_s/r_{s0}$ is normalized by the initial spot size, $Z_R = k_0 r_{s0}^2/2$ is the Rayleigh distance, $p_d = \gamma\eta_L m^2\omega_0^2 c^3 Z_R^2/e^2$, whose dimension is the same as that of the power p, the density width for channel self-guiding is given by

$$\triangle n_c = \frac{r_{\text{ch}}^2}{\pi r_e r_{s0}^4}\cdot\frac{1+\omega_0^2\tau_e^2}{\omega_0^2\tau_e^2}, \quad (20)$$

and the total critical power of the nonlinear self-focusing including the third-order intensity-dependent self-focusing and the relativistic self-focusing is given by

$$\frac{1}{p_{\text{crit}}} = \frac{1}{p_a} + \frac{1}{p_p}\frac{\omega_0^2\tau_e^2}{1+\omega_0^2\tau_e^2}. \quad (21)$$

From Eq. (16), Eq. (17), Eq. (18), and Eq. (19), we can obtain all the information of the electric field. p/p_p, p/p_d, $\triangle n/\triangle n_c$, p/p_{crit}, and $\omega_0\tau_e$ represent the effects of the relativistic self-focusing, the electron density perturbation, plasma channel, the nonlinear self-focusing, and plasma collisions, respectively. Furthermore, according to the envelope equation, *i.e.*, Eq. (19) and the magnitude of $\omega_0\tau_e$, the analysis of the laser propagation can be reasonably simplified.

When the collisions are not taken into account, *i.e.*, the electron collision time τ_e is infinite and the refractive index is a real number, the familiar results

$$\alpha_s = Z_R R\dot{R} = \frac{Z_R R^2}{R_c},$$

are obtained, where $R_c = r_s/\dot{r}_s = R/\dot{R}$ is the wavefront radius of curvature,

$$\frac{\partial p}{\partial z} = 0,$$

$$\frac{\partial \theta_s}{\partial z} = \frac{k_0}{2}\left(\eta_L^2 - 1 - \frac{\omega_{pe0}^2}{\omega_0^2} + \frac{p}{p_d}\frac{4}{R^4}\right) + \frac{1}{Z_R R^2}\left(\frac{3p}{p_{\text{crit}}} - 1\right),$$

$$\frac{\partial^2 R}{\partial z^2} = \frac{1}{Z_R^2 R^3}\left(1 - \frac{\triangle n}{\triangle n_c'}R^4 - \frac{p}{p_{\text{crit}}'} - \frac{2p}{p_d}\frac{k_0 Z_R}{R^2}\right),$$

and the density width and the total critical power are

$$\triangle n_c' = \frac{r_{\text{ch}}^2}{\pi r_e r_{s0}^4},$$

and

$$\frac{1}{p_{\text{crit}}'} = \frac{1}{p_a} + \frac{1}{p_p}.$$

When the collisions are great, *i.e.*, $\omega_0\tau_e \ll 1$, the corresponding results are given by

$$\alpha_s = Z_R R\dot{R} + \frac{1}{\omega_0\tau_e}\frac{\triangle n}{2\triangle n_c}R^4,$$

$$\frac{\partial p}{\partial z} = -\frac{p}{\omega_0\tau_e}\frac{\triangle n}{\triangle n_c}\frac{R^2}{Z_R},$$
$$\frac{\partial \theta_s}{\partial z} = \frac{k_0}{2}(\eta_L^2 - 1) + \frac{1}{Z_R R^2}\left(\frac{3p}{p_{\text{crit}}} - 1\right),$$
$$\frac{\partial^2 R}{\partial z^2} = \frac{1}{Z_R^2 R^3} \times\left(1 - \frac{\triangle n}{\triangle n_c}R^4 - \frac{p}{p_{\text{crit}}} - \frac{A_2}{\omega_0\tau_e} - \frac{A_3}{\omega_0^2\tau_e^2} - A_4\right),$$

where $A_{2,3,4}$ are all strongly related to the density width of the plasma channel $\triangle n$. From the above four equations, we conclude that the collisions cause a linear decrease in the laser power, have little effect on the phase of the laser, and lead to a close relationship between the laser spot size and the electron density width in the plasma channel.

In the middle of the two cases mentioned above, there is a complicated case. For example, we take the laser wavelength $\lambda_0 = 0.8\mu$m, the initial spot size $r_{s0} = 100\mu$m, the electron density of the plasma channel axis $n_0 = 10^{18}/\text{cm}^3$, $p_p/p_a \simeq 5\times 10^3$, $\triangle n/\triangle n_c = 1$, and $\omega_0\tau_e \simeq 1$. Then, we have

$$\alpha_s = \frac{5\pi}{4}R\dot{R} + \frac{R^4}{2} + \frac{1.8\times 10^{-2}}{\pi\gamma}\frac{1}{R^2} + \frac{p}{4p_p},$$

$$\frac{\partial p}{\partial z} = -\frac{2\times 10^2}{9}p - \frac{4}{5\pi}pR^2 + \frac{4}{5\pi}\frac{p^2}{p_pR^2},$$

$$\frac{\partial\theta_s}{\partial z} = \frac{\pi\times 10^5}{8}(\eta_L^2 - 1) - \frac{10^2}{9} + \frac{1.2\times 10^4}{\pi}\frac{p}{p_pR^2},$$

$$\frac{\partial^2 R}{\partial z^2} = \frac{0.64}{\pi^2}\frac{1}{R^3}\left(1 - 5\times 10^3\frac{p}{p_p} - \frac{5}{16}\frac{p^2}{p_p^2} - R^4 + \frac{125\pi}{18}\frac{p}{p_p}R^2 + \frac{1}{2\gamma}\frac{p}{p_pR^4} + \frac{9}{\gamma}\frac{p}{p_pR^5} - \frac{4}{\gamma^2R^8}\right),$$

where it has been assumed that the length of the plasma channel is similar to the Rayleigh length of the laser, that the terms related to $\dot{R}$ can be neglected in the study of the laser spot size, and that the initial laser power is the critical power of the plasma relativistic self-focusing. In this case, the spot size of the laser is mainly determined by the density of the plasma and the power of the laser.

V. DISCUSSION AND CONCLUSION

In this paper, using the source-dependent expansion method, we have studied the propagation of an intense laser beam in a partially stripped, pre-formed plasma channel. The effects of diffractions, third-order intensity-dependent self-focusing, relativistic self-focusing, self-guiding of the plasma channel, and collisions in the plasma are analyzed in detail.

The propagation equation in the collisional plasma, *i.e.*, the refractive index equation, is derived. In that equation, the equivalent refractive index includes the linear parts of the medium and the plasma, the nonlinear parts associated with the third-order intensity-dependent, the relativistic self-focusing effects and the electron density perturbation, and the imaginary parts associated with collisions in the plasma corresponding to the plasma absorption. The solutions are derived by using the SDE method introduced in this paper and are four differential equations in the electric-field parameters, *i.e.*, the wave curvature α_s, the spot size r_s, the laser power p, and the phase θ_s. These equations can be simplified based on $\omega_0\tau_e$ and can be reduced to a simple form in a non-collisional plasma, and the solutions are given for two typical cases, *i.e.*, $\omega_0\tau_e \ll 1$ and $\omega_0\tau_e \simeq 1$.

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REFERENCES

[1] E. Esarey, P. Sprangle, J. Krall and A. Ting, IEEE J. Quantum Electron. **33**, 1879 (1997).
[2] P. Sprangle, B. Hafizi and J. R. Peñano, Phys. Rev. E **61**, 4381 (2000).
[3] Z. M. Sheng, K. Nishihara, T. Honda, Y. Sentoku and K. Mima, Phys. Rev. E **64**, 066409 (2001).
[4] C. D. Decker, W. B. Mon, K. C. Tzeng and T. Katsouleas, Phys. Plasma **3**, 2047 (1996).
[5] P. Sprangle, B. Hafizi and P. Serafim, Phys. Rev. E **59**, 3614 (1999).
[6] B. Hafizi, A. Ting, P. Sprangle and R. F. Hubbard, Phys. Rev. E **62**, 4120 (2000).
[7] Y. R. Shen, *The Principles of Nonlinear Optics* (Wiley, New York, 1984).
[8] P. Sprangle, A. Ting and C. M. Tang, Phys. Rev. A **36**, 2773 (1987).