한국 퍼지 및 지능시스템 학회 2002, Vol. 12, No. 4, pp. 341-346

타입-2 퍼지값의 순위결정

A Ranking Method for Type-2 Fuzzy Values

이승수 · 이광형
Seungsoo Lee and Kwang H. Lee

한국과학기술원 전자전산학과

요 약

주어진 값에 존재하는 불확실성을 표현하기 위하여 타입-1 퍼지값을 사용하듯이, 타입-1 퍼지값의 소속함수를 명확히 정의하기 어려운 경우에 타입-2 퍼지값을 사용할 수 있다. 타입-2 퍼지값은 타입-1 퍼지값에 비해 표현범위가 넓다는 장점이 있지만 타입-2 퍼지값의 사용을 위해서는 기존에 타입-1 퍼지값에서 정의되었던 연산들에 대한 확장된 재정의가 필요하다. 본 논문에서는 타입-2 퍼지값에 대한 비교 및 순위결정에 대한 방법을 제안하였다. 제안된 방법은 타입-2 퍼지값의 실제값과 그 실제값에 대한 가능성을 고려하여 비교결과를 산출하는 만족함수에 기반하고 있으며, 각각의 비교 및 순위결정 결과에 대한 가능성 혹은 신뢰도를 계산한다. 본 논문에서는 제안된 방법이 갖는 몇몇 특성에 대하여도 분석하였다.

Abstract

Type-1 fuzzy set is used to show the uncertainty in a given value. But there are many situations where it needs to be extended to type-2 fuzzy set because it can be also difficult to determine the crisp membership function itself. Type-2 fuzzy systems have the advantage that they are more expressive and powerful than type-1 fuzzy systems, but they require many operations defined for type-1 fuzzy sets need to be extended in the domain of type-2 fuzzy sets. In this paper, comparison and ranking methods for type-2 fuzzy sets are proposed. It is based on the satisfaction function that produces the comparison results considering the actual values of the given type-2 fuzzy sets with their possibilities. Some properties of the proposed method are also analyzed.

Key Words : Type-2 fuzzy set, Fuzzy comparison, Fuzzy ranking.

1. Introduction

Type-1 fuzzy set is used to show the uncertainty in a given value. But there are many situations where it needs to be extended to type-2 fuzzy set because it can be also difficult to determine the crisp membership function itself.

Intrinsically type-2 fuzzy sets are more expressive than type-1 fuzzy sets. For this advantage, there have been many researches in various fields such as fuzzy control systems to extend its frameworks from type-1 to type-2 [5][6][7]. But this also makes it necessary to extend the operations defined on type-1 fuzzy sets to be extended in the domain of type-2 fuzzy sets [1][2]. Extension of comparison and ranking is one of the important issues because these operations are required in a lot of applications.

In this paper, a ranking method for type-2 fuzzy sets is proposed. It is based on the satisfaction degree that shows the possibility one type-2 fuzzy set is greater than the other type-2 fuzzy set.

접수일자 : 2001년 10월 26일
완료일자 : 2002년 5월 2일
감사의 글 : 본 연구는 첨단정보기술 연구센터를 통하여 과학재단의 지원을 받았습니다.

The content of this paper is divided into two sections: comparison and ranking. In section 2, some preliminary definitions are introduced and a comparison method for continuous type-2 fuzzy sets are proposed including its properties. A ranking method based on the proposed comparison method can be found in section 3. Properties of the proposed ranking method are also analyzed in the same section.

2. Comparison

2.1 Fuzzy value

In this paper we use the term *fuzzy value* instead of *fuzzy number* because of the following reasons:

i) A fuzzy set is called a fuzzy number if it is both convex and normalized. These two concepts, however, are more or less difficult to be extended in the domain of type-n fuzzy set.

ii) Even if a fuzzy set is not a shape of a fuzzy number, it is possible to compare it with another fuzzy set if the two fuzzy sets satisfy some conditions which are more general than the conditions of fuzzy number.

Definition 1 A *type-n fuzzy value* is a type-*n* fuzzy set that satisfies the following conditions.
- It is defined on a domain that has a precedence order
- Its support is a finite interval

The second condition is given due to a property of the proposed comparison method. It can be removed depending on the comparison method used in ranking.

To define a type-2 fuzzy value, the definition of support of a type-2 fuzzy set is necessary. We use the following definition [1].

Definition 2 The *support* of a type-2 fuzzy set $\tilde{\tilde{A}}$, $\mathrm{Support}(\tilde{\tilde{A}})$, is the crisp set of all x ∈ X such that $\mathrm{Support}(\tilde{\mu}_{\tilde{\tilde{A}}}(x)) \neq \varnothing$ and $\mathrm{Support}(\tilde{\mu}_{\tilde{\tilde{A}}}(x)) \neq \{0\}$.

There are two kinds of membership function in type-2 fuzzy values: primary and secondary. We will classify the continuity of a type-2 fuzzy value depending on the continuity of its membership functions.

Definition 3 A type-2 fuzzy value is called *continuous* if all of its membership functions are continuous and *discrete* if all of them are discrete. A type-2 fuzzy value that is neither continuous nor discrete is called *semi-continuous*.

A continuous or semi-continuous type-2 fuzzy value can be converted into a discrete type-2 fuzzy value using a discretization method.

2.2 Comparison of continuous type-2 fuzzy values

Comparing fuzzy values is an operation closely related to ranking fuzzy values. There are many different kinds of fuzzy comparison methods, but in the majority of case, they are only applicable to type-1 fuzzy values [4]. To rank type-2 fuzzy values, the comparison method must be defined on type-2 fuzzy values. We proposed a comparison method for discrete type-2 fuzzy values that is an extension of a comparison method for type-1 fuzzy values proposed by Lee et al. [1][4]. We will extend this comparison method to be applicable to continuous type-2 fuzzy values in this paper.

Proposed comparison method is based on the possibility theory. The difficulty in comparing fuzzy values comes from the fact that a fuzzy value is corresponding to a range of crisp values. Depending on its actual value, a fuzzy number can be greater or less than the other. Because it is difficult to compare two fuzzy values directly, all the possible actual values of two fuzzy values are compared in this approach.

If there are two type-2 fuzzy values $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, any possible combination of actual values (x_i, y_j), $x_i \in \tilde{\tilde{A}}$, $y_j \in \tilde{\tilde{B}}$ will be lain within the rectangle bounded by the support of $\tilde{\tilde{A}}$ and that of $\tilde{\tilde{B}}$ as shown in Fig. 1.

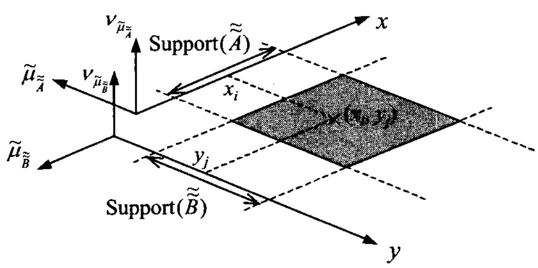


Fig. 1. Comparing the actual values of type-2 fuzzy values

Each possible combination of actual values can be mapped into one of the three sets: $G(\tilde{\tilde{A}} > \tilde{\tilde{B}})$, $G(\tilde{\tilde{A}} = \tilde{\tilde{B}})$, and $G(\tilde{\tilde{A}} < \tilde{\tilde{B}})$. That is, for every (x_i, y_j), $x_i \in \tilde{\tilde{A}}$, $y_j \in \tilde{\tilde{B}}$,

- $(x_i, y_j) \in G(\tilde{\tilde{A}} > \tilde{\tilde{B}})$ if $x_i > y_j$
- $(x_i, y_j) \in G(\tilde{\tilde{A}} = \tilde{\tilde{B}})$ if $x_i = y_j$
- $(x_i, y_j) \in G(\tilde{\tilde{A}} < \tilde{\tilde{B}})$ if $x_i < y_j$

There is no uncertainty in this mapping because every actual value is all crisp.

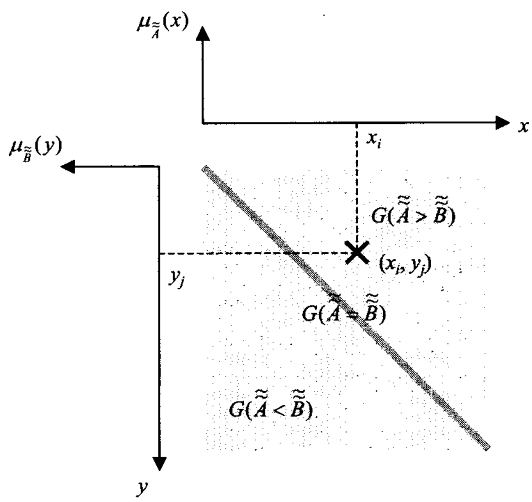


Fig. 2. Grouping actual value pairs

To compare two fuzzy values, we need to calculate the possibility, or confidence degree, of each set. This possibility is called a *satisfaction degree* of the given comparison [4].

Definition 4 Let A, B be fuzzy values and $*$ an arithmetic comparison relation. The *satisfaction degree* $s(A * B)$ for $A * B$ denotes the degree to which the arithmetic comparison relation $*$ for A and B is satisfied, in other words, the degree to which the proposition $A * B$ is true.

The satisfaction degree has a value within [0, 1]. The degree 1 represents full satisfaction (truth) of the relation $A * B$, while the degree 0 represents the dissatisfaction (falsity). The larger the value of the degree, the greater the satisfaction.

To estimate the satisfaction degree of given comparison relation, a measure called *satisfaction function*, $S(A * B)$, is used. The satisfaction function of a comparison relation can be defined as the summation of the possibilities satisfying the given comparison relation.

$$S(A*B)=\frac{\sum_{\forall(x_i,y_j)\in G(A*B)}\text{Possibility}((x_i,y_j))}{\sum_{\forall(x_i,y_j)}\text{Possibility}((x_i,y_j))}$$

The possibility of a point (x_i, y_j) is proportional to the possibility of each actual value, that is,

$$\text{Possibility}((x_i,y_j))\propto\text{Possibility}(x_i)$$

$$\text{Possibility}((x_i,y_j))\propto\text{Possibility}(y_j)$$

In the case of discrete type-2 fuzzy values, this possibility of an actual value x_i is calculated as

$$\text{Possibility}(x_i)\propto\sum_u u\otimes v_{\tilde{\mu}(x_i)}(u)$$

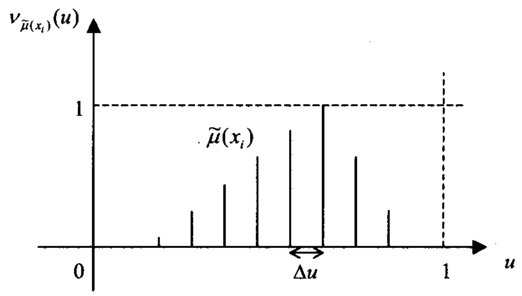


Fig. 3. Possibility of a point x_i in a discrete type-2 fuzzy value

This possibility can be extended for the case of continuous type-2 fuzzy values as

$$\text{Possibility}(x_i)\propto\lim_{\Delta u\to 0}\sum_u u\otimes v_{\tilde{\mu}(x_i)}(u)=\int_0^1 u\otimes v_{\tilde{\mu}(x_i)}(u)du$$

Then the possibility of an actual value pair (x_i, y_j) can be calculated as

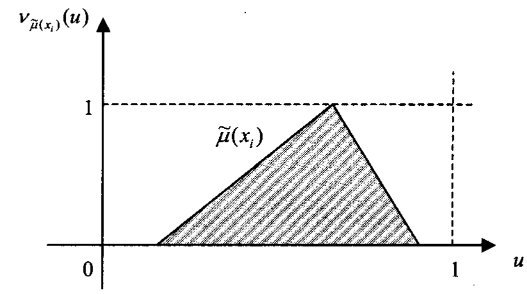


Fig. 4. Possibility of a point x_i in a continuous type-2 fuzzy value

$$\text{Possibility}((x_i,y_j))\propto\int_0^1\int_0^1 u\otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x_i)}(u)\otimes w\otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y_j)}(w)dudw$$

Based on this possibility calculation, we propose the satisfaction function for continuous type-2 fuzzy values.

Definition 5 If we denote the primary and secondary membership function of a type-2 fuzzy value $\tilde{\tilde{A}}$ as $\tilde{\mu}_{\tilde{\tilde{A}}}$ and $v_{\tilde{\mu}_{\tilde{\tilde{A}}}}$ respectively, the satisfaction functions $S(\tilde{\tilde{A}}>\tilde{\tilde{B}})$, $S(\tilde{\tilde{A}}=\tilde{\tilde{B}})$ and $S(\tilde{\tilde{A}}<\tilde{\tilde{B}})$ for continuous type-2 fuzzy values are defined as

$$S(\tilde{\tilde{A}}>\tilde{\tilde{B}})$$
$$=\lim_{\gamma\to 0}S_\gamma(\tilde{\tilde{A}}>\tilde{\tilde{B}})$$
$$=\frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\int_0^1\int_0^1 u\otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u)\otimes w\otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^1\int_0^1 u\otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u)\otimes w\otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy}$$

$$S(\tilde{\tilde{A}}=\tilde{\tilde{B}})$$
$$=\lim_{\gamma\to 0}S_\gamma(\tilde{\tilde{A}}=\tilde{\tilde{B}})$$
$$=0$$

$$S(\tilde{\tilde{A}}<\tilde{\tilde{B}})$$
$$=\lim_{\gamma\to 0}S_\gamma(\tilde{\tilde{A}}<\tilde{\tilde{B}})$$
$$=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{y}\int_0^1\int_0^1 u\otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u)\otimes w\otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^1\int_0^1 u\otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u)\otimes w\otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy}$$

where $S_\gamma(\tilde{\tilde{A}}>\tilde{\tilde{B}})$, $S_\gamma(\tilde{\tilde{A}}=\tilde{\tilde{B}})$, and $S_\gamma(\tilde{\tilde{A}}<\tilde{\tilde{B}})$ is the satisfaction functions for discrete type-2 fuzzy values and $\otimes$ is a t-norm operator that satisfies the following restriction:

$$\forall x,y\in[0,1],\ \ x\neq 0, y\neq 0\to x\otimes y\neq 0$$

Satisfaction functions for continuous type-2 fuzzy values has the following properties.

Proposition 1 $S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 1$.

Proof

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{1}\int_{0}^{1} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy$$
$$= \int_{-\infty}^{\infty}\int_{y}^{\infty}\int_{0}^{1}\int_{0}^{1} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{y}\int_{0}^{1}\int_{0}^{1} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dudwdxdy$$

$\therefore\ S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 1$.

Proposition 2 If $\text{Support}(\tilde{\tilde{A}}) \cap \text{Support}(\tilde{\tilde{B}}) = \varnothing$, then $S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = 1$ or $S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 1$.

Proof

$\text{Support}(\tilde{\tilde{A}}) \cap \text{Support}(\tilde{\tilde{B}}) = \varnothing$ means that

$\text{Sup}(\text{Support}(\tilde{\tilde{B}})) < \text{Inf}(\text{Support}(\tilde{\tilde{A}}))$ or

$\text{Sup}(\text{Support}(\tilde{\tilde{A}})) < \text{Inf}(\text{Support}(\tilde{\tilde{B}}))$.

In the case of $\text{Sup}(\text{Support}(\tilde{\tilde{B}})) < \text{Inf}(\text{Support}(\tilde{\tilde{A}}))$,

$G(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = \varnothing$ that means $S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 0$.

And in the case of $\text{Sup}(\text{Support}(\tilde{\tilde{A}})) < \text{Inf}(\text{Support}(\tilde{\tilde{B}}))$,

$G(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = \varnothing$ and $S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = 0$.

Because $S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) + S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 1$,

$\therefore\ S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = 1$ or $S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 1$.

Proposition 3 $S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) = S(\tilde{\tilde{B}} = \tilde{\tilde{A}})$.

Proof

$S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) = 0$ and $S(\tilde{\tilde{B}} = \tilde{\tilde{A}}) = 0$.

$\therefore\ S(\tilde{\tilde{A}} = \tilde{\tilde{B}}) = S(\tilde{\tilde{B}} = \tilde{\tilde{A}})$.

Proposition 4 If $\tilde{\tilde{A}} \equiv \tilde{\tilde{B}}$, then $S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 0.5$.

Proof

$\tilde{\tilde{A}} \equiv \tilde{\tilde{B}}$ means $\forall x \in X,\ \tilde{\mu}_{\tilde{\tilde{A}}}(x) \equiv \tilde{\mu}_{\tilde{\tilde{B}}}(x)$. Therefore,

$$S(\tilde{\tilde{A}} > \tilde{\tilde{B}})$$
$$= S(\tilde{\tilde{A}} > \tilde{\tilde{A}})$$
$$= \frac{1}{2}(S(\tilde{\tilde{A}} > \tilde{\tilde{A}}) + S(\tilde{\tilde{A}} < \tilde{\tilde{A}}))$$
$$= 0.5$$

In a similar way, we can show that $S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 0.5$.

$\therefore\ S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = S(\tilde{\tilde{A}} < \tilde{\tilde{B}}) = 0.5$.

2.3 Semi-continuous type-2 fuzzy values

A type-2 fuzzy value that is neither continuous nor discrete is called semi-continuous. An example of a semi-continuous type-2 fuzzy value comes from the extension from type-1 to type-2. If we extend a type-1 fuzzy value to a type-2 fuzzy value, the secondary membership function is always discrete. If it is a continuous type-1 fuzzy value, it will be extended to a semi-continuous type-2 fuzzy value because the primary membership function is continuous and the secondary membership function is discrete.

The satisfaction functions for semi-continuous type-2 fuzzy values are in the form of the mixture of each function for continuous and discrete values. For example, if the primary membership functions are all continuous and the secondary membership functions are all discrete, the satisfaction function $S(\tilde{\tilde{A}} > \tilde{\tilde{B}})$ will be

$$S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\sum_{w}\sum_{u} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{w}\sum_{u} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dxdy}$$

Especially if both $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are type-2 extensions of type-1 fuzzy values, the following equation is satisfied for every (x_i, y_j)

$$\sum_{w}\sum_{u} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x_i)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y_j)}(w) = \mu_{\tilde{A}}(x_i) \otimes \mu_{\tilde{B}}(y_j)$$

where $\mu_{\tilde{A}}(x_i)$ and $\mu_{\tilde{B}}(y_i)$ is the membership value of x_i and y_j in type-1 fuzzy values, and the satisfaction function $S(\tilde{\tilde{A}} > \tilde{\tilde{B}})$ can be simplified as

$$S(\tilde{\tilde{A}} > \tilde{\tilde{B}}) = \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\sum_{w}\sum_{u} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{w}\sum_{u} u \otimes v_{\tilde{\mu}_{\tilde{\tilde{A}}}(x)}(u) \otimes w \otimes v_{\tilde{\mu}_{\tilde{\tilde{B}}}(y)}(w)dxdy} = \frac{\int_{-\infty}^{\infty}\int_{y}^{\infty}\mu_{\tilde{A}}(x) \otimes \mu_{\tilde{B}}(y)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu_{\tilde{A}}(x) \otimes \mu_{\tilde{B}}(y)dxdy}$$

which is identical to the satisfaction function defined for continuous type-1 fuzzy values [3].

In some situations where this approach is not appropriate or too much complex, the semi-continuous type-2 fuzzy values can be discretized and compared using the satisfaction function for discrete type-2 fuzzy values.

3. Ranking

In this section, a ranking method is proposed using the comparison method proposed in 2.2. As in the case of comparison, it is not intuitive if ranking fuzzy values produce one and only one crisp result. So we will propose a method to calculate the confidence degree of each ranking result.

Before describing the ranking method, we will introduce the concept of preference function of fuzzy values [4].

Definition 6 The preference function $R(\tilde{\tilde{A}},\tilde{\tilde{B}})$ of type-2 fuzzy values $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is defined as

$$R(\tilde{\tilde{A}},\tilde{\tilde{B}})=S(\tilde{\tilde{A}}>\tilde{\tilde{B}})+\frac{1}{2}S(\tilde{\tilde{A}}=\tilde{\tilde{B}})$$

If $\tilde{\tilde{A}}$ or $\tilde{\tilde{B}}$ is a continuous type-2 fuzzy value, this preference function can be simplified as

$$R(\tilde{\tilde{A}},\tilde{\tilde{B}})=S(\tilde{\tilde{A}}>\tilde{\tilde{B}})$$

because $S(\tilde{\tilde{A}}=\tilde{\tilde{B}})$ is always 0 as long as one of them is continuous. We can consider the value of preference function as the confidence degree of the statement "$\tilde{\tilde{A}}$ is greater than $\tilde{\tilde{B}}$" or "$\tilde{\tilde{A}}$ is preferred to $\tilde{\tilde{B}}$."

The preference function satisfies the following properties.

Proposition 5 $R(\tilde{\tilde{A}},\tilde{\tilde{B}})+R(\tilde{\tilde{B}},\tilde{\tilde{A}})=1$.

Proof

$$\begin{aligned}&R(\tilde{\tilde{A}},\tilde{\tilde{B}})+R(\tilde{\tilde{B}},\tilde{\tilde{A}})\\&=S(\tilde{\tilde{A}}>\tilde{\tilde{B}})+\frac{1}{2}S(\tilde{\tilde{A}}=\tilde{\tilde{B}})+S(\tilde{\tilde{B}}<\tilde{\tilde{A}})+\frac{1}{2}S(\tilde{\tilde{B}}=\tilde{\tilde{A}})\\&=S(\tilde{\tilde{A}}>\tilde{\tilde{B}})+S(\tilde{\tilde{A}}=\tilde{\tilde{B}})+S(\tilde{\tilde{A}}<\tilde{\tilde{B}})\\&=1.\end{aligned}$$

Proposition 6 $R(\tilde{\tilde{A}},\tilde{\tilde{A}})=0.5$.

Proof

$$\begin{aligned}R(\tilde{\tilde{A}},\tilde{\tilde{A}})&=S(\tilde{\tilde{A}}>\tilde{\tilde{A}})+\frac{1}{2}S(\tilde{\tilde{A}}=\tilde{\tilde{A}})\\&=\frac{1}{2}(S(\tilde{\tilde{A}}>\tilde{\tilde{A}})+S(\tilde{\tilde{A}}=\tilde{\tilde{A}})+S(\tilde{\tilde{A}}<\tilde{\tilde{A}}))\\&=0.5.\end{aligned}$$

When we compare n fuzzy values, there are totally $n!$ possible ranking results. If we denote the ith ranking result as r_i, the result of ranking n fuzzy values can be expressed as a fuzzy set where each element is r_i and each membership function is the confidence degree of r_i. Therefore we will use the membership function $\mu(r_i)$ as the confidence degree of a ranking result r_i as follows.

$$\{(r_1,\mu(r_1)),(r_2,\mu(r_2)),\dots,(r_{n!},\mu(r_{n!}))\}$$

Definitions 7 The confidence degree of ith ranking result r_i, $\mu(r_i)$, is defined as

$$\mu(r_i)=\min(R(\tilde{\tilde{A}}_j,\tilde{\tilde{A}}_k))$$

where $\tilde{\tilde{A}}_j>\tilde{\tilde{A}}_k$ in the ranking result r_i.

Example 1 If the values of preference functions on four fuzzy values $\tilde{\tilde{A}}_1$, $\tilde{\tilde{A}}_2$, $\tilde{\tilde{A}}_3$, and $\tilde{\tilde{A}}_4$ are given as

R	$\tilde{\tilde{A}}_1$	$\tilde{\tilde{A}}_2$	$\tilde{\tilde{A}}_3$	$\tilde{\tilde{A}}_4$
$\tilde{\tilde{A}}_1$	–	0.998	1.000	1.000
$\tilde{\tilde{A}}_2$	0.002	–	0.183	0.681
$\tilde{\tilde{A}}_3$	0.000	0.817	–	0.992
$\tilde{\tilde{A}}_4$	0.000	0.319	0.008	–

then the confidence degree of a ranking result $\tilde{\tilde{A}}_1>\tilde{\tilde{A}}_3>\tilde{\tilde{A}}_2>\tilde{\tilde{A}}_4$ can be calculated as follows.

$$\begin{aligned}&\mu(\tilde{\tilde{A}}_1>\tilde{\tilde{A}}_3>\tilde{\tilde{A}}_2>\tilde{\tilde{A}}_4)\\&=\min(R(\tilde{\tilde{A}}_1,\tilde{\tilde{A}}_3),R(\tilde{\tilde{A}}_1,\tilde{\tilde{A}}_2),R(\tilde{\tilde{A}}_1,\tilde{\tilde{A}}_4),R(\tilde{\tilde{A}}_3,\tilde{\tilde{A}}_2),R(\tilde{\tilde{A}}_3,\tilde{\tilde{A}}_4),R(\tilde{\tilde{A}}_2,\tilde{\tilde{A}}_4))\\&=\min(0.988,1.000,1.000,0.817,0.992,0.681)\\&=0.681\end{aligned}$$

Confidence degrees of other ranking results can be calculated in the same way. The following examples show the confidence degrees of two other ranking results.

$$\mu(\tilde{\tilde{A}}_1>\tilde{\tilde{A}}_3>\tilde{\tilde{A}}_4>\tilde{\tilde{A}}_2)=0.319$$

$$\mu(\tilde{\tilde{A}}_2>\tilde{\tilde{A}}_1>\tilde{\tilde{A}}_3>\tilde{\tilde{A}}_4)=0.002$$

After assigning a confidence degree to each ranking result, final ranking result will be given as a form of a fuzzy set. For example, the final ranking result for the type-2 fuzzy values of Example 1 will be

$$\{(A_1>A_3>A_2>A_4,0.681),(A_1>A_3>A_4>A_2,0.319),\dots\}$$

Furthermore, an α-cut of the result fuzzy set can be used as an alternative to get a candidate of crisp ranking result.

For the application that requires a crisp ranking result, the representative ranking result can be used.

Definition 8 The ranking result with the largest confidence degree is called the *representative ranking result.*

In the example above, $\tilde{\tilde{A}}_1>\tilde{\tilde{A}}_3>\tilde{\tilde{A}}_2>\tilde{\tilde{A}}_4$ is the representative ranking result.

4. Conclusion

A ranking method for type-2 fuzzy values is proposed in this paper. In comparison, the comparison method for discrete type-2 fuzzy values is extended for continuous and semi-continuous type-2 fuzzy values. In ranking, a confidence degree is assigned to each ranking result based on the proposed comparison method. The result of ranking is given as a form of a fuzzy set so that it can provide flexibility in its appliance.

The drawback of the proposed method is that it has high computational complexity. This problem can be more or less evaded using discrete fuzzy values although it may sacrifice some precision. And the restriction of finite support of a fuzzy value can be a disadvantage in some special applications.

For more usability, the development of approximation algorithm based on heuristics is considered as a further work.

Reference

[1] S. Lee and K. H. Lee, "Comparison of type-2 fuzzy sets with satisfaction function," Proc. 2nd International Symposium on Advanced Intelligent Systems, pp. 436-439, 2001.

[2] N. N. Karnik and J. M. Mendel, "Operations on type-2 fuzzy sets," Fuzzy Sets and Systems, vol. 122, pp. 327-348, 2001.

[3] J.-H. Lee and H. Lee-Kwang, "Comparison of fuzzy values on a continuous domain," Fuzzy Sets and Systems, vol. 118, no. 3, pp. 419-428, 2001.

[4] K.-M. Lee, C.-H. Cho, and H. Lee-Kwang, "Ranking fuzzy values with satisfaction function," Fuzzy Sets and Systems, vol. 64, no. 3, pp. 295-309, 1994.

[5] N. N. Karnik and J. M. Mendel, "Applications of type-2 fuzzy logic systems to forecasting of time-series", Information Sciences, vol. 120, pp. 89-111, 1999.

[6] R. I. John, "Type-2 inferencing and community transport scheduling", Proc. 4th Euro. Congress Intelligent Techniques Soft Computing, pp. 1369-1372, 1996.

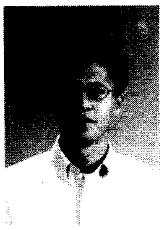
[7] M. Wagenknecht and K. Hartmann, "Application of fuzzy sets of type-2 to the solution of fuzzy equation systems", Fuzzy Sets and Systems, vol. 25, pp. 183-190, 1988.

[8] G. Bortolan and R. Degani, "A review of some methods for ranking fuzzy subsets", Fuzzy Sets and Systems, vol. 15, pp. 1-19, 1985.

[9] N. N. Karnik and J. M. Mendel, "An introduction to type-2 fuzzy logic systems," University of Southern California Report, 1998.

[10] D. Dubois and H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, 1980.

저 자 소 개



이승수(Seungsoo Lee)
1996년 : 한국과학기술원 전산학 학사
1998년 : 한국과학기술원 전산학 석사
1998년~현재 : 한국과학기술원 전자전산학과 박사과정 재학중

관심분야 : 퍼지 그래프, 타입-2 퍼지 시스템
E-mail : sslee@if.kaist.ac.kr

이광형(Kwang H. Lee)
1978년 : 서울공대 산업공학 학사
1980년 : 한국과학기술원 산업공학 석사
1982년 : INSA 전산학 석사(DEA)
1985년 : INSA 전산학 공학박사
1985년~2000년 : 한국과학기술원 전산학과 조교수, 부교수, 교수
2000년~현재 : 한국과학기술원 미래산업 석좌교수
관심분야 : 퍼지 이론 및 응용, 정보보안, 바이오정보
E-mail : khlee@if.kaist.ac.kr