OPTIMUM SPEED REDUCTION RATIO FOR D.C. SERVO DRIVE SYSTEMS

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Abstract--This report investigates a method of selecting the optimum speed reduction ratio between motor and load for d.c. servo drive systems under heavy external load conditions. The optimum reduction ratio is determined to minimise heat dissipation in the armature coil of d.c. servo motors and, therefore, to enhance the operating performance of the total servo system. In addition, based upon the optimum reduction ratio, a strategy of selecting the best d.c. motor is discussed from the viewpoint of torque requirements of the servo system. Finally, an example is illustrated to demonstrate how the optimum strategy can be applied to actual design problems.

NOMENCLATURE

$a, b, c, c(i)$	constants
I_a	armature current
J	system inertia
J_m	motor inertia
J_l	load inertia or mass
K_b	back-emf constant
K_t	motor torque constant
N	reduction ratio
N_{opt}	optimum reduction ratio
N^*_{opt}	no load optimum reduction ratio
p	pitch of lead screw
r	constant
R	external load parameter
R_a	motor resistance
t	time
t_c	duty cycle time
T	motor torque
T_i	inertia torque
T_f	friction torque
T_l	load torque reflected to motor shaft
T_L	external load (torque or force)
T_{Lequ}	equivalent external load (torque or force)
T_{max}	maximum motor torque
T_{min}	minimum motor torque
V	motor input voltage
W_d	total heat dissipation in armature coil
W_{d1}	heat dissipation by inertia load
W_{d2}	heat dissipation by external load
ω	motor rotational speed
ω_L	external load speed (rotational or linear)
ω_{Lmax}	maximum external load speed (rotational or linear)
$\frac{\partial \omega}{\partial t}$	motor acceleration

INTRODUCTION

HEAT DISSIPATION in the armature coil of d.c. motors critically limits the performance of servo drive systems. This is especially the case when the motor is operated repeatedly under heavy load conditions. Figure 1 shows typical torque-speed characteristics of a d.c. servo motor. The continuous operating zone is determined as the limit in which motor can be driven safe from thermal damage. If the required torque exceeds the

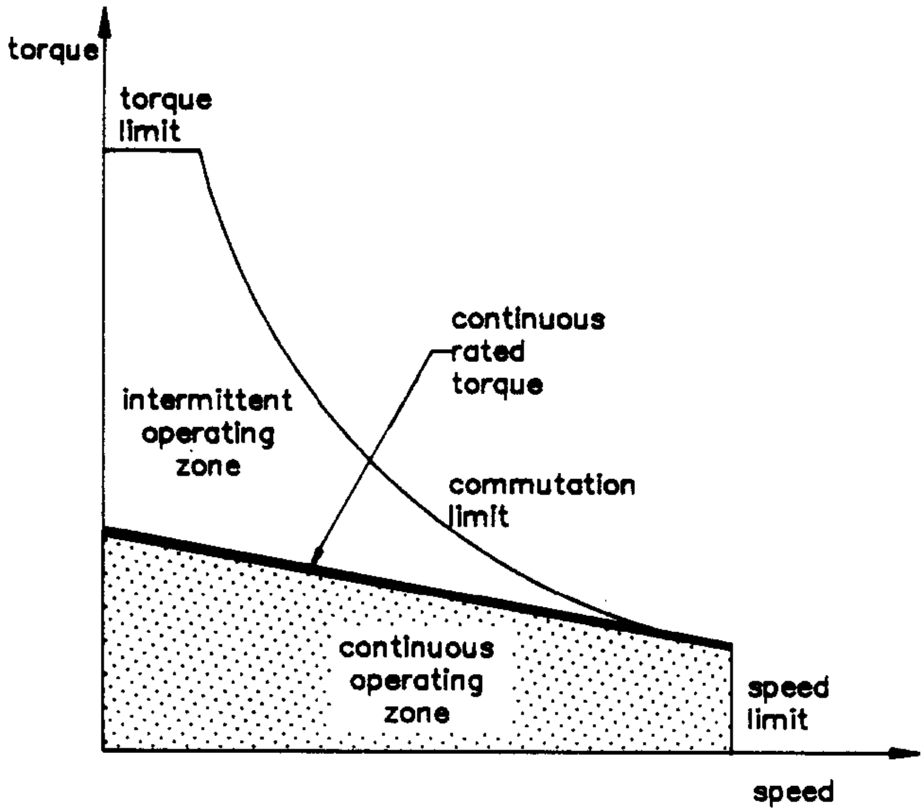

FIG. 1. Torque and speed characteristics of d.c. servo motors.

continuous rated torque, the operating time should be restricted to allow off-time for releasing heat [1,2].

The armature current is a crucial parameter on heat dissipation and directly proportional to the required motor torque. Therefore, the required torque should be minimised for given load conditions and this is generally realised by adopting torque reduction mechanisms between motor and load, such as gears or lead screws of fine pitch. A rule of thumb for deciding the reduction ratio says that it should be matched so that motor inertia equals load inertia when both are referred to a common base such as the motor shaft. However, this practice of matching the reduction ratio minimises the total system inertia, but does not guarantee minimum torque in the case where heavy external loads are applied. This is why it is very often found that a mismatched reduction ratio produces better performance [3]. Motivated by this, a method of selecting the optimum reduction ratio has been derived in this work, which can minimise heat dissipation of d.c. motors under given load conditions and therefore enhance the overall performance of servo systems.

MOTOR HEAT DISSIPATION

As shown in Fig. 2, the equivalent circuit of a d.c. servo motor is composed of inductance L, resistance Ro and a back-emf (electromotive force) constant Kb [4]. The inductance, L, can generally be neglected for engineering analysis since it is small in

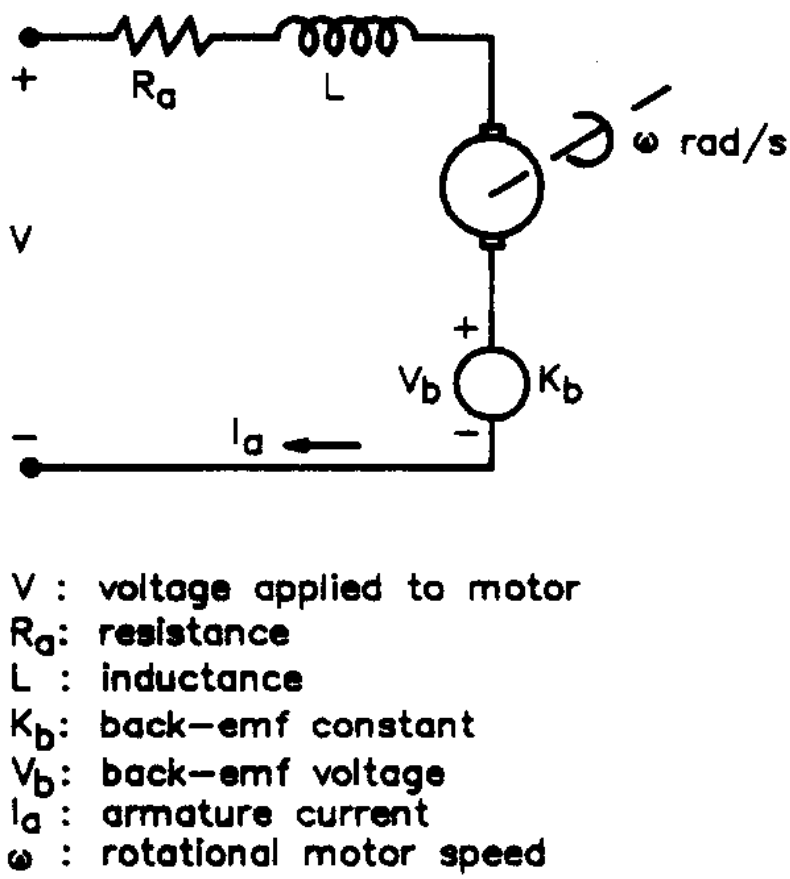

FIG. 2. Equivalent circuit of a d.c. servo motor.

most servo motors. Thus, when an input voltage V is applied, the instantaneous armature current I, and speed to may be derived as

$$V = R_a I_a + K_b \omega . \tag{1}$$

The torque generated by a d.c.motor is linearly proportional to the current 1,, the constant of proportionality being expressed as K,

$$T = K_t I_a . \tag{2}$$

The motor torque T can then be equated as

$$T = T_i + T_f + T_l \ . \tag{3}$$

In equation (3), T~ represents the inertia torque required to accelerate or decelerate the total system moment of inertia referred to the motor shaft. T r denotes the friction torque demanded to resist mechanical coulomb or viscous friction inherent in servo drive systems. The friction torque becomes insignificant in well manufactured drive systems and can be ignored for the simplification of analysis. Finally, T~ is the available torque to drive the external load and is normally referred to as the load torque.
The inertia torque Ti is decided by the system inertia and acceleration as

$$T_i = J \frac{\partial \omega}{\partial t}. \tag{4}$$

In obtaining the system inertia J, long and tiresome calculations are required for moving load, shafts, bearings, gears, plus the motor and tacho [5], but it can be written in a simple form as

$$J = J_m + (1/N^2)J_1 \tag{5}$$

in which N is the reduction ratio. Jl represents the load inertia which is affected by introducing a torque reduction mechanism, while Jm is the motor inertia.

The load torque Tt is the total external load reflected to the motor shaft. Thus, when the external load is summed as *TL*, the load torque is calculated as

$$T_l = (1/N)T_L. \tag{6}$$

From equations (2)-(6) the armature current I, is derived as

$$I_a = \frac{1}{K_t}\left[\left\{J_m + (1/N^2)J_l\right\}\frac{\partial \omega}{\partial t} + (1/N)T_L\right]. \tag{7}$$

Then the total heat dissipation in the motor coil is determined by the armature current and resistance as

$$W_d = \int_0^{t_c} (I_a^2 R_a)\, \mathrm{d}t \tag{8}$$

where *tc* is the duty cycle time of the servo drive system.

DETERMINATION OF OPTIMUM REDUCTION RATIO

The simple duty cycle of a servo system is mostly given by a trapezoidal speed profile as illustrated in Fig. 3a. The speed toL represents the external load speed which may be the rotational or linear velocity of the load. If the load is driven in rotational motion the load speed COL is converted to the motor speed to as

$$\omega = N\,\omega_L \tag{9}$$

in which N represents the non-dimensional reduction ratio of rotational motion between the motor and the load. On the other hand, in the case where the load is to perform linear motion by adopting a lead screw, its equivalent reduction ratio is evaluated by

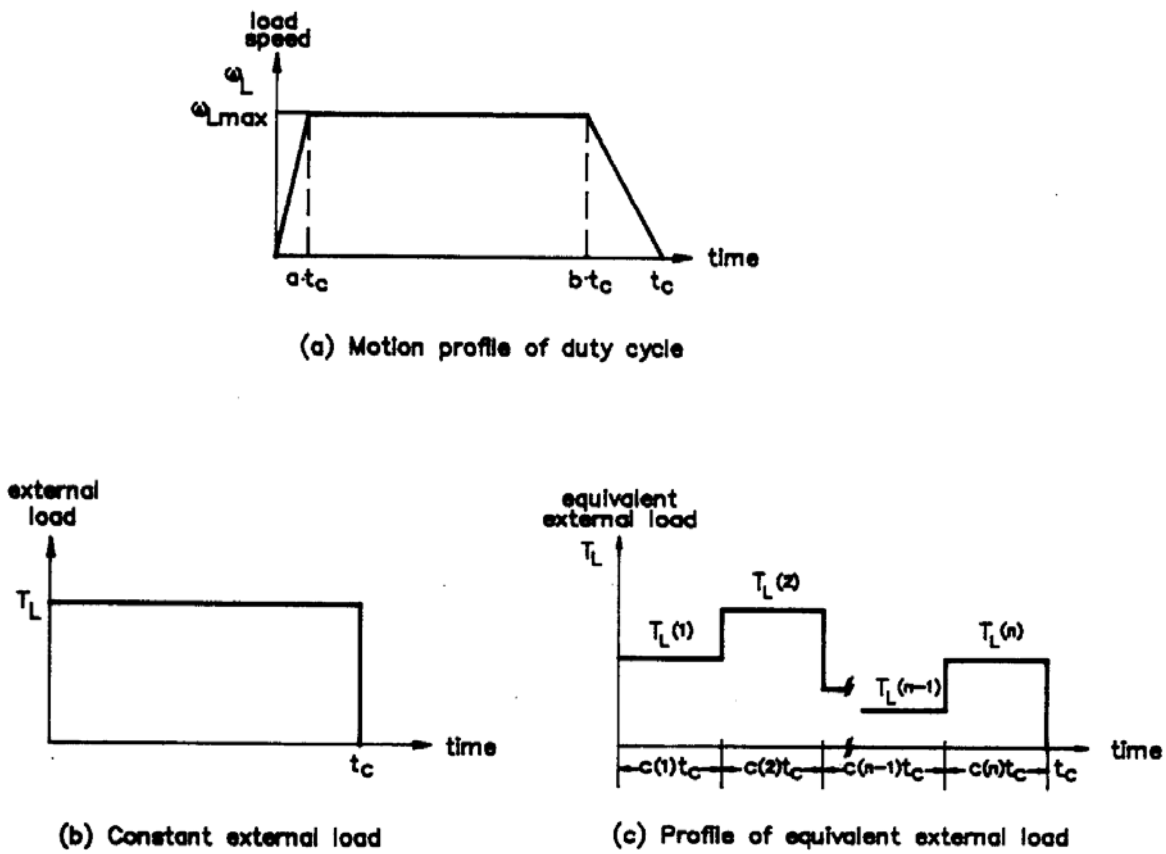

FIG. 3. Duty cycle of a servo drive system.

$$N = \frac{2\pi}{p} \tag{10}$$

where p is the pitch of the lead screw. In addition, the load inertia given by equation (5) and the external load TL of equation (6) are replaced with their equivalent mass and external force, respectively. Then, the acceleration is determined during the duty cycle of Fig. 3a as

$$\frac{\partial\omega}{\partial t} = \frac{N\,\omega_{Lmax}}{a\,t_c} \qquad (0 \le t < a\,t_c) \tag{11}$$

$$\frac{\partial\omega}{\partial t} = 0 \qquad (a\,t_c \le t < b\,t_c) \tag{12}$$

$$\frac{\partial\omega}{\partial t} = \frac{N\,\omega_{Lmax}}{(1-b)t_c} \qquad (b\,t_c \le t \le t_c)\ . \tag{13}$$

If the external load TL remains constant during the duty cycle as shown in Fig. 3b the heat dissipation can be obtained by substituting equations (11)--(13) into equation (8) as

$$W_d = W_{d1} + W_{d2}$$
$$= \frac{c\, R_a\, \omega_{Lmax}^2 J_l^2}{K_t^2\, t_c}\left[N^2\left(\frac{J_m}{J_l} + \frac{1}{N^2}\right)^2 + \frac{r}{N^2}\right] \tag{14}$$

where

$$c = \frac{1}{a} + \frac{1}{1-b} \tag{15}$$

and

$$r = \frac{T_L^2\, t_c^2}{c\, \omega_{Lmax}^2 J_l^2}\,. \tag{16}$$

The first term of equation (14), *Wd~*, represents the heat dissipation caused by the inertia load of the total system, while the other term, Wd2, by the external load T/. Figure 4 shows how the heat dissipation varies with the reduction ratio. When N is small, both *Wa~* and *Wd2* become large, as does the total sum *Wd.* AS N increases *Wd~* decreases to a certain minimum and then begins to increase, while Wd2 decreases steadily to zero. As a result, a unique optimum reduction ratio exists which minimises the total heat dissipation, *Wd. The* optimum reduction ratio is then determined by differentiating the armature coil heat dissipation with respect to the reduction ratio and equating this to zero to obtain the optimum reduction ratio, i.e.

$$\frac{\partial \omega_d}{\partial (N^2)} = 0 \tag{17}$$

giving

$$N_{opt}^2 = \frac{J_l}{J_m}\sqrt{1+r}\,. \tag{18}$$

It becomes clear that when the external load *TL* is negligible, i.e. r=0, the optimum ratio is simply evaluated only by the inertia ratio as

$$N_{opt}^2 = N_{opt}^{*2} = \frac{J_l}{J_m}\,. \tag{19}$$

However, when TL becomes large the optimum reduction ratio increases following an asymptotic line as illustrated in Fig. 5 in which the locus of optimum ratio is plotted against TL.

If the external load TL varies within a duty cycle as in the more practical case represented in Fig. 3c, integration of equation (8) leads to the concept of an equivalent external load *Tz, eq~*, which is defined as

$$T_{Lequ}^2 = \sum_{i=1}^{n} T_L(i)^2\, c(i) \quad \text{with} \sum_{i=1}^{n} c(i) = 1\,. \tag{20}$$

Then the optimum reduction ratio is determined by equation (18) in the same manner, except that TL is replaced with *TLeq,*.

The generalised duty cycle of a servo system can be represented by combination of a series of trapezoidal speed profiles whose individual cycle time is denoted by *to(i)*, as shown in Fig. 6. The external load condition is presented by the equivalent external loads of each speed profile, *TLequ(i).* The total heat dissipation of the generalised duty

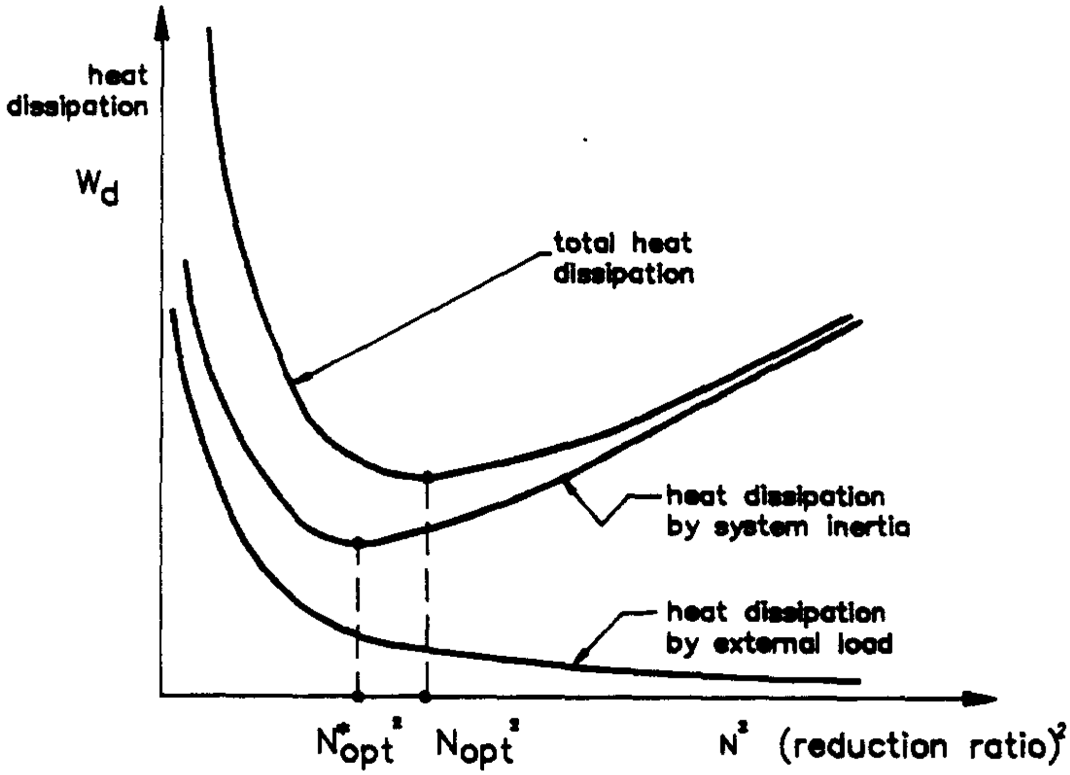

FIG. 4. Variation of motor heat dissipation with reduction ratio.

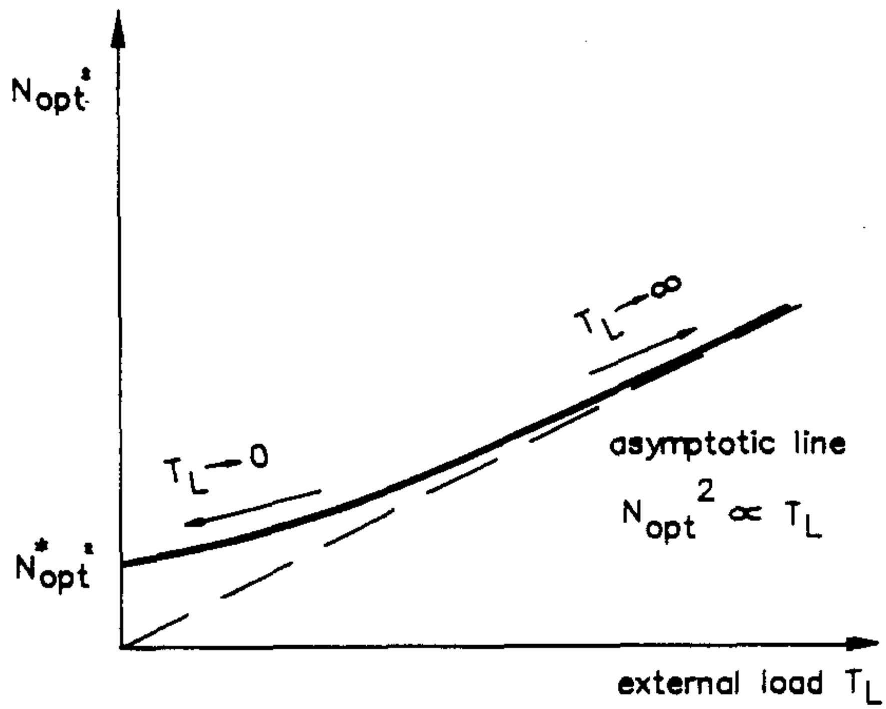

FIG. 5. Locus of optimum reduction ratio against external load.

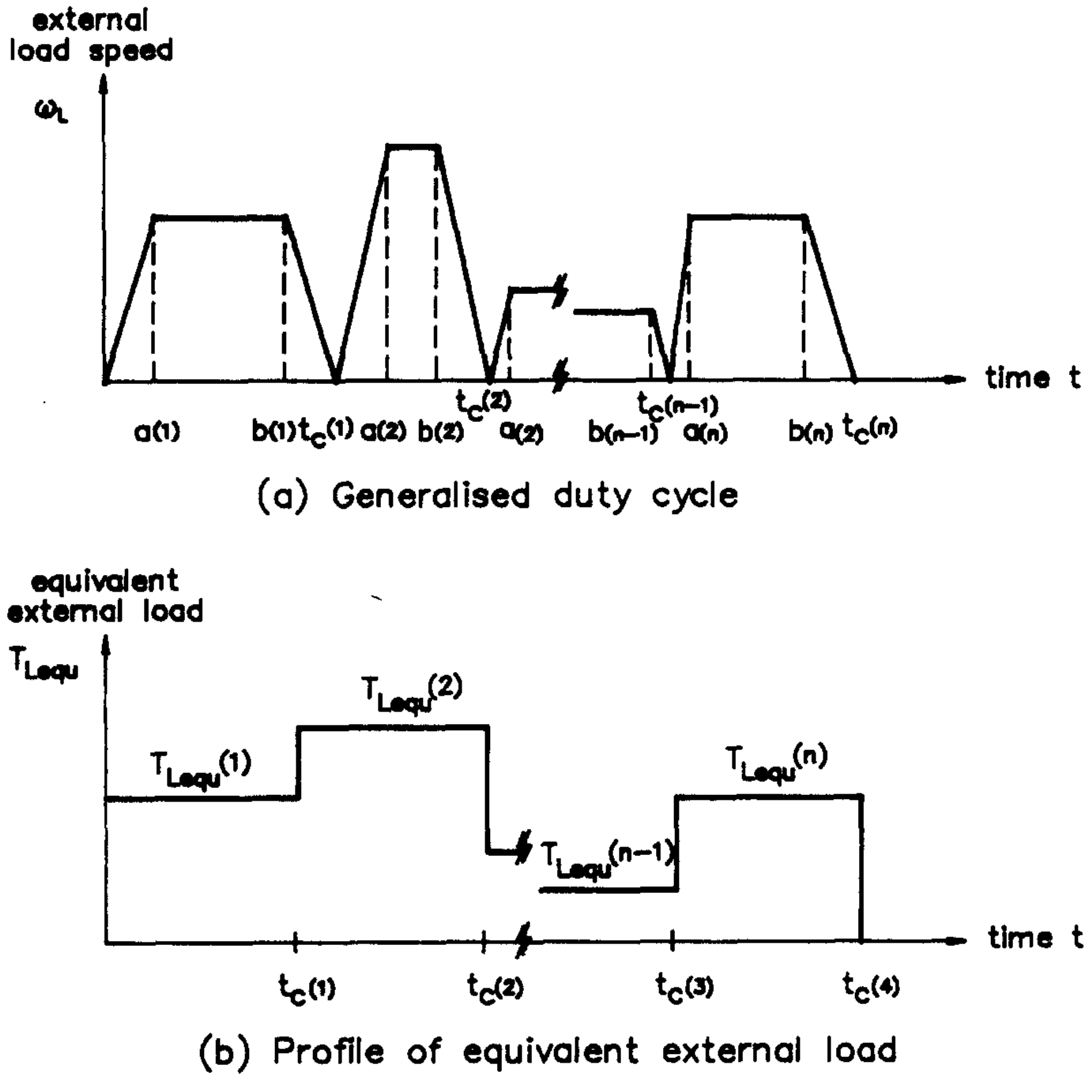

FIG. 6. Generalised duty cycle with varying external load.

cycle is then obtained by the summation of the individual heat dissipation of each speed profile, giving

$$W_d = \sum_{i=1}^{n} k(i) \left[N^2 \left(\frac{J_m}{J_l} + \frac{1}{N^2} \right)^2 + \frac{r(i)}{N^2} \right] \tag{21}$$

where

$$k(i) = c(i) \frac{R_a \, \omega_{Lmax}^2(i) \, J_l^2}{K_t^2 \, t_c(i)} \tag{22}$$

$$r(i) = \frac{T_{Lequ}^2(i) \, t_c^2(i)}{c(i) \, \omega_{Lmax}^2(i) \, J_l^2} \tag{23}$$

and

$$c(i) = \frac{1}{a(i)} + \frac{1}{1 - b(i)}. \tag{24}$$

Then the generalised optimum reduction ratio is solved by applying equation (17) as

$$N_{opt}^2 = \frac{J_l}{J_m} \sqrt{1 + R} \tag{25}$$

where

$$R = \sum_{i=1}^{n} s(i)\, r(i) \Bigg/ \sum_{i=1}^{n} s(i) \tag{26}$$

and

$$s(i) = \frac{t_c(l)\, c(i)\, \omega_{Lmax}^2(i)}{t_c(i)\, c(l)\, \omega_{Lmax}^2(l)}. \tag{27}$$

The optimum ratio of equation (25) for the generalised duty cycle is the extended form of equation (18) in which R, as defined in equation (26), denotes the total sum of the effects of external load conditions.

OPTIMUM SELECTION OF A D.C. MOTOR

When the optimum reduction ratio *Nop*, is adopted, the motor torque T is obtained by substituting equations (25) and (7) into equation (2), giving

$$T\Big|_{N=N_{opt}} = J_m \left(1 + \frac{1}{\sqrt{1+R}}\right) \frac{\partial\omega}{\partial t} + \sqrt{\frac{J_m}{J_1}}\, \frac{1}{\sqrt[4]{1+R}}\, T_L. \tag{28}$$

It is noted that the motor torque varies with the acceleration $\frac{\partial\omega}{\partial t}$ and the external load TL during the duty cycle. In general, the maximum peak of the motor torque, *Tmax*occurs during an acceleration period when $\frac{\partial\omega}{\partial t}$ becomes most significant and also TL is applied in the same direction of the acceleration. If $\frac{\partial\omega^*}{\partial t}$ and T*L are the acceleration and load of the acceleration period respectively when T~,x occurs, T,,,,~ is obtained as

$$T_{max} = J_m\left(1 + \frac{1}{\sqrt{1+r}}\right) \frac{\partial\omega^*}{\partial t} + \sqrt{\frac{J_m}{J_1}}\, \frac{1}{\sqrt[4]{1+r}}\, T_L^*. \tag{29}$$

The minimum motor torque, Tmi,, is determined when $\frac{\partial\omega}{\partial t}$ = 0 and *TL* becomes minimum by the equation

$$T_{min} = \sqrt{\frac{J_m}{J_1}}\, \frac{1}{\sqrt[4]{1+r}}\, T_{Lmin}. \tag{30}$$

Now i.t is worthwhile to note that, even though the maximum torque is decided dominantly by the acceleration, the heat dissipation of equation (21) is affected more by the external load T/_ since the acceleration time is short compared with the total duty cycle time, i.e. *a(i)* ,~ 1. For this reason the optimum reduction ratio of equation (25) is determined not only by the inertias but also by the external load condition.

When the torque requirements of a servo drive system have been identified as in equations (29) and (30), it then becomes important to choose the best d.c. motor to satisfy the requirements. In general, two torque parameters are specified for d.c. motors as design data; one is the peak torque and the other is the rated torque. The peak torque indicates the maximum torque which the specified motor can produce instantaneously only for acceleration and deceleration. The rated torque is the limit of the continuous operating zone of the motor. Therefore, the peak torque of the motor to be selected should be higher than the maximum required torque given by equation (29) and, at the same time, its rated torque should also be higher than the minimum required torque of equation (30).

Another important fact to be considered in selecting a d.c. motor is that the maximum and minimum torque requirements are affected by the inertia of the chosen motor itself. Figure 7 illustrates the effects of the motor inertia Jm on the maximum and minimum torques of equations (29) and (30). As Jm increases, both Tmox and T,,,i, increase, but T,,,ox increases more rapidly. Generally, a d.c. motor with a large inertia can produce more torque, but it also increases the torque requirements due to its own inertia, J,,,. Therefore a larger motor, chosen simply because of its high torque producing capability, does not guarantee the safety margin of torque requirements. The optimal selection should be made to find a motor which has the smallest motor inertia among the candidate motors satisfying the maximum and minimum torque requirements, as shown in Fig. 7.

EXAMPLE AND DISCUSSION

An example is illustrated and discussed to demonstrate how the optimum developed strategy can be applied to actual design problems. Figure 8 shows the schematic drawing of a wheel in-feed servo system.built in a creep-feed grinding machine. The carriage is driven linearly by a d.c. torque motor via a ball screw. Figure 9a describes the duty cycle of the servo system composed of three trapezoidal linear speed profiles for the feeding of the grinding wheel, actual creep-feed grinding process and wheel retraction respectively. The external load condition is also presented in Fig. 9b in terms of external forces during the grinding process due to the friction of guideways and the cutting forces. From the specified speed profile and external load condition the maximum and minimum motor torques given by equations (29) and (30) are respectively obtained as

$$T_{max} = 372\,J_m + 1.7\sqrt{J_m} \qquad (31)$$

and

$$T_{min} = 0.12\sqrt{J_m} \qquad (32)$$

where the motor torques and inertias are expressed in kgf m and kg m 2, respectively.

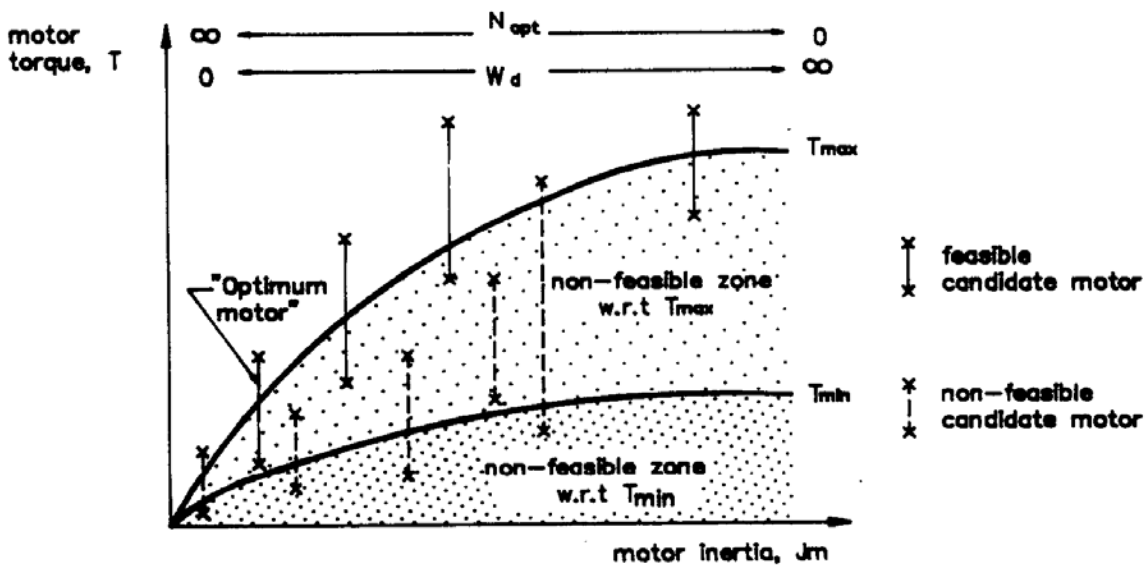

FIG. 7. Optimum selection of d.c. motor.

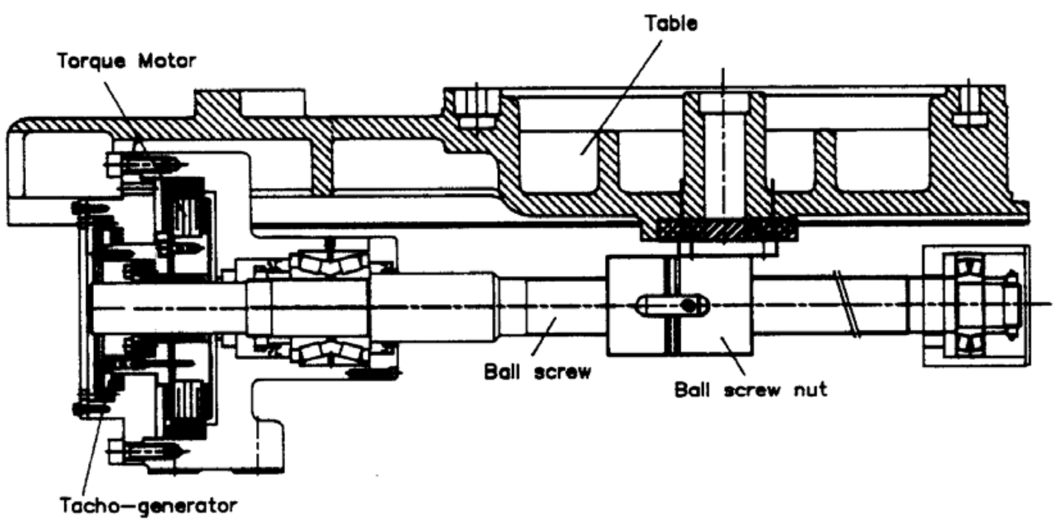

FIG. 8. Design example—grinding wheel in-feed drive servo.

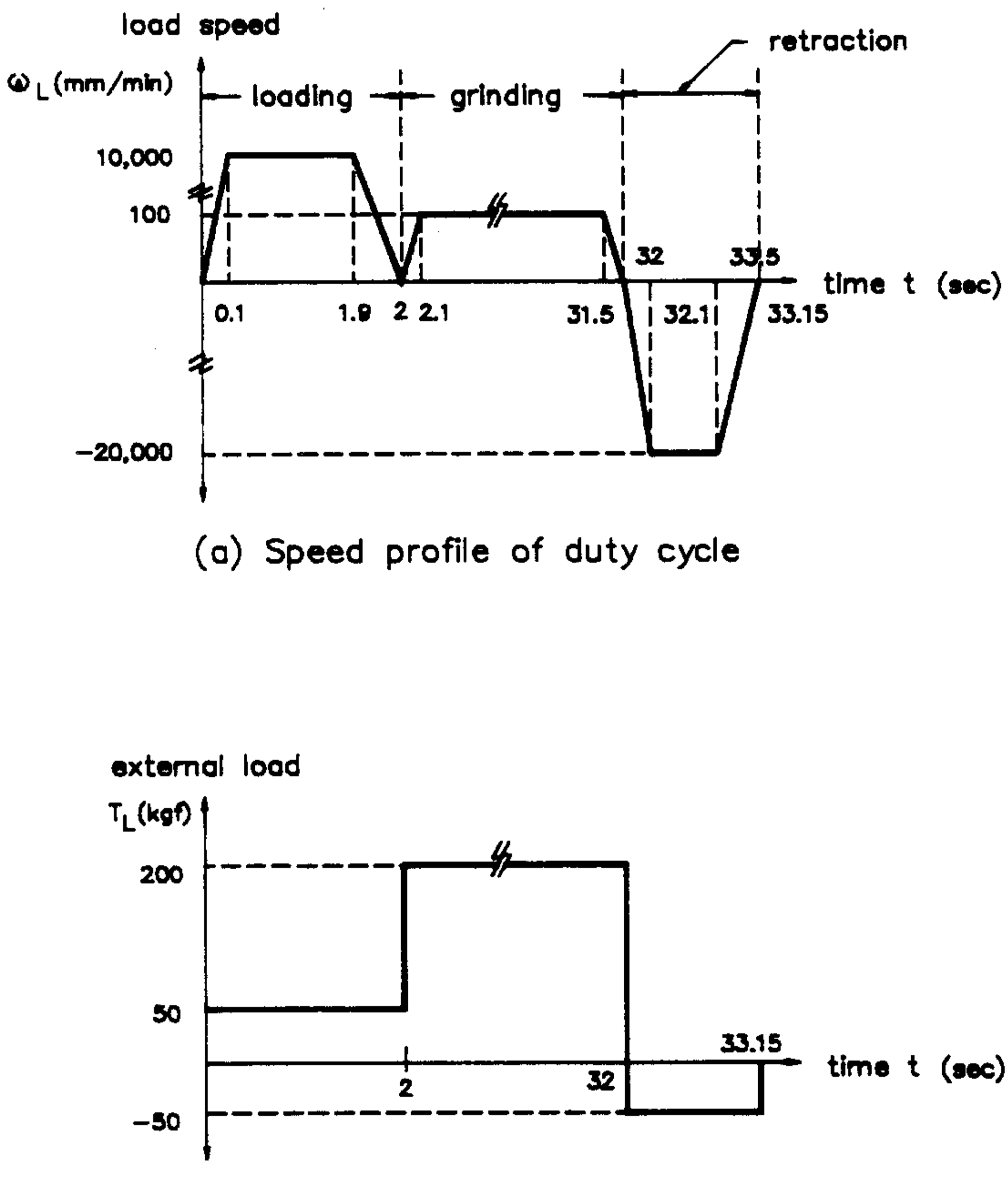

FIG. 9. Duty cycle of example servo.

Based on the relationships between the required torques and the motor inertia of equations (31) and (32), the Inland type QT-6202 torque motor with the smallest motor inertia among the candidates satisfying the torque requirements is chosen. Then, the optimum reduction ratio, *Nopt* is calculated from equation (25) and, finally, the pitch of the ball screw is determined by equation (10) as 5.5 mm.

The total amount of heat dissipation of armature coil when *Nopt* is taken in this application is 2.33 kJ per duty cycle. On the other hand, if the reduction ratio is decided simply by the inertia ratio of equation (19) in which the external load conditions are neglected, the heat dissipation reaches 7.0 kJ per duty cycle. It can therefore be seen that, for this exam'ple, a saving of 200 per cent of the heat generated can be achieved by optimising the reduction ratio.

CONCLUSION

A method of selecting the optimum reduction ratio between motor and load for d.c. servo drive systems has been derived. When the external load is not significant the optimum reduction ratio may be determined simply by the inertia ratio such that motor inertia equals load inertia. However, when heavy external loads exist, the optimum ratio is greatly influenced by load conditions and then should be determined by not only system inertia but also by external load. A design example has been demonstrated in which the performance of a servo drive can be improved significantly by adopting the optimum reduction ratios decided by the optimal strategy developed in this work.

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