LETTER

Reduced-Complexity Vector Channel Estimation for Systems with Receive Diversity

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SUMMARY We consider a blind estimation of the vector channel for systems with receive diversity. The objective of this paper is to reduce the complexity of the conventional subspace-based method in vector channel estimation. A reduced-complexity estimation scheme is proposed, which is based on selecting a column of the covariance matrix of the received signal vectors. The complexity and performance of the proposed scheme is investigated via computer simulations.

key words: *vector channel estimation, complexity, receive diversity*

1. Introduction

Systems with receive diversity, which employ an antenna array for receiving signals, have been proven to increase system capacity and cell coverage through diversity-combining [1]. The systems acquire increased diversity gains from the proper use of the spatial diversity. To achieve maximum diversity gains, the receiver should be capable of estimating a vector channel which characterizes the unique propagation pattern between mobile users and the antenna array.

In general, a pilot-aided channel estimation scheme, based on the pre-known symbols at the receiver, is utilized for channel estimation. However, when the pilot symbols are costly or impracticable, blind estimation has to be employed. Among various blind estimation schemes, a subspace decomposition method used in multiple signal classification (MUSIC) [2] can be adopted in the blind vector channel estimation. Based on the covariance matrix averaged over the set of received signal vectors for certain time duration, the subspace-based scheme is known to be robust compared with the decision-directed blind schemes, which depend on the instantaneous received symbol. The subspace-based scheme, however, requires intensive computations for the computation of a covariance matrix and of its eigenvalue decomposition (EVD), which increase complexity. This computational burden, together with the increase of hardware components, can be one of the major obstacles for systems with receive diversity to be adopted in practice.

The complexity reduction can be attempted in the diversity reception technique as in [3]. This paper focuses on reduction of the complexity in the vector channel estimation in order to provide a possible solution to the complexity problem.

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2. System Model

A receiver with an antenna array can be modelled as a single-input-multiple-output(SIMO) system assuming the user separation is completed through other multiplexing schemes. The received signal vector at the k'th time instant, $\mathbf{x}[k]$, which has the dimension of the number of antenna elements, M, consists of the transmitted signal, $s[k]$, attenuated by the vector channel, $\mathbf{h}$, and the additive white Gaussian noise vector, $\mathbf{n}[k]$:

$$\mathbf{x}[k] = \mathbf{h}s[k] + \mathbf{n}[k] \tag{1}$$

It is assumed that the noise vector is white in time and space and is independent of $s[k]$. It is also assumed that a narrow band signal is transmitted from the user's terminal. The vector channel $\mathbf{h}$ is expressed as a constant assuming the terminal is stationary for the time duration, NT_s, where N is the number of observations for estimation and T_s denotes the sampling interval.

3. Conventional Subspace-Based Estimation

In the subspace-based blind estimation scheme [4], the vector channel, $\mathbf{h}$ in (1), is estimated from a set of received vectors, $\{\mathbf{x}[k] \in \mathbb{C}^M : k = 1, \cdots, N\}$ assuming the transmitted signal is unavailable at the receiver, where $\mathbb{C}^M$ is an M dimensional complex vector space. The subspace-based scheme is based on the covariance matrix of the received signal vectors, defined and expressed as

$$\mathbf{R} \triangleq E\{\mathbf{x}[k]\mathbf{x}^H[k]\} = \sigma_s^2\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I}, \tag{2}$$

where $(.)^H$ denotes a Hermitian transpose, σ_s^2 is the signal variance, and σ_n^2 is the noise variance. An eigenvalue decomposition of $\mathbf{R}$ in (2) can be expressed as

$$\mathbf{R} = \sum_{i=1}^{M} \lambda_i \mathbf{e}_i \mathbf{e}_i^H, \tag{3}$$

where the eigenvalues are ordered in a descending manner in magnitude as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. It can be readily proved from (2) that the eigenvector associated with the largest eigenvalue spans the signal subspace and the rest of the eigenvectors span the noise subspace. Furthermore, all the eigenvalues are equal to the noise variance, σ_n^2, except for the largest eigenvalue, λ_1. Thus, the basis for the signal subspace, which is the eigenvector, $\mathbf{e}_1$, associated with λ_1, can be an estimate of the vector channel: $\hat{\mathbf{h}} = \mathbf{e}_1$.

However, the real covariance matrix is not generally available and is approximated by a sample covariance matrix:

$$\hat{\mathbf{R}} \triangleq \frac{1}{N}\sum_{k=1}^{N}\mathbf{x}[k]\mathbf{x}^H[k], \tag{4}$$

where N is the number of observations. It is readily derived that the construction of $\hat{\mathbf{R}}$, requires $O(NM^2)$ multiplications. Recall that M denotes the number of antenna elements. Adding the computation of EVD for $M \times M$ matrix, $O(M^3)$, the total computations reaches $O(NM^2 + M^3)$.

4. Reduced-Complexity Estimation

4.1 Algebraic Solution

The following proposition can be formulated assuming that the variance of the noise is known to the receiver.

Proposition 1: The scaled vector channel is expressed as the l'th column of the covariance matrix minus the variance of the noise in the l'th element, where $l \in \{1, \cdots, M\}$.

Proof: Let us rewrite the expression for the covariance matrix in (2).

$$\mathbf{R} = [\mathbf{r}_1, \cdots, \mathbf{r}_M] = \sigma_s^2\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I} \tag{5}$$

From (5), the expression for $\mathbf{h}$ is given by

$$\sigma_s^2\mathbf{h}[h_1^*, \cdots, h_M^*] = [\mathbf{r}_1 - \sigma_n^2\mathbf{u}_1, \cdots, \mathbf{r}_M - \sigma_n^2\mathbf{u}_M], \tag{6}$$

where h_i is the i'th element of $\mathbf{h}$, $\mathbf{r}_i$ is the i'th column of $\mathbf{R}$, and $\mathbf{u}_i$ denotes the $M \times 1$ unit vector with one in the i'th position.

If we take the l'th column of the matrix in (6), we get

$$\sigma_s^2 h_l^*\mathbf{h} = \mathbf{r}_l - \sigma_n^2\mathbf{u}_l \tag{7}$$

for any $l \in \{1, 2, \cdots, M\}$. □

Proposition 1 and its straightforward proof show that the vector channel can be estimated from a column of the covariance matrix, $\mathbf{R}$, if σ_n^2 is available. As we will see in the next section, the variance of the noise can also be estimated from the received signals. Thus, we propose new estimates:

$$\hat{\mathbf{h}}_{prop} = \hat{\mathbf{r}}_l - \hat{\sigma}_n^2\mathbf{u}_l, \tag{8}$$

where $\hat{\mathbf{r}}_l$ is the l'th column of the sample covariance matrix in (4), $\hat{\sigma}_n^2$ is an estimate of the variance of the noise, and $l \in \{1, 2, \cdots, M\}$. Note that the scale of the channel estimate is not of interest here as in other subspace-based blind methods. The scale is sometimes not required as in differentially modulated systems, or it is acquired from a further estimation step.

The proposed estimate in (8) does not require the calculation of the entire covariance matrix, which can result in a drastic reduction of computations: $O(NM)$. There arise, however, issues on how to estimate the noise variance without intensive calculations and how to select the column, l, in an optimal way.

4.2 Noise Variance Estimation

The first method we propose is the estimation by projection. This approach is based on the observation that the selected column in (7) equals the scaled vector channel except l'th element, that is, $\breve{\mathbf{h}} = \alpha\breve{\mathbf{r}}_l$, where each vector is formed from $\mathbf{h}$ and $\mathbf{r}_l$, respectively, by skipping the l'th element. Let us define a $(M-1)\times 1$ vector, $\breve{\mathbf{x}}[k]$, in the same way and consider a projection onto the orthogonal complement of the space spanned by $\breve{\mathbf{h}}$:

$$\mathbf{P}_{\breve{\mathbf{h}}}^{\perp} = \mathbf{I} - \mathbf{P}_{\breve{\mathbf{h}}} = \mathbf{I} - \breve{\mathbf{h}}[\breve{\mathbf{h}}^H\breve{\mathbf{h}}]^{-1}\breve{\mathbf{h}}^H = \mathbf{I} - \breve{\mathbf{r}}_l[\breve{\mathbf{r}}_l^H\breve{\mathbf{r}}_l]^{-1}\breve{\mathbf{r}}_l^H. \tag{9}$$

If we project the received signal, $\breve{\mathbf{x}}[k]$, onto the orthogonal complement of the signal space, given in (9), the noise variance can be approximated by

$$\hat{\sigma}_n^2 = \frac{1}{N(M-2)}\sum_{k=1}^{N}||\mathbf{P}_{\breve{\mathbf{h}}}^{\perp}\breve{\mathbf{x}}[k]||^2. \tag{10}$$

As shown in [5], $\hat{\sigma}_n^2$ converges to the actual variance of the noise, σ_n^2, as N goes to infinity. Thus, the estimated value can be used for the estimation of the vector channel as proposed in (8). It can be calculated that implementation of the algorithm requires $O(3N(M-1))$ computations by appropriately ordering the calculation.

The subspace decomposition discussed in the previous section is another approach for the estimation of the variance of the noise. Here, we propose to take signals from only two antenna elements to estimate the variance of the noise to avoid the increase of computations associated with the formation of the whole covariance matrix. The proposed scheme is based on the assumption that the variance of the noise of each antenna is identical.

Let us form a 2×2 matrix which we call a sub-covariance matrix as

$$\tilde{\mathbf{R}} \triangleq E\{\tilde{\mathbf{x}}[k]\tilde{\mathbf{x}}^H[k]\} = \sigma_s^2\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H + \sigma_n^2\tilde{\mathbf{I}}, \tag{11}$$

where $\tilde{\mathbf{x}}$ is 2×1 vector formed by any two elements of the original vector of received signals. As shown in the previous section, the smallest eigenvalue of the sub-covariance matrix equals the variance of the noise:

$$\hat{\sigma}_n^2 = \lambda_2, \tag{12}$$

where λ_2 is the smallest eigenvalue of $\tilde{\mathbf{R}}$. It can be calculated that this method requires $O(N\times4+8)$ calculations regardless of the number of antenna elements.

4.3 Column Selection

In Sect. 4.1, we have seen that any column of the covariance matrix, $\mathbf{R}$, compensated by the noise variance can be an estimate of the vector channel if the noise variance is perfectly known. In this section, we propose that there is an optimal way of selecting a column in $\mathbf{R}$ to minimize the estimation errors when errors exist in the estimate of the noise variance, which is the more realistic situation.

First, let us define a measure called the *normalized orthogonal distance* (ND) of two vectors, $\mathbf{x}_1$ and $\mathbf{x}_2$, which is given by

$$d(\mathbf{x}_1, \mathbf{x}_2) \triangleq \frac{1}{\|\mathbf{x}_2\|} \left\| \frac{\mathbf{x}_1^H \mathbf{x}_2}{\mathbf{x}_1^H \mathbf{x}_1} \mathbf{x}_1 - \mathbf{x}_2 \right\|, \tag{13}$$

where $\|\mathbf{x}\| = \sqrt{\mathbf{x}^H \mathbf{x}}$. The ND has the maximum value, which is unity, when two vectors are orthogonal to each other and the minimum value, which is zero, when they are parallel to each other regardless of their magnitude.

The following proposition shows that the selection of a specific column yields the minimal estimation errors when the estimate of the noise variance includes estimation errors.

Proposition 2: Let $\bar{\hat{\mathbf{h}}}(l) \triangleq E\{\hat{\mathbf{h}}(l)\}$ and $\Delta_d \triangleq \sigma_n^2 - E\{\hat{\sigma}_n^2\}$. Then, $\bar{\hat{\mathbf{h}}}(l)$ has the minimal ND from the actual vector channel, $\mathbf{h}$, over the M column indices, if (i) $l = \arg\max_{1 \le i \le M} |\mathbf{u}_i^T \mathbf{r}_i|^2$ and (ii) $|\Delta_d| \le \frac{\mathbf{h}^H \mathbf{h} \sigma_s^4}{2}$.

Proof: Let x_l and h_l be the l'th elements of the column vectors, $\mathbf{x}$ and $\mathbf{h}$ respectively. Then, it can be readily seen that $|\mathbf{u}_l^T \mathbf{r}_l|^2 = E\{x_l x_l^*\}$ and $|\mathbf{u}_l^T \mathbf{h}|^2 = h_l h_l^*$. From (5), $E\{x_l x_l^*\}$ can be expressed in terms of $h_l h_l^*$.

$$E\{x_l x_l^*\} = \sigma_s^2 h_l h_l^* + \sigma_n^2 \tag{14}$$

Equation (14) shows that $|\mathbf{u}_l^T \mathbf{r}_l|^2$ is maximum when $|\mathbf{u}_l^T \mathbf{h}|^2$ is maximum.

Let us consider the squared normalized orthogonal distance of $\bar{\hat{\mathbf{h}}}(i)$ and $\mathbf{h}$.

$$d^2(\mathbf{h}, \bar{\hat{\mathbf{h}}}(i)) = \frac{1}{\|\bar{\hat{\mathbf{h}}}(i)\|^2} \left\| \frac{\mathbf{h}^H \bar{\hat{\mathbf{h}}}(i)}{\mathbf{h}^H \mathbf{h}} \mathbf{h} - \bar{\hat{\mathbf{h}}}(i) \right\|^2 \tag{15}$$

Let us derive the numerator and the denominator of $d^2(\mathbf{h}, \bar{\hat{\mathbf{h}}}(i))$ using the following relationship from (7).

$$\bar{\hat{\mathbf{h}}}(i) = h_i^* \sigma_s^2 \mathbf{h} + \Delta_d \mathbf{u}_i, \tag{16}$$

where h_i is the i'th element of the $\mathbf{h}$ vector and σ_s^2 is the signal power. It can be derived that the numerator part is expressed as

$$\frac{\| \mathbf{h}^H \bar{\hat{\mathbf{h}}}(i) \mathbf{h} - \mathbf{h}^H \mathbf{h} \bar{\hat{\mathbf{h}}}(i) \|^2}{(\mathbf{h}^H \mathbf{h})^2} = \frac{\Delta_d^2 (\mathbf{h}^H \mathbf{h} - |h_i|^2)}{\mathbf{h}^H \mathbf{h}} \tag{17}$$

Similarly, the denominator part is expressed as

$$\bar{\hat{\mathbf{h}}}(i)^H \bar{\hat{\mathbf{h}}}(i) = \Delta_d^2 + |h_i|^2 (\mathbf{h}^H \mathbf{h} \sigma_s^4 + 2\Delta_d) \tag{18}$$

If the column index l is chosen to satisfy condition (i), it is shown that $|h_l|^2$ is the maximum over $|h_i|^2$ for $i \in \{1, \cdots, M\}$ in the first part of this proof. From this, it can be readily verified from (17) and (18) that the numerator of $d^2(\mathbf{h}, \bar{\hat{\mathbf{h}}}(l))$ is minimum and the denominator of it is maximum under the condition (ii). This proves that $d(\mathbf{h}, \bar{\hat{\mathbf{h}}}(l))^2$ is minimum. □

This proposition illustrates an important characteristic of the proposed estimation scheme to provide a way to select the optimal estimate over M candidates. Condition (i) is equivalent to finding a channel with the maximum power. Condition (ii) can only be violated when the average error in the noise variance estimation is greater than M times the signal power, which is an unrealistic case in practice.

Finally, the proposed estimation scheme is summarized as follows:

Step 1: Given a set of received signal vectors, $\{\mathbf{x}[k] \in \mathbb{C}^M : k = 1, \cdots, N\}$, find the index, l, satisfying $l = \arg\max_{1 \le i \le M} \sum_{k=1}^{N} x_i[k] x_i^*[k]$.
Step 2: Calculate the column of covariance matrix by $\hat{\mathbf{r}}_l = \frac{1}{N} \sum_{k=1}^{N} \mathbf{x}[k] x_l[k]^*$.
Step 3: Calculate the variance of the noise, $\hat{\sigma}_n^2$, using one of methods given in Sect. 4.2.
Step 4: The estimate is given by $\hat{\mathbf{h}}_{prop} = \hat{\mathbf{r}}_l - \hat{\sigma}_n^2 \mathbf{u}_l$.

5. Simulations and Discussion

To evaluate the performance of the proposed scheme, we investigate four cases: (i) the conventional subspace-based method, (ii) the proposed method *with* column selection - using the projection and (iii) the EVD for noise variance estimation, and (iv) the proposed method *without* column selection. For the proposed method without selection, we choose the EVD method for the noise variance estimation which is derived to be less complex than the projection method. The computations required for each estimation method are listed in Table 1. As we expect, the proposed schemes are much less complicated than the conventional scheme. Among the proposed schemes, the method without selection (Case(iv)) requires the least computations since the calculation for the average power is unnecessary.

Numerical results are displayed in Fig. 1. We fix the number of antenna elements M to 8 and plot the floating point operations (FLOPS) versus the number of samples N. Note that for a give N, the number of FLOPS is smaller for

Table 1 Complexity for each estimation method.

Methods	Complexity
Conventional Subspace	$O(M^2 N + M^3)$
Proposed - Projection with Selection	$O((5M - 3)N)$
Proposed - EVD with Selection	$O(2(M + 2)N + 8)$
Proposed - EVD without Selection	$O((M + 4)N + 8)$

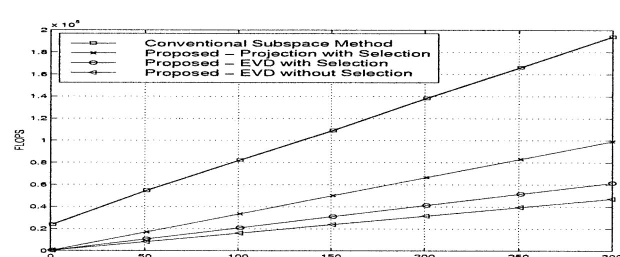

Fig. 1 Floating point operations.

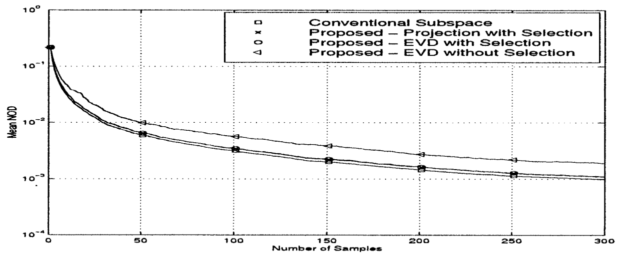

Fig. 2 Mean ND with SNR = 5 [dB].

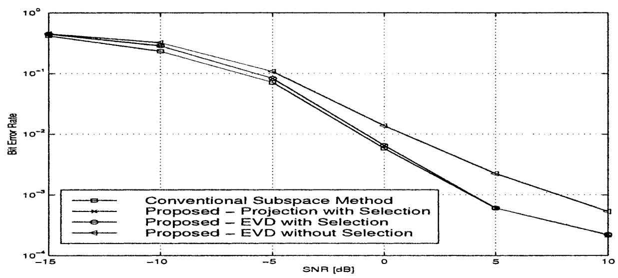

Fig. 3 Bit error rate vs. SNR with N=150.

proposed methods (Case (ii),(iii), and (iv)) compared to the conventional method (Case (i)). Furthermore, the rate of increase of FLOPS vs. N is also smaller. These two observations are consistent with complexity estimates in Table 1.

The estimation performance of each method is investigated for an SNR = 5 [dB]. The SNR is defined to each branch. As a performance measure, the mean of ND defined in (13) is used. In Fig. 2, the mean ND is displayed versus the number of data samples. The performance of the proposed estimation methods with column selection exhibits slightly larger error (i.e., larger mean ND) than those of the conventional subspace-based method, while those of the proposed method *without* selection yields relatively larger errors. Among the proposed methods, the two methods *with* selection have smaller error than those of the method *without* selection. This implies that the selection methods results in a decrease of error in the proposed methods.

The bit error rate (BER) versus SNR is presented in Fig. 3 with a fixed number of data samples, $N = 150$. The quadrature phase shift keying (QPSK) modulation is utilized and the uncorrelated flat-fading channel is assumed. The conventional method performs better than the proposed methods especially for low SNR. However, as the SNR increases, the BER performance of the proposed methods with selection approach that of the conventional method. Hence, the proposed methods with selection can achieve the BER performance of the conventional method with significantly less computations under reasonably high SNR.

6. Conclusion

A reduced-complexity blind estimation scheme for the vector channel of systems with receive diversity is presented. The proposed scheme is based on an algebraic solution of the statistical system model equation. The reduction in computations results from the use of one column of the covariance matrix while the conventional subspace-based method requires the calculation of the whole covariance matrix. An optimal selection scheme is also proposed to improve the estimation performance in the proposed scheme. Computer simulations show that the proposed scheme yields an estimation performance close to that of the conventional subspace method with significantly less computations.

The proposed estimation scheme can be applied for systems with an antenna array combined with any multiplexing method. However, orthogonal frequency division multiplexing (OFDM) will be especially suitable for the application due to the narrow-band channel assumption, required for the subspace-based estimation schemes considered in this paper.

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