Formation Flying of small Satellites Using Coulomb Forces

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Abstract: The formation flying of satellites has been identified as an enabling technology for many future space missions. The application of conventional thrusters for formation flying usually results in high cost, limited life-time, and a large weight penalty. Various methods including the use of coulomb forces have been considered as an alternative to the conventional thrusters. In the present investigation, we investigate the feasibility of achieving the desired formation using Coulomb forces. This method has several advantages including low cost, light weight and no contamination. A simple controller based on the relative position and velocity errors between the leader and follower satellites is developed. The proposed controller is applied to circular formations considering the effects of disturbance forces, disturbances in initial formation conditions as well as system nonlinearity. Results of the numerical simulation state that the proposed controller is successful in establishing circular formations of leader and follower satellites, for a formation size below 100 m.

Keywords: Formation flying, Coulomb forces, Lyapunov stability.

1. Introduction

The formation flying of satellites has been identified as an enabling technology for many future space missions. The application of Coulomb forces for formation flying of satellites is examined in the present paper. By accumulating or discharging electrons/ions, a positive or negative charge on the satellite is developed. With the variation of this charge, the Coulomb force between the two satellites can be varied and thereby, the relative distance between them can be controlled.[1]

This paper explores a simple control law based on the relative position and velocity errors of the two Coulomb satellites. The stability of the control law is discussed using Lyapunov theorem.

2. System Equations of Motion

The system consists of a leader satellite and a follower satellite (Fig. 1).

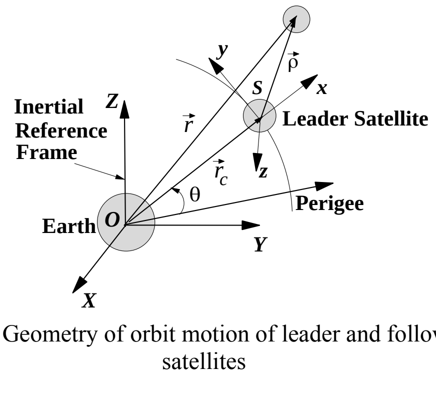


Fig. 1 Geometry of orbit motion of leader and follower satellites

Assume that the leader and follower satellites are point masses to derive the governing equations of motion. The orbital motion of the leader satellite is defined by a radial distance r_c from the center of the Earth and a true anomaly θ.

The orbital motion of the follower satellite is also defined by a radial distance r from the center of the Earth and a true anomaly. Then, the orbital motions of leader and follower satellite are described with respect to the Earth centered coordinate as

$$\ddot{\vec{r}}_c = -\frac{\mu}{r_c^3}\vec{r}_c$$
$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} \qquad (1)$$

The motion of the follower satellite is described with respect to the motion of the leader satellite by a relative frame S-xyz fixed at the center of the leader satellite. The x-axis points along the local vertical, the z-axis is taken along normal to the orbital plane, and the y-axis represents the third axis of this right handed frame taken. The relative equations of motion of the follower satellite with respect to the leader satellite are as follows:

$$\ddot{x} - \dot{\theta}^2 x - 2\dot{\theta}\dot{y} - \ddot{\theta}y = -\frac{\mu}{r^3}(r_c + x) + \frac{\mu}{r_c^2}$$
$$\ddot{y} - \dot{\theta}^2 y + 2\dot{\theta}\dot{x} + \ddot{\theta}x = -\frac{\mu}{r^3}y \qquad (2)$$
$$\ddot{z} = -\frac{\mu}{r^3}z$$

Referring to Eq. (2), if the orbit of the leader satellite is assumed to be circular, $\ddot{\theta}$ is zero, $\dot{\theta}$ is a constant, and the relative equations of motion of the follower satellite with respect to the Leader satellite can be written as

$$\ddot{x} - \dot{\theta}^2 x - 2\dot{\theta}\dot{y} = -\frac{\mu}{r^3}(r_c + x) + \frac{\mu}{r_c^2}$$
$$\ddot{y} - \dot{\theta}^2 y + 2\dot{\theta}\dot{x} = -\frac{\mu}{r^3}y \qquad (3)$$
$$\ddot{z} = -\frac{\mu}{r^3}z$$

Note that the above equations are nonlinear equations. Due to the gravitational force terms we cannot find a closed-form solution of differential equations. However, we can obtain a solution using numerical simulations applying six initial conditions. With suitable approximations, we can derive linear equations of relative motion, which is called the Hill's Equations [6]. The initial conditions for Hill's equations for a circular formation are given by

$$\begin{aligned} y_0 &= \frac{2\dot{x}_0}{\dot{\theta}} \\ \frac{\dot{y}_0}{\dot{\theta}} &= -2x_0 \\ z_0 &= \pm\sqrt{3}x_0 \\ \dot{z}_0 &= \pm\sqrt{3}\dot{x}_0 \end{aligned} \tag{4}$$

3. Coulomb Force and Controller Design

Let E_1 be the electrostatic field experienced by a satellite, say satellite 1 and Q_2 is the charge of satellite 2. Then we can write as [2]

$$E_1 = K_c Q_2 \frac{1}{|\rho|^2}\exp\left(-\frac{|\rho|}{\lambda_d}\right) \tag{5}$$

where ρ is the relative distance between two satellites and K_c is the Coulomb's constant, $8.99\times10^9\ Nm^2/C^2$ and λ_d is the Debye length which varies from 140 to 1400 m.

If the satellite 1 has a charge Q_1, the electrostatic force F_1 applied to the satellite 1 is given by

$$\begin{aligned} F_1 &= Q_1 E_1 \\ a_1 &= \frac{1}{m_1}F_1 \end{aligned} \tag{6}$$

where m_1 is the mass of the satellite 1 and a_1 is its acceleration. The components of the Coulomb force along x, y, and z directions can be written as

$$f_x = F\frac{x}{\rho} \quad f_y = F\frac{y}{\rho} \quad f_z = F\frac{z}{\rho} \tag{7}$$

We establish a control law with relative distance errors (e) and velocity errors ($\dot{e}$) defined as

$$\begin{aligned} e &= \rho - \rho_d \\ \dot{e} &= \dot{\rho} - \dot{\rho}_d \\ \hat{Q} &= \mu e + \nu\dot{e} \end{aligned} \tag{8}$$

where e is an error of the relative distance and the derivative of e is an error of the relative velocity. μ and ν are control gains and the subscript '*d*' represents the desired value. We consider $\dot{\rho}_d$ =0 and assume that each satellite can be charged to Q_{max}=10×10^{-5}. For simplification, a new notation is defined as

$$\hat{Q} = \frac{Q_1\cdot Q_2}{(10\times10^{-10})} \tag{9}$$

where Q_1 and Q_2 are charges accumulated on the leader and follower satellites. $\hat{Q}$ is a normalize value of the $Q_1\times Q_2$.

Finally, we can obtain Coulomb control force using equations (5)-(8) and (9).

$$\begin{aligned} F &= K_c Q_1 Q_2 \frac{1}{|\rho|^2} e^{-\frac{|\rho|}{\lambda_d}} = K_c(\hat{Q}\times10^{-10})\frac{1}{|\rho|^2}\exp\left(-\frac{|\rho|}{\lambda_d}\right) \\ &= K_c\cdot[\mu e + \nu\dot{e}]\times10^{-10}\frac{1}{|\rho|^2}\exp\left(-\frac{|\rho|}{\lambda_d}\right) \\ &\approx k_1 e + k_2\dot{e} \end{aligned} \tag{10}$$

where $k_1 = K_c\mu(10^{-10})(\frac{1}{|\rho_d|^2}\exp(-\frac{|\rho_d|}{\lambda_d}))$ and

$k_2 = K_c\nu(10^{-10})(\frac{1}{|\rho_d|^2}\exp(-\frac{|\rho_d|}{\lambda_d}))$

Substituting these forces into equation (2), the both forces applied to satellite are same magnitude with opposite direction. Note that force sign is positive when the force applies to outer direction.

4. Lyapunov Stability

To prove the stability of the closed system applied the proposed control law, Let us consider a Lyapunov candidate function as

$$V = \begin{bmatrix} e & \dot{e} \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = 2e^2 + 2e\dot{e} + 2\dot{e}^2 = E^T P E \tag{11}$$

where $E = \begin{bmatrix} e & \dot{e} \end{bmatrix}^T$ is the state error vector and P is a positive definite matrix. It is easily observed the stability of the closed-system by differentiating the Lyapunov candidate such that the time derivative of the function is given by

$$\begin{aligned} \dot{V} &= 4e\dot{e} + 2\dot{e}^2 + 2e\ddot{e} + 4\dot{e}\ddot{e} \\ &= 4e\dot{e} + 2\dot{e}^2 + 2e\ddot{\rho} + 4\dot{e}\ddot{\rho} \\ &= 4e\dot{e} + 2\dot{e}^2 + 2e(-\frac{\mu}{r^2} + -\frac{\mu}{r_c^2} - 2F) + 4\dot{e}(-\frac{\mu}{r^2} + -\frac{\mu}{r_c^2} - 2F) \\ &\simeq 4e\dot{e} + 2\dot{e}^2 + 2e(-2F) + 4\dot{e}(-2F) \\ &\simeq e\dot{e}(4 - 4k_2 - 8k_1) - 4k_1 e^2 + \dot{e}^2(2 - 8k_2) \end{aligned} \tag{12}$$

Note that that r_c is much greater than ρ. Thus, it is assumed that the following terms can be negligible:

$$-\frac{\mu}{(|\vec{r}_c + \vec{\rho}|)^2} + -\frac{\mu}{|\vec{r}_c|^2} \simeq 0 \tag{13}$$

For achieving stable response, the gains k_1 and k_2 are taken such that $\dot{V}$ is always negative. Thus, the conditions of stability are obtained as

$$\begin{aligned} &1)\ k_2 = 1 - 2k_1 \\ &2)\ k_1 > 0 \\ &3)\ k_2 > 0.25 \end{aligned} \tag{14}$$

By theorem of Lyapunov stability [3], we conclude that the closed-system is asymptotically stable at $e = \dot{e} = 0$.

5. Numerical Study

In this paper, a numerical simulation is used to illustrate the performance of the proposed control law. A leader satellite is in a circular orbit of radius 42241 km. Initially the follower satellite is positioned at a radial distance of 70 m from the leader satellite, i.e., x_0 = 70 m. Using equation (2), this case is simulated with no control force. As per the system response shown in Fig. 2, the relative distance increases until the true anomaly difference is 180 degree, because the leader satellite revolves the Earth faster than the follower satellite. If we do not control the position of two satellites, the relative distance could diverse with some velocity.

Referring to equation (1), even if we apply the corrected initial conditions, the relative distance between the two grows satellites is varies not only perturbed force but also nonlinearity and eccentricity perturbation [4].

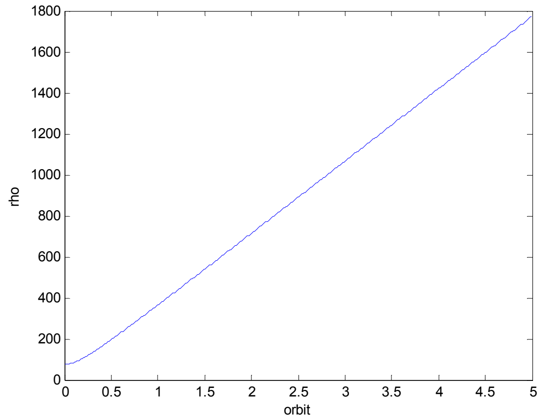


Fig. 2 Relative distance without control input.

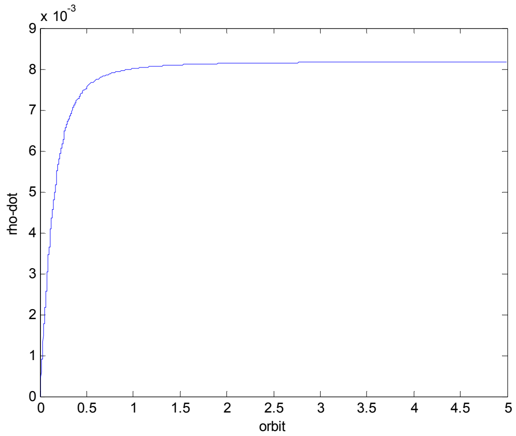


Fig. 3 Relative velocity without control input.

Next applying the Coulomb control force into equations (1) and (2), we can find the system equations of motion to be

$$\begin{aligned} &\ddot{X}_c + \frac{\mu}{r_c^3} X_c = F_x \qquad \ddot{X} + \frac{\mu}{r^3} X = -F_x \\ &\ddot{Y}_c + \frac{\mu}{r_c^3} Y_c = F_y \qquad \ddot{Y} + \frac{\mu}{r^3} Y = -F_y \\ &\ddot{Z}_c + \frac{\mu}{r_c^3} Z_c = F_z \qquad \ddot{Z} + \frac{\mu}{r^3} Z = -F_z \end{aligned} \tag{15}$$

where (X_c, Y_c, Z_c) and (X, Y, Z) are position vectors of leader satellite and follower satellite with respect to Earth centered coordinate, respectively. F_x, F_y and F_z are magnitudes of component of the coulomb force, respectively.

$$\begin{aligned} &\ddot{x} - \dot{\theta}^2 x - 2\dot{\theta}\dot{y} = -\frac{\mu}{r^3}(r_c + x) + \frac{\mu}{r_c^2} + f_x \\ &\ddot{y} - \dot{\theta}^2 y + 2\dot{\theta}\dot{x} = -\frac{\mu}{r^3} y + f_y \\ &\ddot{z} = -\frac{\mu}{r^3} z + f_z \end{aligned} \tag{16}$$

We at first analyze the planar motion of the leader and follower satellites, i.e., along x and y directions, z=0, derivative of z =0. It is assumed that the two satellites have a mass of 20 kg and the Debye length is 1000 m. The gain k_1 and k_2 are taken as 0.75 and 0.125, respectively. Fig. 4 shows the system response for the desired relative distance of 70 m. Comparing Fig. 2 with Fig. 4, we find that the proposed control law using Coulomb force results in the relative distance of 70m and the relative velocity converging to null.

Note that we consider the control gains as per equation (14). However, we find that some values of the control gains that do not satisfy the stability conditions (14) also result in stable system response. For example, taking control gains μ = 500 and ν = 10, the system response converges faster and the error is small. Table 1 shows appropriate results of response with varying desire distance from 10 m to hundreds of meters. In Table 1 the mark 'o' represents that the response follows the desire distance successfully. The symbol 'x' represents the response does not follow and finally diverge since the magnitude of Coulomb force diminish with ($1/r^2$) of relative distance. This is the clue the other reference content says "Coulomb force is effective for tight formation of 10~100 m" [5].

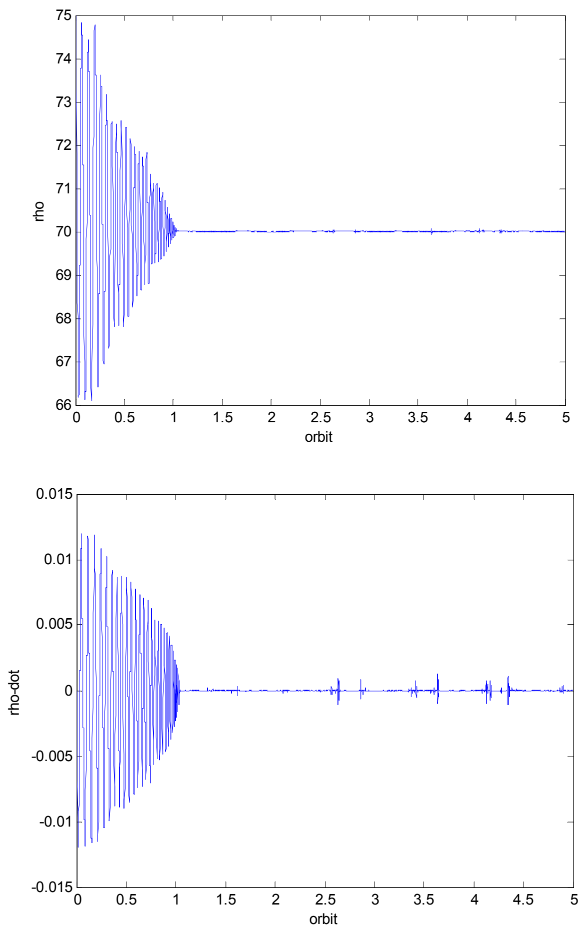


Fig. 4 Relative distance and velocity using control force.(($Q_{1\max} Q_{2\max}$ = 10^{-10})

Table 1. Effect of $Q_{\max}$ and ρ on the system response.

$Q_{1\max}Q_{2\max}$ \ ρ (m)	10	40	70	100	200	500
10^{-12}	o	x	x	x	x	x
10^{-11}	o	o	x	x	x	x
10^{-10}	o	o	o	o	x	x
10^{-9}	o	o	o	o	o	x

Next three-dimensional relative motion is studied using equation (16). By applying the corrected initial conditions which make the orbit circle to Hill's equations, then the desire distance is 70 meters, we examine the relative distance. In this simulation, the leader satellite is also at 42241km. we can see that the relative distance is controlled within the error bound of 0.0015m. This is very reasonable value for the formation flying of satellites. The applied initial conditions are as follows:

$$x(0)=0 \quad y(0)=70 \quad z(0)=0$$

$$\dot{x}(0)=(70\dot{\theta}/2) \quad \dot{y}(0)=0 \quad \dot{z}(0)=\left(70\sqrt{3}/2\right)\dot{\theta} \tag{17}$$

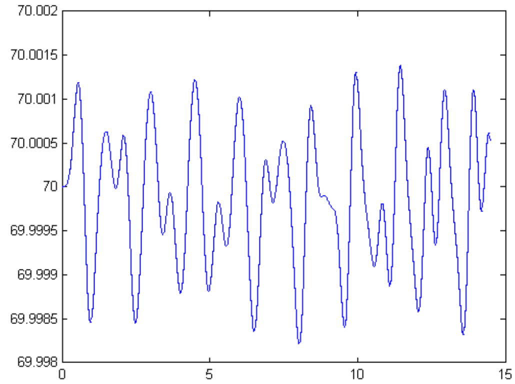
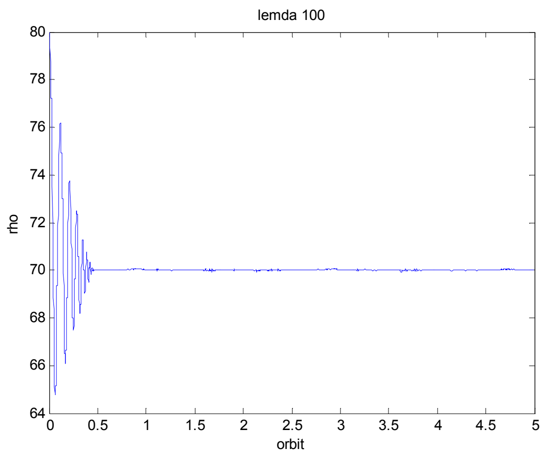
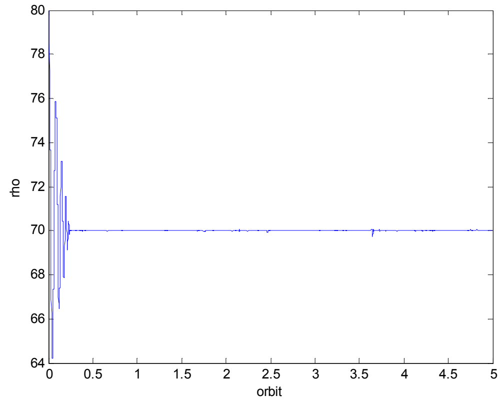


Fig. 5 Control relative distance using Coulomb force. ($Q_{1\max}Q_{2\max}$ = 10^{-9})

Next we examine the effect of Debye length illustrated in Fig. 6. In the simulation, if we take Debye length be equally to 100 m, the system response takes about 0.5 orbit to reach steady-state, on the other hand when we assume Debye length of 1000 m, the steady-state is reached in about 0.25 orbit. The smaller the value of Debye length is, the longer it takes the system response to reach a steady-state which is expected as the decrease in Debye length results in the decrease of Coulomb force.



(a)100m



(b)1000m

Fig. 6 Various Debye length cases ($Q_{1\max}Q_{2\max}$ = 10^{-10})

6. Conclusions

In this paper, a satellite formation flying using Coulomb forces has been examined. The proposed simple control law based on relative position and velocity errors has been developed and its stability has been examined through Lyapunov theorem. Numerical simulations show that this control law results in bounded relative motion between the two satellites. The effect of Debye length on the desired formation was also examined. In future work, the case of projected circular formation will be investigated.

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