Analysis of Perilune Altitude for Optimal Lunar Landing Trajectory

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Abstract: Generally, the lunar landing stage is divided into two distinct phases: de-orbit and descent, and this descent phase is usually divided into two sub-phases: braking and approach. And the optimization problem of minimal energy is usually focused on descent phases. In these approaches, the energy of de-orbit burning is not considered. Therefore, as low as possible perilune altitude can be chosen to save the fuel for the descent phase. Usually, the perilune altitude is chosen between 10 to 15 km because of the mountainous lunar terrain and possible guidance errors. However, it is required more de-orbit burning energy for the lower perilune altitude. Therefore, in this paper, the perilune altitude of the intermediate orbit is also considered with optimal thrust programming for minimal energy. Furthermore, these perilune altitude and optimal thrust programming can be expressed by a function of the radius of a parking orbit by using continuation method and co-state estimator.

Keywords: Lunar landing, Optimal trajectory, Perilune altitude, Continuation method

1. INTRODUCTION

Many kinds of lunar exploration missions are accomplished in the 60s and 70s in the last century by the United States and Soviet Union. However, these lunar exploration missions are discontinued during several decades. Recently, these lunar exploration missions are restarted by various space agencies with the revival of interest in the scientific exploration of the Moon. Especially, the discovery of the tritium of Moon is very interested. So, the lunar landing missions are also being considered.

Generally, the lunar landing stage is divided into two distinct phases: de-orbit and descent, and this descent phase is usually divided into two sub-phases: braking and approach. And the optimization problem of minimal energy is usually focused on descent phases. To find this optimal solution, the 2 dimensional approach is already studied by Ramanan[2] and Liu[3]. And same approach is studied by Shan[4] under the variable thrust level. Using these optimal lunar landing trajectory, Liu also studied the landing guidance control.[5]

However, in these approaches, the energy of de-orbit burning is not considered. Therefore, as low as possible perilune altitude can be chosen to save the fuel for the descent phase. Usually, the perilune altitude is chosen between 10 to 15 km because of the mountainous lunar terrain and possible guidance errors. However, it is required more de-orbit burning energy for the lower perilune altitude. In the Fig. 1, the cost values history is plotted as changing the perilune altitude conditions. Therefore, in this paper, the perilune altitude of the intermediate orbit is also considered with optimal thrust programming for minimal energy. Furthermore, these perilune altitude and optimal thrust programming can be expressed by a function of the radius of a parking orbit by using continuation method and co-state estimator.

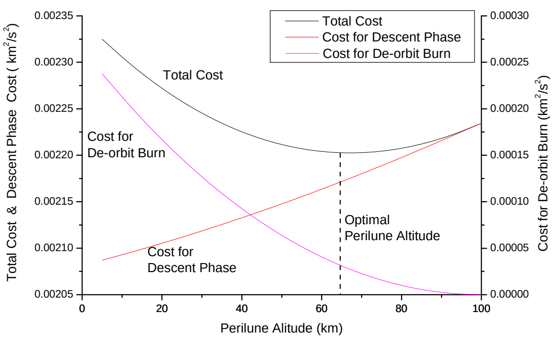

Fig. 1 Cost function values history for the perilune altitude

2. SOLUTION PROCESS

2.1 Problem description

In this paper, the minimum energy trajectory for lunar landing will be discussed. And the energy for the de-orbit phase can be describe as function of r_0 (perilune radius) as follows.

$$\Delta V = \sqrt{\mu\left(\frac{2}{r_p} - \frac{2}{r_p + r_0}\right)} - \sqrt{\frac{\mu}{r_p}} \tag{1}$$

where, r_p represents the radius of lunar parking orbit and is given.

Therefore, the cost function for minimum energy landing can be written as follows.

$$J = \underbrace{\frac{1}{2}\Delta V^2}_{\text{De-orbit burn Phase}} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f}\left(\frac{T}{m}\right)^2 dt}_{\text{Descent Phase}} \tag{2}$$

where, t_0 represents the initiated time of descent phase.

2.2 Assumptions

In this paper, following assumptions are used.

i) The lunar gravity field is uniform and lunar is entirely sphere body. And, the lunar rotates on its own axis with constant angular velocity.

ii) The lunar parking orbit (circular orbit) and the lunar equator are placed in same plane, and each rotation direction is same.

iii) The orbit transfer strategy at the de-orbit phase is Hofmann method.

iv) The thrust level of the lunar lander is constant for descent phase.

2.3 Governing Equations

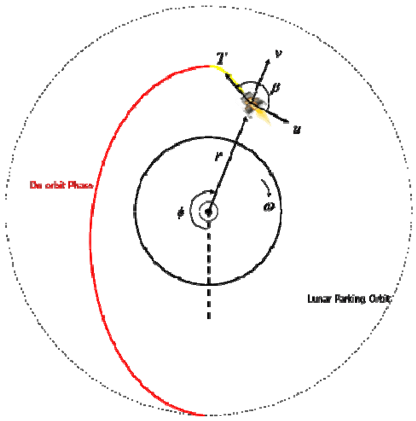

Fig. 2 Polar coordinate of lunar landing

The planar motion of the lunar lander is described in Fig. 1. In this figure, r and θ represent radial distance and position angle, u and v represent transverse and radial velocity, ω represents the lunar rotation velocity, T represents a thrust vector of lunar lander and β represents a thrust vector angle which is control command, respectively. Using these parameters, the governing equations of motion can be obtained as follows.

$$\dot{r} = v \tag{3}$$

$$\dot{\phi} = \frac{u}{r} \tag{4}$$

$$\dot{u} = -\frac{uv}{r} + \frac{T}{m}\cos\beta \tag{5}$$

$$\dot{v} = \frac{u^2}{r} - \frac{\mu}{r^2} + \frac{T}{m}\sin\beta \tag{6}$$

$$\dot{m} = -\frac{T}{I_{sp}g} = \text{Const} \tag{7}$$

where, I_{sp} and g represent a specific impulse and gravitational acceleration on the Earth, respectively. These parameters are constant values.

2.4 Optimal control Problem

In order to find the control variable profile for the minimum energy lunar landing trajectory, the calculus of variation will be used. Therefore, the Hamiltonian is formed as follows from cost function and governing equations.

$$H = \frac{1}{2}\left(\frac{T}{m}\right)^2 + \lambda_r \dot{r} + \lambda_\phi \dot{\phi} + \lambda_u \dot{u} + \lambda_v \dot{v} + \lambda_m \dot{m} \tag{8}$$

where, $\lambda = \begin{bmatrix}\lambda_r & \lambda_\phi & \lambda_u & \lambda_v & \lambda_m\end{bmatrix}^T$ are co-state variables.

And the time derivative of these costate variables can be written as follow using optimal control theory. ($\dot{\lambda} = -\partial H / \partial x$)

$$\dot{\lambda}_r = \lambda_\phi \frac{u}{r^2} - \lambda_u \frac{uv}{r^2} + \lambda_v \left(\frac{u^2}{r^2} - 2\frac{\mu}{r^3}\right) \tag{9}$$

$$\dot{\lambda}_\phi = 0 \tag{10}$$

$$\dot{\lambda}_u = -\frac{\lambda_\phi}{r} + \lambda_u \frac{v}{r} - \lambda_v \frac{2u}{r} \tag{11}$$

$$\dot{\lambda}_v = -\lambda_r + \lambda_u \frac{u}{r} \tag{12}$$

$$\dot{\lambda}_m = \frac{T}{m^2}\left(\frac{T}{m} + \lambda_u \cos\beta + \lambda_v \sin\beta\right) \tag{13}$$

The optimal control variable profile can be also obtained by optimal control theory. Therefore, the following two equations are satisfied.

$$\frac{\partial H}{\partial \beta} = \frac{T}{m}\left(-\lambda_u \sin\beta + \lambda_v \cos\beta\right) = 0 \tag{14}$$

$$\frac{\partial^2 H}{\partial \beta^2} = \frac{T}{m}\left(-\lambda_u \cos\beta - \lambda_v \sin\beta\right) \geq 0 \tag{15}$$

Therefore, the optimal control variable profile is obtained as follows.

$$\beta = \tan^{-1}\left(\frac{-\lambda_v}{-\lambda_u}\right) \tag{16}$$

2.5 Two-Point Boundary Value Problem (TPBVP)

For the lunar landing mission the following terminal constraints have to be satisfied.

$$r_f = r_{moon} \quad , \quad u_f = r_{moon}\omega \quad , \quad v_f = 0 \tag{17}$$

where, r_{moon} is the radius of moon, and the subscript f means the values at the final time. For the inertia frame, horizontal velocity is not zero at the lunar surface.

Besides these final constraints, there are some initial constraints because the initial state is not fixed. As previously mentioned, the Hofmann transfer strategy is used at the de-orbit burn phase, and the start point of optimal control problem is the perilune of this orbit. Therefore, these initial states and constraints can be written as follows.

$$\phi_0 = 180^\circ \;,\; u_0 = \sqrt{\mu\left(\frac{2}{r_0} - \frac{2}{r_0 + r_p}\right)} \;,\; v_0 = 0 \;,\; m_0 = M \tag{18}$$

where, M is the total mass of the lunar lander, and the subscript 0 means the values at the initial time.

Therefore, the augmented constraints function can be written as follows.

$$G = \frac{1}{2}\Delta V^2 + \nu^T \psi + \xi^T \theta \tag{19}$$

where, ν and ξ are Lagrange multipliers, and θ and ψ are the initial and final state constraints, respectively, and these constraints can be expressed as follows.

$$\psi = \begin{bmatrix} r_f - r_{moon} & u_f - r_{moon}\omega & v_f \end{bmatrix} \tag{20}$$

$$\theta = \left[u_0 - \sqrt{\mu\left(\frac{2}{r_0} - \frac{2}{r_0 + r_p}\right)} \right] \tag{21}$$

Usually, the augmented constraints function involves the final state in the optimal control problem. However, we want to also find the perilune altitude to minimize total lunar landing energy. So, the cost function dependent the initial state, and the augmented constraints function also dependent the initial state. Therefore, the following boundary conditions can be derived from the optimal control theory.[3]

$$H_f = -G_{t_f} \tag{22}$$

$$\lambda_f = G_{x_f}^T \tag{23}$$

$$\lambda_0 = -G_{x_0}^T \tag{24}$$

From these equations, the boundary conditions for co-state variables can be written as follows.

$$H_f = 0 \tag{25}$$

$$\lambda_\phi(tf) = \lambda_m(tf) = 0 \tag{26}$$

$$\lambda_\phi(0) = 0 \ , \ \lambda_r(0) = -\lambda_u(0)\frac{\partial u_0}{\partial r_0} - \Delta V \frac{\partial \Delta V}{\partial r_0} \tag{27}$$

Therefore, the optimal energy lunar landing problem can be solved by finding the initial state and co-state values to satisfy these boundary conditions.

2.6 Solution of TPBVP – Shooting Method

In the previous section, the optimal control problem can be solved by appropriate values for the initial state and co-state variables. This approach is usually solved by using some parameter optimization methods. There are many kind of the parameter optimization techniques like SQP, Evolutionary algorithm, Genetic algorithm, CEALM, PSO, etc.. Among them, the Shooting method is used in this paper, because the simplicity for programming and fast convergence. Actually, the results for the co-state initial values are very small values. So, choosing the boundary range for these parameters is very difficult for stochastic processes.

For the shooting method, the constraints matrix(h) is the function of the initial and final state variables and final time. So, the differential of this constraints matrix can be written as follows.

$$dh = \dot{h}_f dt_f + h_{z_0}\delta z_0 + h_{z_f}\delta z_f + H.O.T \tag{28}$$

where, the $z = \begin{bmatrix} x^T & \lambda^T \end{bmatrix}^T$ represents augmented state matrix and t_f represents final time. And the subscript 0 and f mean the values at the initial and final time. For simplicity the high order term is neglected in this paper. This equitation can be rewritten for only initial augmented state matrix and final time by using the state transition matrix Φ .

$$dh = \begin{bmatrix} \dot{h}_f & h_{z_0} + h_{z_f}\Phi_f \end{bmatrix} \begin{bmatrix} dt_f \\ \delta z_0 \end{bmatrix} \tag{29}$$

To reduce these constraints, we want to satisfy the following equation.

$$dh = -\alpha h \quad , \quad 0 < \alpha \le 1 \tag{29}$$

Table. 1 Simulation results for the optimal lunar landing

Optimal Perilune altitude (km)	Optimal Terminal time for the descent phase (sec)	Initial Co-state variables for the descent phase
61.0147	1019.7998	$\lambda_r(0) =$ 0.00238096568550 $\lambda_\phi(0) =$ 0 $\lambda_u(0) =$ 1.09106631082654 $\lambda_v(0) =$ 0.06977679327318 $\lambda_m(0) = -3.64929196248789$

Therefore, we can obtain the update law of the augmented state variables at the initial time as follows.

$$\begin{bmatrix} dt_f \\ \delta z_0 \end{bmatrix} = -\alpha \begin{bmatrix} \dot{h}_f & h_{z_0} + h_{z_f}\Phi_f \end{bmatrix}^{-1} h \tag{30}$$

Using this update law, the optimal solution to satisfy the TPVBP can be obtained by iterative process.

2.7 Continuation Method

In the previous section, we can find the solution for the optimal lunar landing trajectory. However, this optimal solution can be changed as changing the lunar parking orbit altitude. Therefore, we can obtain the optimal solutions as function of the lunar parking orbit altitude. For this process, the initial condition estimation is mostly important because we want to use the shooting method. However, these dynamics is not very fast system. So, the state and co-state variables are not suddenly changed. Therefore, the optimal solution is very smooth function for the lunar parking orbit altitude. For this reason, we can easily find the solutions by choosing the initial state and co-state variables from the solutions which is already obtained.

3. RESULTS AND DISCUSSION

For the numerical simulation, we use the same conditions from Ref.[2]. So, the initial parking orbit altitude is 100km from the lunar surface and the initial mass of the lunar lander at the starting point of the descent phase is 300kg. And the constant thrust level is 440N, the specific impulse of the thruster is 310sec.

The simulation results are described in the Table 1. In this simulation, the perilune altitude is 61.0147km. This perilune altitude is very higher than 15 km which is used in the previous researches. So, the terminal time is increase and the final landing mass is decrease. This means the cost values for the descent phase is increase. However, the cost values for the Hofmann transfer is more decrease than this increment. Therefore, the total cost value is decrease. And the initial co-state values for the descent phase are described in the same table. In this result, the initial co-state values are very small, especially, co-state for the radial distance. So, choosing the boundary range for these parameters is very difficult for stochastic processes.

The descent phase profiles are shown in the Fig. 3 to Fig. 6. In the trajectory figure, Fig. 3, there is not altitude increment which is obtained in the Ref.[2]. In the Ref.[2], there is altitude increment to maximize horizontal braking that minimized the gravity loss. However, in this simulation result,

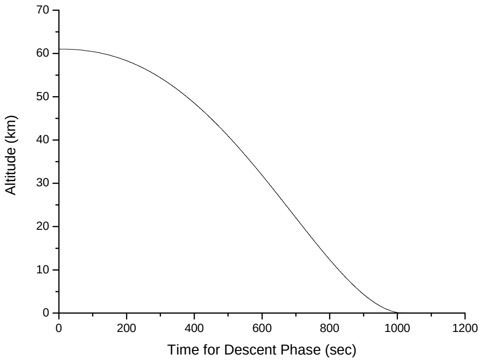

Fig. 3 Trajectory of the lunar lander

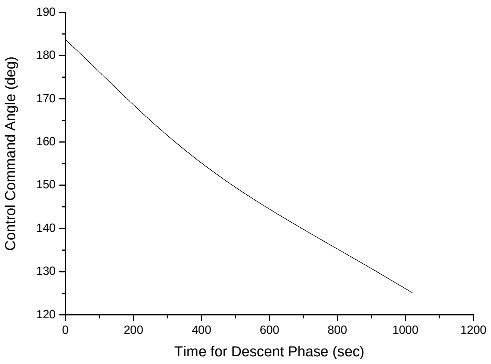

Fig. 4 Control command profile for the optimal lunar landing

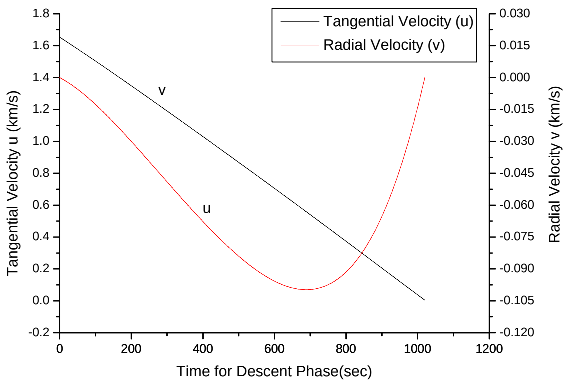

Fig. 5 Velocity history for the optimal lunar landing

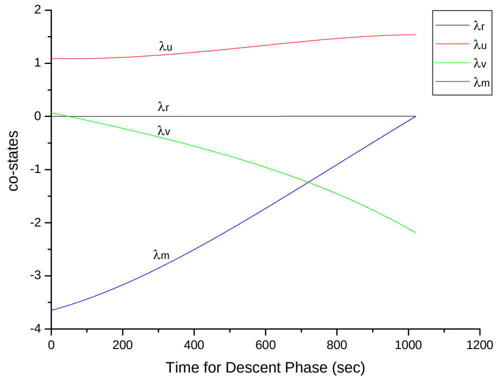

Fig. 6 Co-state profile for the optimal lunar landing

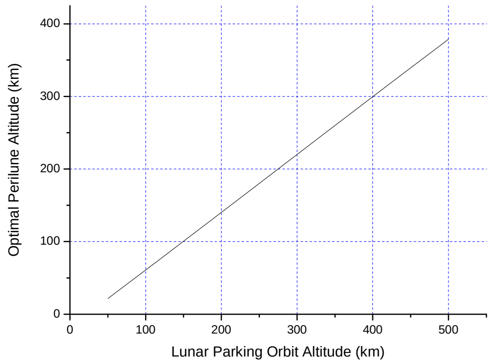

Fig. 7 Optimal perilune altitude for the various lunar parking orbit altitude

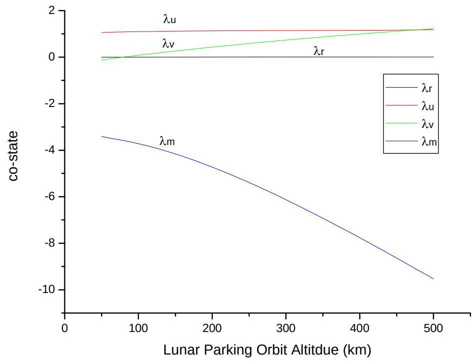

Fig. 8 Initial co-state values at the descent phase for the various lunar parking orbit altitude

this altitude increment does not exist. Therefore, the cost value is decrease. In the Fig. 4, the control input angle is displayed. In this figure, the control input is very smooth function, so, this control input can be apply to the lunar lander. Finally, the radial and tangential velocity history during the descent phase with this control input profile are displayed in the Fig. 5, and the co-state variables history also be plotted in the Fig. 6. In the Fig. 5, the two kind of velocity have some different velocity range. So, the range is divided in the left and right side. Therefore, the magnitude of the radial velocity history, red line, is indicated in the right side.

In the previous simulation, the optimal solution is easily obtained for the 100km lunar parking orbit. However, this parking orbit altitude can be changed as changing the mission requirements. So, the optimal perilune altitude must be changed as changing the lunar parking orbit altitude. In this reason, it is useful that the optimal perilune altitude is described as a function of the lunar parking orbit. So, same simulations are done with various lunar parking orbit altitude conditions. The simulation results are plotted in Fig. 7 and Fig. 8. In the Fig. 7, the optimal perilune altitude is similar to linear function of the lunar parking orbits. So, the curve fitting for the first order is accomplished and the result is obtained as follows.

$$r(0) = 0.79547498416898 r_p - 18.69375349429545 \quad (31)$$

In the Fig. 8, the initial co-state values are plotted at the descent phase as changing the lunar parking orbit altitude.

These results are dependent on the simulation conditions as thrust level, specific impulse and the lunar lander initial mass.

However, the optimal perilune altitude can always be obtained using this process.

4. CONCLUSION

In this paper, we can find the optimal lunar landing trajectory. For this optimal control problem, the perilune altitude is important to reduce the total energy for the lunar landing. Therefore, the initial state is free in this optimal control problem, and we can solve this problem using optimal control theory. For the simulation, we use shooting method. Using this method and the continuation method, we can easily find the optimal solution under various parking orbit conditions, and we can describe this result as a function of parking orbit altitude. However, these results are dependent on the thruster on the lunar lander.

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